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FISCAL SUSTAINABILITY IN JAPAN

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Fiscal sustainability in Japan*

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Abstract

This paper investigates fiscal sustainability of Japan by providing a dynamic stochastic general equilibrium (DSGE) model that features the low interest rate of the government bond relative to the economic growth rate to mimic the actual data. We evaluate fiscal sustainability by investigating whether the expected path of the debt-to-GDP ratio stabilizes or increases without bound. The debt-to-GDP ratio depends crucially on the projected growth rate and the fiscal policy rule. If the government does not react to the current fiscal crisis, the debt-to-GDP ratio will increase without bound, and then the fiscal policy is not sustainable. If the fiscal rule uses Bohn's (1998) idea that involves the response of the primary surplus to the debt, sustainability improves. This rule provides a useful and realistic reform plan in the short and long runs.

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1. Introduction

Whether government debt is sustainable in Japan is a great concern. In 2011 Japan's government debt outstanding is close to the double of annual GDP, which is the highest among the developed countries and comparable to its own peak during World War II. Though the Japanese government (Cabinet Decision, 2010) declared its target at turning from primary deficit to primary surplus by 2020 and lowering the debt-to-GDP ratio from 2021, the weak economic recovery and unstable political situations are undermining its feasibility and credibility.

We evaluate sustainability by investigating whether the debt-to-GDP ratio will stabilize or increase without bound, following Ball et al. (1998) and Bohn (1998), among others[1]. Ball et al. (1998) used the historical joint distribution of interest rates and growth rates to calculate the probability that the government can run temporary budget deficits and then roll over the resulting government debt forever. Bohn (1998) proposed a simple test to check whether the debt-to-GDP ratio displays a mean-reversion property. Bohn (2005) applies his test to the historical data of the US and finds evidence supporting fiscal sustainability.

The path of debt-to-GDP ratio depends on the interest and growth rates as well as the fiscal policy rule. It is therefore of vital importance how to treat the interest and growth rates. Unlike most of the preceding studies that treat the interest and/or growth rates as exogenous, we develop a dynamic stochastic general equilibrium (DSGE) model that incorporates financial friction and endogenizes both the interest and growth rates. The introduction of the intermediation cost into an economy where agents are heterogeneous in the access to production gives rise to declines in both the economic

[1] Mendoza and Ostry (2008) apply Bohn's test to industrial and emerging countries and finds evidence of fiscal solvency in both types of countries. Galí and Perotti (2003) apply Bohn's test to European countries.

growth rate and the interest rate of government bond, enabling the model to mimic the actual data.

Figure 1 illustrates the time series of the financing bill (FB) rate and the GDP growth rate for the period of 1981–2009. Both rates are measured in real terms in terms of GDP deflator. The average of the FB rate is 1.9 percent, while the average of the growth rate is 2.2 percent. Standard growth models that incorporate a positive subjective discount rate, typically set at annual 2 or 3 percent, face difficulty in reproducing a negative gap between the short-term interest and the growth rate unless we assume some form of financial market frictions. The intermediation cost is a tractable way to formulate them.

Our results show that fiscal sustainability depends crucially on the projected growth rate and the fiscal policy rule. We evaluate fiscal sustainability by testing whether the expected path of the debt-to-GDP ratio stabilizes or increases without bound. If the government does not react to the current fiscal crisis but follows the rule observed over the last three decades, which features the reaction of the primary balance to the GDP growth rate and the lagged primary balance, then the expected debt-to-GDP ratio will reach 3.2 in 20 years and 11.5 in 100 years. The probability that the debt-to-GDP ratio diverge is 93.9 percent in 20 years and 99.7 percent in 100 years. The fiscal policy is hardly sustainable. If the fiscal rule adopts Bohn’s (1998) idea that primary surplus should increase when debt outstanding was high in the previous year, sustainability improves. If the current primary deficit turns to a surplus in 10 years and increases up to 2.2 percent surplus of GDP in 20 years, the debt-to-GDP ratio will stabilize at 2.3.

This paper differs from other literature that investigates the fiscal problem in that it applies the model that endogenizes interest and growth rates to the debt problem. Broda and Weinstein (2005) and Doi et al. (2011) investigate several scenarios of the Japanese debt problem by using exogenous pairs of growth and interest rates. Mendoza and Oviedo (2004, 2006) and Arellano (2008) apply dynamic stochastic general equilibrium (DSGE) models to sovereign default risk. Implications of their small-open-economy model, in which the interest rate is exogenous, differ from those of the closed economy that will explain well the debt problem of a large economy like Japan. Sakuragawa and Hosono (2010) develop a model that endogenizes the interest rate but their model is silent on implications on sustainability through the route of endogenous growth rate.

This paper is organized as follows. In Section 2, we outline the model. In Section 3, we develop the theoretical analysis. In Section 4, we describe the simulation procedure. In Section 5, we investigate the sustainability of the Japanese public debt. In Section 6, we report the estimated consumption tax rate required to attain sustainability. Section 7 concludes.

2. Model

Consider an economy made up of two types of agents that live infinitely, with the number of each normalized to be unity, and the third type of agents that live for two periods and act as intermediaries. To provide implications for low interest rates relative to the growth rate, we consider a financial friction and two types of agents, who are supposed to supply and demand funds (e.g., Woodford, 1990). Our model is simple and tractable, serving as an alternative to multi-period overlapping-generations models.

Type E agents have access in all *even* periods to an AK production technology but cannot access to it in any odd periods, while Type O agents have access only in all *odd* periods. Type E agents buy capital from type O agents for production in even periods and sell it to type O agents in odd periods. The AK technology transforms K_t units of the final good into random $(1+x_{t+1})K_t$ units after one period. x_{t+1} is the stochastic rate of return on capital net of the depreciation rate, where the expected value $E(x_{t+1})$ is positive for $x_{t+1} \in (-1,+\infty)$. Two reasons motivate the introduction of the AK model. First, fiscal sustainability is a long-run problem. Second, the AK model enables one to have the positive link between interest and growth rates that is observed in the time series. The rate of return on capital, x_{t+1}, is a random variable that is observed at the beginning of the period. It follows a first-order Markov process and takes values in a finite set, $X = \{x_1,....,x_n\}$. Denote the probability of a variable, x_{t+1}, given x_t, by $\pi(x_{t+1}|x_t)$.

To simplify the notation, let there be one representative agent of each type so that the individual income $(1+x_{t+1})K_t$ denotes the aggregate income. There is no population growth. Both types have identical preferences over consumption and maximize $E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\alpha}}{1-\alpha}$, where α is the inverse of the elasticity of substitution of consumptions across periods, β (0< β <1) is the discount factor, and E_0 is the expectation operator. We impose the relevant condition on bounded utility by $E_t\{\beta(1+g_{t+1})^{1-\alpha}\} < 1$, where g_t is the growth rate of the aggregate income argued below. The government spending, G_t, is a constant share of GDP to meet $G_t = zY_t$. The government finances its spending by imposing lump-sum taxes and by issuing public debt.

At each period, finite N agents who act as intermediaries are born and live for two periods. They are endowed with a specific skill of intermediating finance between private agents, and maximize the second-period consumption. The intermediary accumulates assets and debts just across two periods, which simplifies our analysis.

We introduce financial friction by supposing that these agents have to bear a proportional intermediation cost κ per unit of funds. To lend 1 unit of fund to firms, the intermediary raises $1+\kappa$ units of funds from investors and spends κ units of funds as the intermediation cost. Funds are measured in terms of goods. One may interpret the intermediation cost as a cost of monitoring or identifying a borrower, or of verifying credit.

3. Theoretical Analysis

The intermediary issues securities that request the rate of repayment $r^b(x_t)$ to firms and guarantee the rate of return $r(x_t)$ to investors, both of which are contingent on x_t. In a world of competitive intermediation, intermediaries finally have to earn zero profitto satisfy

(1) $$\{1+r(x_t)\}(1+\kappa)=1+r^b(x_t)$$

for any x_t. Note that both assets are risky in the sense that the rate of return depends on the productivity. We take an approach of the incomplete bond market where the government can issue only one-period bonds and private agents cannot insure away the income uncertainty.

At the beginning of an odd period t, type E agents face a shock x_t, receive capital income $x_t K_{t-1}$, sell capital stock K_{t-1}, repay $(1+r^b(x_t))B_{t-1}$, consume C_t, pay taxes T_t^E, and invest the remaining in the private security W_t and the public bond D_t to satisfy the budget constraint, $(1+x_t)K_{t-1} - \{1+r^b(x_t)\}B_{t-1} = C_t + W_t + D_t + T_t^E$. On the other hand, at the odd period t, type O receives interest incomes from the private security $\{1+r(x_t)\}W_t$ and the public bond $\{1+R(x_t)\}D_{t-1}$, consume $\tilde{C}_t$, pay taxes T_t^O, and invest the remaining in capital to produce in the odd period to satisfy $\{1+r(x_t)\}W_{t-1} + \{1+R(x_t)\}D_{t-1} + B_t = \tilde{C}_t + K_t + T_t^O$, where $R(x_t)$ is the interest rate on the government bond. Type O agents do not have an incentive to buy the security issued by the intermediary or the government since they do not insure away the risk of the return on capital.

Now we are ready to derive the relationships among the return on private securities, government bonds, and physical assets. The securities issued by the intermediary and the government are perfect substitutes for the agents who invest in financial assets, implying that

(2) $R(x_t) = r(x_t)$ for any x_t.

Combining this with (1), we have $R(x_t) < r^b(x_t)$: the government can borrow at a lower rate than private agents. The reason behind this finding is that loans to the government can be monitored with no cost, while loans to private agents need intermediation cost.

We also establish that

(3) $x_t = r^b(x_t)$ for any x_t.

Otherwise, the agent who can access to the production technology either raises funds from the intermediary and invests in physical capital indefinitely (in the case of

$x_t > r^b(x_t)$) or raises no fund and does not invest in physical capital at all (in the case of $x_t < r^b(x_t)$), neither of which is consistent with the equilibrium.

The first-order conditions of the agents with respect to K_{t+1}, and D_{t+1} lead to[2]

(4) $$1 = \beta \sum_{x_{t+1} \in S} \pi(x_{t+1} | x_t)(1 + x_{t+1}) \frac{\{\tilde{C}(x_t)\}^\alpha}{\{C(x_{t+1} | x_t)\}^\alpha},$$

and

(5) $$1 = \beta \sum_{x_{t+1} \in S} \pi(x_{t+1} | x_t)\{1 + R(x_{t+1})\} \frac{\{C(x_t)\}^\alpha}{\{\tilde{C}(x_{t+1} | x_t)\}^\alpha}.$$

Market clearing in the good implies $\kappa B_t + C_t + \tilde{C}_t + K_t + zY_t = (1 + x_t)K_{t-1}$ and the one in the credit markets implies $W_t = (1+\kappa)B_t$, respectively.

The government's budget constraint is given by

(6) $$D_t = \{1 + R(x_t)\}D_{t-1} + G_t - T_t^E - T_t^O.$$

As Barro (1979) have argued, we impose an additional feasibility constraint restricting the government's taxable income to be limited to some fraction of the aggregate income. We simply call a fiscal policy *feasible* if the tax revenue does not exceed a fraction τ of GDP, that is,

(7) $$T_t^E + T_t^O \le \tau Y_t$$

for any state x_t.

Following Ball et al. (1998) and Bohn (1998), we evaluate *sustainability* by investigating whether the expected path of the debt-to-GDP ratio stabilizes or increases without bound. Strictly, satisfying the government's intertemporal budget constraint only does not guarantee the bounded debt-to-GDP ratio to be a necessary condition for

[2] The first-order conditions of W_{t+1}, and B_{t+1} confirm our discussion leading (2) and (3), and are skipped here.

sustainability (e.g., McCallam, 1984). The additional imposition of the constraint (7) enables the unbounded debt-to-GDP ratio to become a necessary condition (e.g., Bohn 1991). The formal analysis is left to Sakuragawa and Hosono (2011).

Let $C_t / \tilde{C}_t \equiv \theta_t$ denote the consumption ratio between two different types of agents. Limiting focus on an economy with θ_t being constant through time, we define the consumption growth rate as

(8) $$\frac{C(x_{t+1}|x_t)}{C(x_t)} = \frac{\tilde{C}(x_{t+1}|x_t)}{\tilde{C}(x_t)} \equiv 1 + g(x_{t+1}|x_t) .$$

We use (8) to rewrite (5) as

(9) $$1 = \beta \sum_{x_{t+1} \in X} \pi(x_{t+1}|x_t)\{1 + g(x_{t+1}|x_t)\}^{-\alpha} \theta^{\alpha} \{1 + R(x_{t+1})\} ,$$

which embodies the relationship between the growth and interest rates. On the other hand, (1)-(4), (8) and (9) jointly imply $\theta = (1+\kappa)^{1/2\alpha} > 1$[3]. In the presence of the intermediation cost, agents consume more when they receive income and consume less when they do not. Plugging (1) and $\theta = (1+\kappa)^{1/2\alpha}$ into (9), we have the following:

(10) $$1 = \beta \sum_{x_{t+1} \in X} \pi(x_{t+1}|x_t)\{1 + g(x_{t+1}|x_t)\}^{-\alpha} (1 + x_{t+1})(1+\kappa)^{-1/2} ,$$

which relates the growth rate with the return on capital.

We are permitted to write $g(x_j|x_t) = g_j$, which implies that the growth rate of consumption depends only on the current rate of return on capital. This arises from the

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[3] Substituting (1)-(3) and (8) into (4) yields

$1 = \beta \sum_{x_{t+1} \in S} \pi(x_{t+1}|x_t)\{1 + g(x_{t+1}|x_t)\}^{-\alpha} \theta^{-\alpha} (1+\kappa)\{1 + R(x_{t+1})\}$. Combining this with (9) leads to

$\theta = (1+\kappa)^{1/2\alpha}$.

nature of the AK production technology and the fact that the proportion of government expenditure in output is constant. When the transition probability function is rewritten as $\pi(x_j | x_i) = \phi_{ij}$, and $R(x_j) = R_j$, (9) and (10) are

$$(11) \qquad 1 = \beta \sum_{j=1}^{n} \phi_{ij} (1+g_j)^{-\alpha} (1+R_j)(1+\kappa)^{1/2} \quad \text{for } i = 1,..., n,$$

and

$$(12) \qquad 1 = \beta \sum_{j=1}^{n} \phi_{ij} (1+g_j)^{-\alpha} (1+x_j)(1+\kappa)^{-1/2} \quad \text{for } i = 1,..., n.$$

Equations (11) and (12) constitute $2n$ equations and solve n growth rates and n interest rates given the exogenous sequence of $\{x_1,..., x_n\}$.

The deterministic version of the model will reveal the implication of the model clearly. Letting g and R denote the growth rate and the interest rate of the deterministic model, it follows that $1+g = \beta^{\frac{1}{\alpha}} (1+x)^{\frac{1}{\alpha}} (1+\kappa)^{-\frac{1}{2\alpha}}$ and $1+R = (1+x)(1+\kappa)^{-1}$. Both rates declines as the intermediation cost goes up, but with different speed. We have

$$\log(1+R) - \log(1+g) = -\frac{1}{\alpha}\log\beta + (1-\frac{1}{\alpha})\log(1+x) + \frac{1-2\alpha}{2\alpha}\log(1+\kappa).$$

The gap between two rates may or may not increase as the intermediation cost goes up, depending on the inverse of α, the elasticity of intertemporal substitution. If the elasticity is above 2, the gap increases, while if it is less than 2, the gap decreases.

4. Calibration

In this section we explain the procedure for simulation. The procedure is standard except for using (11) and (12) that endogenize the interest and growth rates. First, we specify the equations used for simulation. We use the fiscal policy rule and the government's budget constraint as well as (11) and (12). Secondly, we choose parameters, and finally set initial conditions.

4.1. Specifying Equations

First, we specify the Markov process for the gross rate of return on capital, which is the primary driving force of the interest and growth rates. The AR (1) in the logarithm form describes its process:

(13) $$\log(1+x_{t+1}) = \rho\log(1+x_t) + (1-\rho)\log(1+x^a) + e_{t+1},$$

where ρ is the serial correlation coefficient, x^a is the average value, and e_{t+1} is a random shock that is independent and identically distributed as a normal distribution with standard deviation σ_e.

We next specify the fiscal policy rule that determines the primary balance-to-GDP ratio, denoted by s_t, as

(14) $$s_{t+1} = \gamma_0 + \gamma_1 s_t + \gamma_2 g_{t+1},$$

The GDP growth rate is expected to capture the business cycle effects. When the economic boom comes, an increase in tax revenues improves the fiscal stance. Actually, Figure 2 illustrates a positive correlation between the two. The lagged variable captures the persistency of the government expenditure and tax revenues.

Finally, the debt-to-GDP ratio, denoted by d_t, evolves from the government's budget constraint (6) as

(15) $$d_{t+1} = \frac{(1+R_{t+1})}{(1+g_{t+1})}d_t - s_{t+1}.$$

Remember that R_{t+1} is the return on one-period government bond. If the return on government bond with two or more period maturity is the expected value of the future returns on one-period bond, the maturity mix of the actual government bonds does not matter. To the extent that the return on government bonds with longer maturity has a positive premium, our simulation result would underestimate the future d_{t+1}. We consider this potential bias to be conservative; if our simulation result shows that d_{t+1} grows without bound, our conclusion does not change even if we consider the positive premium on long-run government bonds.

Given the predetermined triplet, (x_t, s_t, d_t), (11),(12), (13)-(15) determine the next period triplet $(x_{t+1}, s_{t+1}, d_{t+1})$ recursively. In the first step, a random variable x_{t+1} is drawn from the distribution (13)[4] that is a Markov of x_t, which, in turn, determines g_{t+1} and R_{t+1} through (11) and (12). In the second, (14) determines s_{t+1}. In the third, (15) determines d_{t+1}. We repeat those three steps 10,000 times to obtain the expected value of d_{t+1} and the probability that d_{t+1} exceeds its initial value.

4.2 Parameters

First, we choose the preference parameters, β and α. We set the annual discount factor β at 1/1.02=0.980. Based on the relationship between interest and growth rates, (11i), we choose α by regressing the nominal government bond yield on the nominal GDP growth rate. The sample used for the estimation below covers the period of 1981-2009 except for otherwise mentioned. Data used to set parameters is described in Data Appendix. The result for the OLS estimation is

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[4] We discretize (13) with 9 states following Tauchen and Hussey (1991) to generate a series of x_{t+1}.

$$R_t = 0.023 + 0.565 g_t, \quad Adj.R^2 = 0.651,$$
$$(0.003) \quad (0.077)$$

where the numbers in parentheses are standard errors. Following the above result, we set α at 0.565, implying that the elasticity of intertemporal substitution is set at 1.770. This figure is in the range where a rise in the intermediation cost reduces the gap between the two rates..[5]

Next we choose the technology parameters, ρ and σ_e. Based on the relationship between the growth rate and the return on capital, (12), we conduct the OLS estimation of the AR(1) process of the GDP growth, obtaining

$$g_{t+1} = 0.002 + 0.780 g_t, \quad Adj.R^2 = 0.4130.$$
$$(0.006) \quad (0.175)$$

We set ρ at 0.780 and σ_ε at 0.01996, where the latter is the root mean squared error of the regression. We use the chosen σ_ε and α to set σ_e. We set the average return on capital, x^a, at a value that yields 1 percent average GDP growth rate given the chosen values of β, α, and κ in (12i). This growth rate seems reasonable given that the average GDP growth rate over the period of 1990-2009 is 1.1 percent.

Third, we set the financial intermediation cost, κ, at 0.015, the average of net interest margins between the bank loans and the bank deposits over the period of 2000–2009. We choose the average over the last decade because, as Figure 3 shows, the net interest margins tended to decline over the last two decades especially at a high rate in the 1990s.

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[5] This figure is consistent with much empirical studies reporting that the elasticity of intertemporal substitution is almost between 1 and 2. See Abe and Yamada (2005), among others.

Finally, we choose parameters of the fiscal policy rule. The fiscal rule is interpreted as a consequence of the conflict of interests among many pressure groups and so changes little unless the political situation changes drastically. To capture the inertia of the policy rule, we specify the rule by the regression. The estimation result that covers the period of 1981-2008 is[6]

$$s_t = -0.021 + 0.658 s_{t-1} + 0.577 g_t, \quad Adj.R^2 = 0.759.$$

$$(0.006)\ (0.160) \quad (0.104)$$

Based on the above result, we set γ_0=–0.021, γ_1=0.658, and γ_2=0.577.

Table 1 summarizes the parameters.

4.3 Initial Conditions

We conduct the stochastic simulation from year 2011. To do so, we need to construct the initial values of the triplet ($x_{2010}, s_{2010}, d_{2010}$). We construct x_{2010} so as to reproduce g_{2010} from the deterministic version of (12), $x_{2010} = \beta^{-1}(1+g_{2010})^{\alpha}(1+\kappa)^{1/2} - 1$ (see Data Appendix for the estimate of g_{2010}).

We obtain s_{2010} from the government's estimate (see Data Appendix). To obtain d_{2010}, we need to construct R_{2010}. We assume perfect foresight and substitute g_{2010} into the deterministic version of (11), $R_{2010} = \beta^{-1}(1+g_{2010})^{\alpha}(1+\kappa)^{-1/2} - 1$. Then, we substitute R_{2010}, g_{2010}, d_{2009} and s_{2010} into (15) to get d_{2010} (=1.860).

5. Simulation Results

In this section we simulate the model to investigate the fiscal sustainability of Japan.

[6] The data of primary balance is available only up to 2008. See Data Appendix.

A. Baseline Forecast

The first row of Table 2 reports the expected debt-to-GDP ratio and the probability that the debt-to-GDP ratio exceeds its initial value as of 2009 (d_0 =1.792)[7]. Under the baseline parameters, the gap between the interest and growth rates is 0.8 percent points[8] and the average primary surplus is -4.5 percent of GDP. The expected debt-to-GDP ratio reaches 11.8 in 100 years and continues to increase afterwards. The probability that the debt-to-GDP ratio goes up from its initial level reaches 93.9 percent in 20 years and 99.7 percent in 100 years. The fiscal policy is hardly sustainable.

B. Alternative GDP Growth Rates

Here we investigate the path of the debt-to-GDP ratio when the economic growth rate changes. Parameters are unchanged except for the average return on capital, x^a , which we change to match the new growth rate. As the growth rate increases from 1 to 2 and 3 percent, the interest rate also goes up from 1.8 to 2.37 and 2.94 percent, but the gap between the two rates shrinks as the growth rate goes up, from 0.8 to 0.27 and -0.06 percent points. The second and third rows of Table 2 show that the debt-to-GDP ratio

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[7] We chose year 2009 as t=0. So, t=20 (“20 years after”), for example, indicates year 2029 in Tables 2 to 3 and Figure 5 below.

[8] The simulated gap is greater than the observed gap in the case of FB, minus 0.3 percent and smaller than the one in the case of long-term government bond, 1.4 percent. Note that the rate of return on long-term government bond is illustrated in Figure 1. Since the maturity of FB is 6 months or less, the gap between the rate of return on the bond with one-year maturity and GDP growth rate is likely to lie between the two observed gaps. Accordingly, the simulated gap will lessen the potential underestimation of the debt-to-GDP ratio arising from ignoring the term premium.

grows more slowly as the growth rate goes up: the debt-to-GDP ratio reaches 3.21(1% case), 2.69 (2% case), and 2.23(3% case). A higher growth contributes to improve sustainability.

C. Alternative Fiscal Policy Rules

In this subsection, we consider alternative fiscal policy rules. The parameters are unchanged except for those of the fiscal policy rule.

One simple way to restore sustainability would be to raise the average primary surplus by raising the value of γ_0 in the rule (14). We find that the primary surplus that is 1.96 percent of GDP on average is enough to stabilize the expected debt-to-GDP ratio at its initial value and, thus, to make debt sustainable.

A more flexible and maybe more interesting way to restore sustainability is to change the fiscal policy rule. As Bohn (1998) addresses, a rational government should increase the primary surplus when the debt-to-GDP ratio is high. We incorporate his idea by specifying the fiscal rule as

$$s_{t+1} = \gamma_0 + \gamma_1 s_t + \gamma_2 g_{t+1} + \gamma_3 d_t \, . \tag{16}$$

We use the same parameters for γ_1 and γ_2 as in the baseline case, but set γ_3 arbitrarily.[9] To compare with the benchmark case, we adjust γ_0 so that the average primary surplus is –4.5 percent of GDP given that d_{t-1} is at the value as of 2010.

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[9] To avoid possible endogeneity, we estimated (16) using d_{t-2} as an instrument and found that the coefficients on d_{t-1} was positive but not significant. Figure 4 illustrates no significant positive relationship between s_t and d_{t-1}, though it does not control for the GDP growth rate. Bohn (1998) estimates the determinants of the US primary surplus, finding that the coefficient of the lagged debt-to-GDP ratio is around 0.03 to 0.05, depending on the sample period (Table 1).

Tables 3A reports the expected debt-to-GDP ratios for several values of γ_3. We find that a sufficiently large positive response to the debt-to-GDP ratio tends to stabilize the debt-to-GDP ratio. If γ_3=0.01, 0.03 and 0.05, the debt-to-GDP ratios stabilize at about 4.9, 2.7 and 2.3, respectively. Table 3B reports the associated expected primary surplus-to-GDP ratios.

Figure 5 depicts the expected primary surplus-to-GDP ratio in the case of γ_3=0.03 and 0.05, as well as the baseline (γ_3=0). If γ_3=0.05, the current primary deficit will turn to a surplus in 10 years, increase up to 2.2 percent surplus of GDP in 20 years. The government can attain this path by decreasing expenditures and/or increasing taxes *gradually* for the coming 20 years. A delayed fiscal consolidation will lead to the lower surplus-to-GDP ratio in early periods but the higher ratio in later periods, resulting in a stable but higher debt-to-GDP ratio.

6. Consumption Tax and Sustainability

A great concern is how much increase in tax rate is needed if fiscal sustainability is to be restored only by raising consumption tax rate.

The baseline parameters for the fiscal policy rule implies that the long-run primary deficit ($\frac{\gamma_0+\gamma_1 g}{1-\gamma_2}$) is 4.5 percent of GDP if $g = 0.01$. We can regard this long-run deficit as structural (i.e., cyclicality- and momentum- adjusted) one. On the other hand, to achieve fiscal sustainability under the 1 percent average GDP growth rate, the primary surplus must be 1.96 percent of GDP. The difference between the structural and targeted primary balances amounts to 6.5 percentage points, and should be eliminated either by a decrease in expenditures or an increase in revenues.

The consumption tax revenue as a proportion of nominal GDP is 2.0 percent on average during the period of 1997-2009, when the tax rate was 5 percent. Because consumption tax does not yield any distortion in our model, we may estimate that raising consumption tax by one percentage point contributes to a 0.4 percentage increase in the primary surplus-to-GDP ratio. For this reasoning, we can mechanically compute that consumption tax rate must be increased by 16 percentage points, i.e., from 5 to 21 percent, to raise primary surplus-to-GDP ratio by 6.5 percentage points.

If the government is to fill in the gap of primary surplus-to-GDP ratios between Bohn's rule with γ_3=0.05 and the past rule (in the baseline simulation) just by raising consumption tax rates, it has to raise the consumption tax rate by 12 percentage points in 10 years and by 16.4 percentage points in 20 years from the current 5 percent level.

7. Conclusion

We have proposed a new framework for investigating fiscal sustainability and applied it to the current Japanese government debt. Our framework follows the literature of investigating whether the debt-to-GDP ratio will stabilize or grow without bounds, but unlike preceding studies, we provide a dynamic stochastic general equilibrium model that can mimic the actual relationship between interest and growth rates by endogenizing the two rates. Our framework also has the advantage that it enables us to simulate transition paths of the debt-to-GDP ratio and the associated primary surplus-to-GDP ratio under various fiscal policy rules.

Our simulation results show that if the government does not react to the current fiscal crisis, the debt-to-GDP ratio will increase without bound, and the fiscal policy is not sustainable. We propose an alternative policy rule that incorporates the reaction of the surplus to the debt. A strong policy message from this simulation is that if the current primary deficit turns to a surplus in 10 years and increases up to 2.2 percent surplus of GDP in 20 years, the debt-to-GDP ratio will stabilize at 2.3. A delayed fiscal consolidation will result in a higher debt-to-GDP ratio and require a larger primary surplus to restore sustainability.

One caveat is that we do not account for the default cost. The default cost would make the sustainability conditions more difficult to be met because the interest rate would be higher and the debt-to-GDP ratio would increase more rapidly as the debt-to-GDP ratio approaches its default level. To quantify this effect is left for future work.

Data Appendix

1. Primary balance is obtained from the current and capital accounts of the system of national accounts (*93SNA*, Economic and Social Research Institute) of the general government as follows:

Primary balance = (Taxes on products and imports + Current taxes on income and wealth + Social burdens + Other current transfers received + Fixed capital depreciation + Capital transfer received) – (Subsidy + Social benefit except for social transfers in kind + Other current transfers paid + Final consumption + Gross fixed capital formation + Increases in inventories + Net purchase of land + Capital transfers paid).

The primary balance data based on *93 SNA* is available only up to 2008. To estimate the primary balance in 2009 and 2010, we used the government's estimate that the primary balances of the central and local governments are -8.1 percent and -6.4 percent of nominal GDP in fiscal years 2009 and 2010, respectively (Cabinet Office, 2010). To convert the government's estimate to the primary balances of the general government (i.e., the total of the central government, local governments, and the social security funds), we assumed that the primary balance of the social security funds as a proportion of nominal GDP as of fiscal year 2008 (-1.1 percent of nominal GDP) did not change up to 2010.

2. Nominal and real GDP are based on *93 SNA,* which is obtained from the website of Economic and Social Research Institute.

Nominal and real GDP data are available up to 2009. For the real GDP growth rate in 2010, we used the government's estimate for fiscal year 2010 (2.6 percent, Cabinet Office, 2010).

3. Interest rate margin is the difference between the deposit rate and the lending rate, both of which are obtained from IMF’s *International Financial Statistics*.

4. Real yield on financial bills and government bonds are nominal yields on each asset minus the change in GDP deflator. Those data are obtained from IMF’s *International Financial Statistics*.

5. Government debt is total debt minus financial bills outstanding. Data source of government debt is *Flow of Funds*, obtained from the website of Bank of Japan.

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Table 1. Paramters

Preference		
β	discount factor	0.980
α	inverse of elasticity of intertemporal substitution	0.565
Technology		
x^a	average return to capital	0.033
ρ	serial correlation of return to capital	0.780
σ	standard deviation of error term in return to capital	0.011
Financial Intermediation		
κ	financial intermediation cost	0.015
Fiscal Policy Rule		
γ_0	Constant	−0.021
γ_1	coefficient on previous-year primary surplus/GDP	0.658
γ_2	coefficient on GDP growth rate	0.577

Table 2. Expected debt-to-GDP ratio under alternative growth rates

After	20 years	50 years	100 years	500 years	1000 years
GDP Growth=1% (baseline)	3.21	5.76	11.84	582.79	49118.59
	(93.9%)	(98.1%)	(99.7%)	(100.0%)	(100.0%)
GDP Growth=2%	2.69	4.00	6.74	74.97	799.67
	(84.7%)	(89.5%)	(95.1%)	(100.0%)	(100.0%)
GDP Growth=3%	2.23	2.64	3.31	9.07	17.60
	(68.5%)	(67.7%)	(70.7%)	(83.7%)	(90.2%)

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.792)

Table 3. Bohn's rule

A. Expected debt-to-GDP ratio

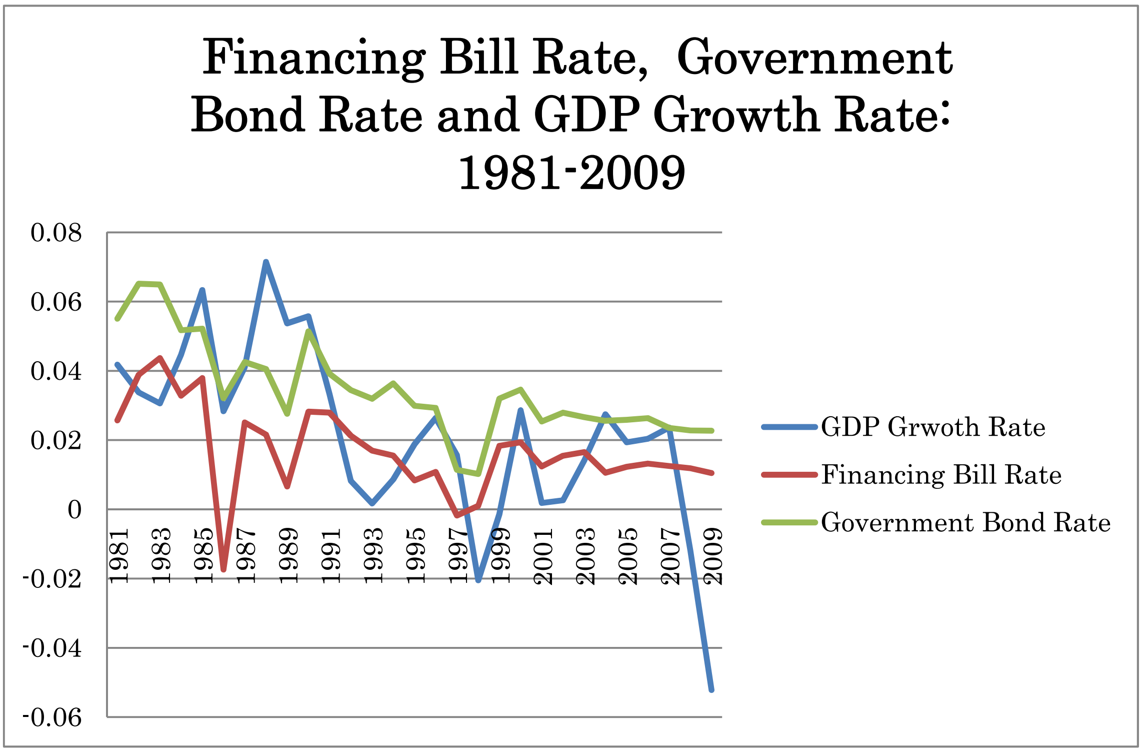
After	20 years	50 years	100 years	500 years	1000 years
γ_3 = 0.001	3.18	5.50	10.42	200.42	4154.41
	(93.9%)	(98.1%)	(99.7%)	(100.0%)	(100.0%)
γ_3 = 0.01	2.94	3.94	4.62	4.96	4.91
	(94.0%)	(97.8%)	(99.0%)	(99.4%)	(99.3%)
γ_3 = 0.03	2.56	2.65	2.66	2.67	2.65
	(93.3%)	(94.2%)	(94.4%)	(94.5%)	(94.2%)
γ_3 = 0.05	2.33	2.32	2.31	2.32	2.32
	(92.1%)	(91.0%)	(91.1%)	(91.3%)	(91.4%)

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.792)

B. Expected Primary Surplus-to-GDP Ratio

After	20 years	50 years	100 years	500 years	1000 years
γ_3 = 0.001	-4.03%	-3.48%	-2.00%	52.63%	1188.60%
γ_3 = 0.01	-1.75%	1.34%	3.47%	4.60%	4.46%
γ_3 = 0.03	1.24%	2.43%	2.48%	2.53%	2.48%
γ_3 = 0.05	2.16%	2.04%	2.13%	2.10%	2.15%

Figure 1



Note. The rates are in real terms.

Figure 2

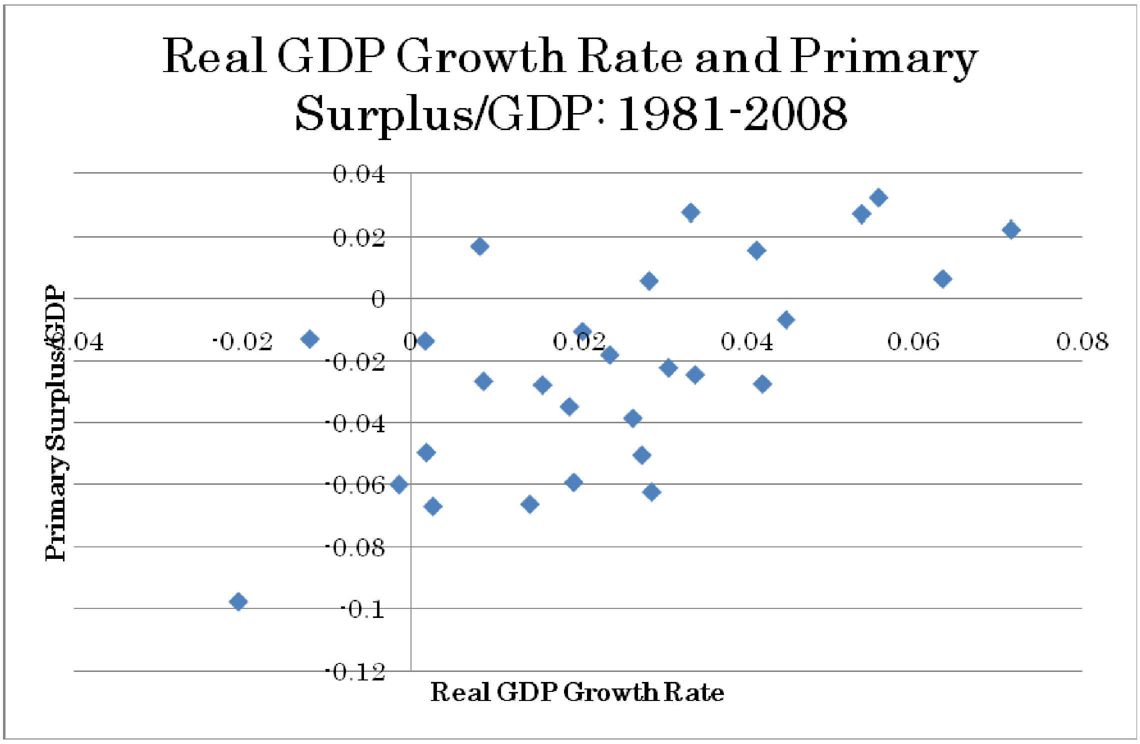
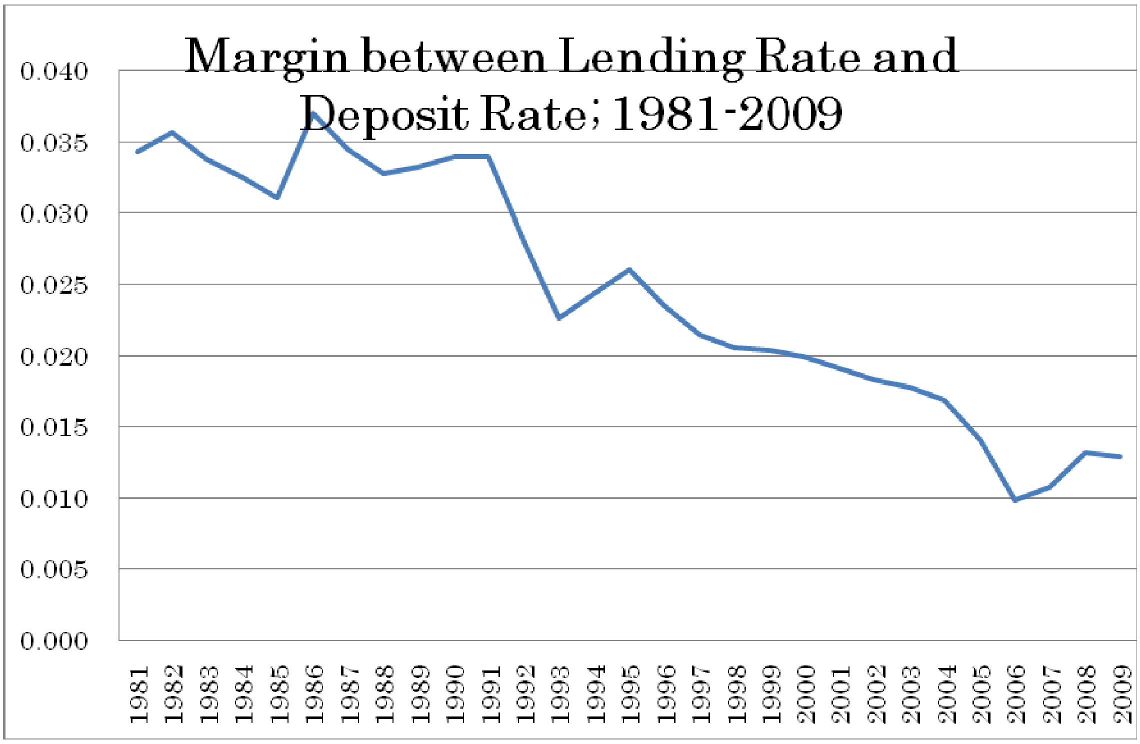

Real GDP Growth Rate and Primary Surplus/GDP: 1981-2008
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.04
-0.02
0
0.02
0.04
0.06
0.08
Primary Surplus/GDP
Real GDP Growth Rate

Figure 3


Margin between Lending Rate and Deposit Rate; 1981-2009
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1981
1982
1983
1984
1985
1986
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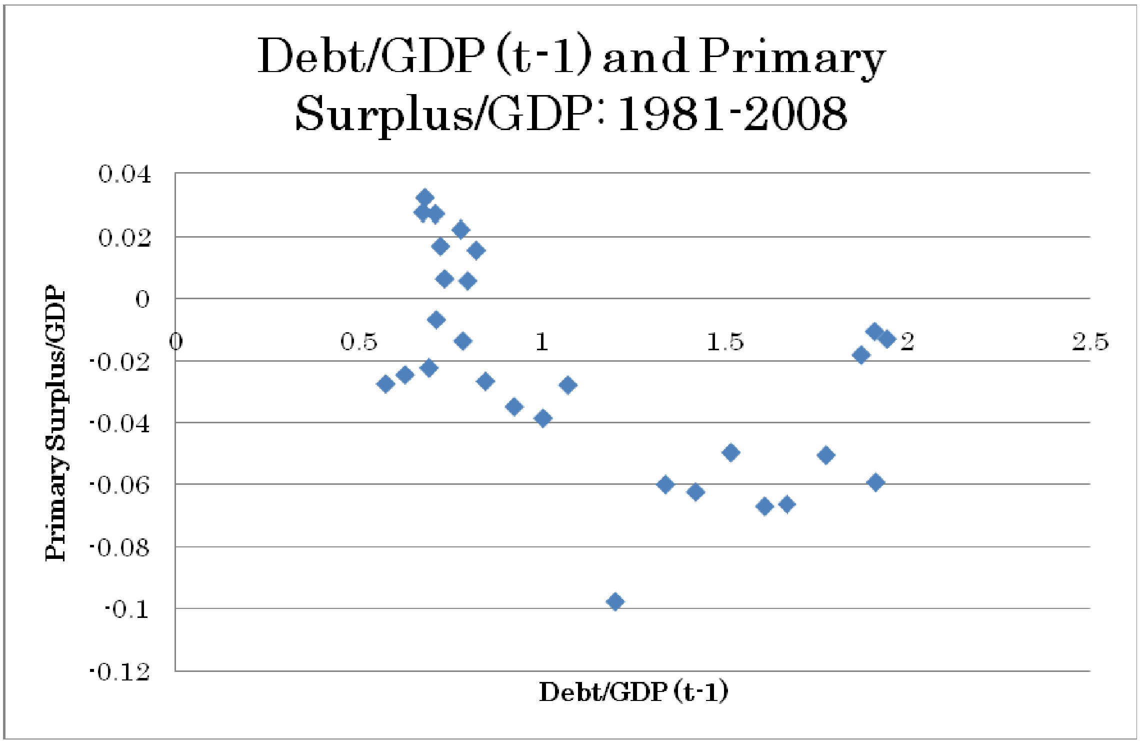
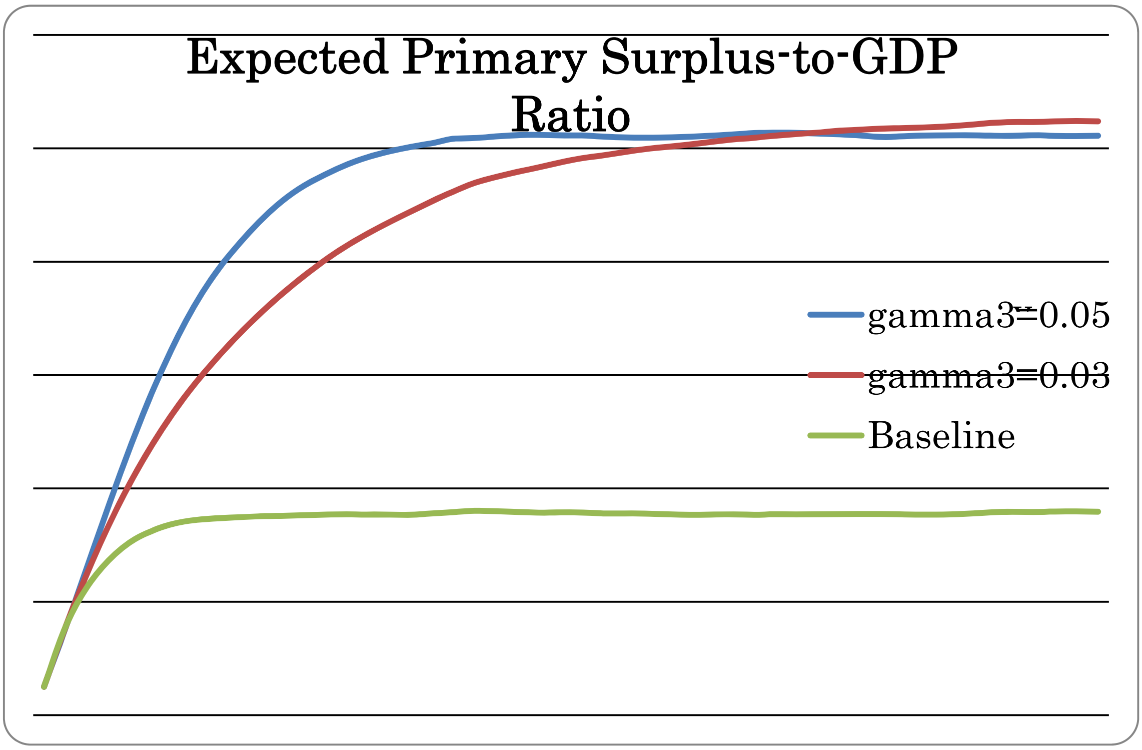
2009

Figure 4



Debt/GDP (t-1) and Primary Surplus/GDP: 1981-2008
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
0
0.5
1
1.5
2
2.5
Primary Surplus/GDP
Debt/GDP (t-1)

Figure 5


Expected Primary Surplus-to-GDP Ratio
gamma3=0.05
gamma3=0.03
Baseline