TCER Working Paper Series

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March 2018

Working Paper E-123
http://tcer.or.jp/wp/pdf/e123.pdf

TOKYO CENTER FOR ECONOMIC RESEARCH
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Abstract

Sustaining economic growth under rapid aging is one of the most important policy issues in Japan. Because of the difficulty of increasing labor force in an aging society, it is desirable to promote human capital accumulation for improvement of labor quality in the long run. However, since human capital accumulated in the young may become obsolete for elder workers, we cannot achieve sufficient level of human capital to sustain economic growth only through education for the young. Thus, we need recurrent education for the elderly or retired female workers in an aging society as in Japan. Hence, this paper investigates whether we can achieve socially optimal level of human capital when the decision to participate in recurrent education is left to the private sector.

To answer this question, this paper studies human capital accumulation through recurrent education as well as primary education and tertiary education in an OLG model. We show that the effects of mortality on recurrent education depends on the relationship between tertiary education and recurrent education. In other words, if they are complements, i.e. if a higher level of tertiary education increases the effects of recurrent education, a decline in mortality rate promotes recurrent education, which improves human capital. On the other hand, if they are substitutes, i.e. if a a higher level of tertiary education decreases the effects of recurrent education, a decline in mortality rate decreases the level of recurrent education, which decreases human capital. In the latter case, we cannot achieve sufficient level of recurrent education to sustain economic growth in an aging society through voluntary choice by the private sector, and hence, we need policies to promote recurrent education.

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JEL codes: J10, I25, O15

*I am grateful to Professor Shin-ichi Fukuda, Professors Jun-ichi Nakamura, Masaya Sakuragawa, Ryuichi Tanaka, Hideki Nakamura, Shigeto Kitano, Hyelim Son, Akira Shimada and the participants of the Financial Group of the Institute of Statistical Research, the 3rd Twin Conference, 79th International Atlantic Economic Conference, the annual meeting of Japanese Economic Association, APEA 2015, the 14th Korea and the World Economy Conference, and the ESRI International Conference. All errors are mine.

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1 Introduction

Sustaining economic growth under rapid aging is one of the most important policy issues in Japan. According to the "Basic Policy on Economic and Fiscal Management and Reform 2016 (Cabinet Office, 2016)," the government determines to deal with growth strategies to achieve 60 billion yen economy. The Cabinet Office also emphasized the importance of productivity revolution to overcome the restrictions of the supply side in an aging society in a leaflet of government's growth strategy entitled "Yawaraka Sveichosenryaku"

One of the possible solutions to increase the labor supply is to increase labor input. Since it is difficult to increase the labor force in an aging society. we should promote human capital accumulation. However, human capital depreciates overtime. This means that we cannot achieve the sufficient level of human capital for retaining economic growth in an aging society. Hence, this paper focuses on the role of recurrent education. The recurrent education is one of the lifetime education proposed by OECD in 1970s as cyclical or recurrent type of educational system in which an individual after he or she finishes formal education takes education again to satisfy his or her needs.

Such type of education is critical for promoting human capital accumulation in an aging society as in Japan. However, it should be noted that even if government invests in educational infrastructure, recurrent education is not promoted, when each worker has no incentive to take recurrent education. As OECD named the title of its paper "Beyond Rhetoric," even if recurrent education is important for the economy, the optimal level of recurrent education is not necessarily achieved.

Human capital is accumulated through higher education and recurrent education, as well as primary education. Among them, the level of primary education and tertiary education is already higher in Japan. In contrast, the level of recurrent education is low from international viewpoints. In this case, government should encourage individuals to take recurrent education. However, there are few previous studies in this literature. Hence, it is important to investigate efficient and effective policy to promote recurrent education by focusing on the relation between tertiary education and recurrent education. In particular, given that human capital depreciates over time, recurrent education for working generations compensates higher education, while recurrent education that substitutes tertiary education would be important for those who have not taken enough education.

In recent years, some universities have introduced recurrent educational program for working generations and retired female workers. The aim of these programs is to encourage the students to accumulate their human capital. Such programs may contribute to economic growth. This paper studies the role of recurrent education for economic growth.

Section 2 investigates the relation between recurrent education and higher education. Section 3 explains the outline of the model and Section 4 describes the model. Section 5 describes the equilibrium conditions in the steady state. Section 6 analyzes the impacts of the decline in mortality rate on tertiary education and recurrent education. Section 7 analyzes the impacts of the decline in mortality rate on human capital. Section 8 concludes.

2 Current situation of recurrent education

This paper focuses on the relation between tertiary education and recurrent education to study the role of recurrent education in human capital accumulation. If tertiary education and recurrent education are complementary, those who completed tertiary education should take recurrent education. In contrast, if tertiary education and recurrent education are substitutes, those who did not take education should take recurrent education. In general, the concepts of complementarity and substitutability are ambiguous, the following subsections surveys the current situation of recurrent education in terms of the relation between tertiary education and recurrent education.

2.1 Participation in recurrent education of each country

This subsection focuses on the current situation of recurrent education from an international point of view. In terms of the importance of recurrent education, OECD (2005) stated that adult learning is an important input for human capital accumulation, which has a positive impact on an individual productivity, innovation and employment opportunity. OECD (2014) also pointed out that it is important to provide formal education system with those who completed compulsory education, especially those who need to adapt to the changes in their career.

However, even if recurrent education is important, the actual level of recurrent education is not necessarily optimal for the society. Figure 1 shows the participation rate in recurrent education [1] of each country in Chart C 6.1 of OECD (2014). According to this figure, the average participation rate among OECD is 51%. The figure also shows that the participation rate in rapidly aging societies tends to be low. For example , Japanese participation rate is 42%, the Korean rate is 50%, and the Italian rate is 25%. This suggests that the participation rate in aging societies is not necessarily optimal.

Such problem occurs both in the cases of formal education and non-formal education. For example, Figure 2 in Chart C 6.6 of OECD (2014) shows the participation rates of each country in 4 cases: participation or non-participation in formal education or non-formal education, respectively. According to this figure, although the participation rate of Japan is

[1]The concept of recurrent education includes formal education which is provided by formal educational institutions such as elementary schools, high schools, universities, and so on, and informal education that is continuous educational activities including private lessons, organized sessions for OJT and workshops.

not so low, the participation rate of formal education is the lowest among major countries. This suggests that Japanese government should promote formal type of recurrent education.

2.2 The relation between recurrent education and higher education

This section surveys whether tertiary education and recurrent education are complements or substitutes by focusing on the participation rate of recurrent education and tertiary education in each advanced country. If tertiary education and recurrent education are complements, those who completed tertiary education should take recurrent education. In contrast, if tertiary education and recurrent education are substitutes, those who did not complete tertiary education should take recurrent education.

Figure 3 shows the participation rates of three categories of educational level in each country in OECD (2003), i.e. high level, middle level and low level of educational attainment by normalizing the participation rate of each country to be one. According to this figure, people with higher educational level tend to take recurrent education. This suggests that tertiary education and recurrent education have positive correlation.

However, from an international point of view, this positive correlation is not necessarily observed. For example, Figure 4 shows the ordering of participation rate in recurrent education of each country by its level of tertiary education in OECD (2015). Both figures focus on the students who are enrolled in higher educational institutions for the first time. In particular, the right figure restricts the students under 25 years old excluding international students [2]. Comparing Figure 1 and Figure 4, participation rate in recurrent education is not necessarily high for the countries with higher level of education. This is true for Finland, Sweden, Netherlands, the U.S., the U.K., Germany, Austria, and Spain. This suggests that tertiary education and recurrent education do not necessarily have positive correlation.

As stated above, although participation rate in tertiary education and recurrent education have positive correlation, participation rate in recurrent education is not necessarily high for the country with high level of education from an international point of view. This suggests that the relation between recurrent education and tertiary education vary from country to country.

2.3 Actual programs of recurrent education

This subsection surveys the actual programs of recurrent education to investigate the relation between tertiary education and recurrent education.

[2] OECD (2015) defines tertiary education as the education system that provides an opportunity to learn the system of special academic fields based on secondary education. Higher educational institutions include community colleges and universities.

2.3.1 Recurrent education in other countries

Sweden had introduced the recurrent educational program named "Adult Education Initiative (AEI)" for 5 years since 1997 to reduced unemployed workers by half by 2000. AEI was a substitute of tertiary education in that it provided an educational opportunity for those who had not completed 3-year senior high school. The evaluation of the program is not necessarily positive. For example, Stenberg (2005) showed that the program reduces unemployed workers, while it prolongs the duration of unemployment. Stenberg et al. (2012) also showed that recurrent education at the level of secondary education for middle-aged workers did not have a significant impact on the retirement age.

On the other hand, Vignoles et al. (2004) studied the impacts of recurrent education related to the jobs provided by employers by using the database called "National Child Development Study (NCDS) ." This paper showed that the employees tended to provide an opportunity of recurrent education with workers who had higher skill or productivity, and that the wages of workers who took recurrent education significantly increase. This result suggests that recurrent education complementary to tertiary education is effective.

OECD (2003) pointed out that since employers in many countries tend to selected employees to take recurrent education, elder part-time workers with low skill at small firms cannot receive support. This suggests that when the choice of taking recurrent education is left to the private sector, the level of recurrent education which is substitute of higher education is not optimal.

2.3.2 Recurrent education in Japan

As in section 2.1, the level of recurrent education in Japan is not high from an international point of view. However, in recent years, some universities have introduced recurrent educational programs for working generations and retired female workers. The Ministry of Education, Culture, Sports, Science and Technology (2016) surveyed such programs. For example, the "smart career program" of Meiji University provides retired women with knowledge of marketing and finance and business skills through lectures by female CEOs, seminars, group discussions, and so on. The one-year program named "Recurrent education Re-employment" of Japan Woman's University provides retired women with immediately adoptable business skills such as English, IT literacy, knowledge of finance and accounting. The program named "Iwate Agriculture frontier school" providing lectures and training by practitioner such as research institutes, agriculture organization, and advanced farmers aims to nurture innovative and productive farm managers who want to activate local farm business. The survey also reports other programs including "Special Course on Global Cyber Security" of Tokyo Denki University and "Education, Training and Research Programs for Nanoscience and Nanotechnology" of Osaka University.

These recurrent educational programs in Japan are complementary to higher education, since the main target is those who have already completed higher education. However, some programs are substitutes in that they target retired workers whose skill or knowledge depreciated.

3 Outline of the model

As showed above, there are two types of recurrent education: a substitute type and a complementary type. The model in the next section analyzes whether the level of recurrent education is optimal or not in an aging society when the decision of participation in recurrent education is left to the individual by focusing on the relation between tertiary education and recurrent education.

In the following analysis, we adopt a three-period OLG model where a new generation is born every period, and the young, the middle, and the old coexist in the same period. Each individual basically chooses the level of consumption and education based on a life-cycle model, but he dies from the middle to the old with probability q. This means that as q decreases, the speed of aging increases.

In the model, the timing and the cost of taking education are the key factors. We consider three types of education: primary education, higher education and recurrent education. Each person succeeds human capital from the previous generation through primary education at the beginning of the young. He takes tertiary education when young, takes recurrent education when middle, and then retires. Primary education increases human capital of the young, while tertiary education and recurrent education increases human capital of the middle. With regard to the cost, we assume that primary education does not incur any cost since primary education is compulsory and its cost is covered by public expenditure. In contrast, tertiary education and recurrent education incur some costs. We assume that tertiary education incurs no pecuniary cost, but causes disutility from an opportunity cost of decreasing working hours by taking education. On the other hand, recurrent education incurs no disutility from an opportunity cost of decreasing working hours by taking education, but causes pecuniary cost, since the cost is not covered by public expenditure.

Hence, we assume that the young allocates a fraction of their time to higher education, and the rest of their time to labor. We also assume that the middle take recurrent education with pecuniary cost. Each person chooses the levels of tertiary education and recurrent education to maximize his lifetime utility, which increases human capital of the middle.

Under these settings, we analyze the impacts of a decline in mortality rate at the beginning of the old q, i.e. aging accompanying longevity, on the levels of tertiary education and recurrent education, and human capital accumulation.

In general, the progress of aging affects human capital accumulation. It should be noted, however, that its impact on each agent's incentive to take education depends on a type of educational cost. In other words, if disutility occurs as an opportunity cost accompanying a decline in working hours, aging promotes human capital accumulation by decreasing educational cost. This is because as mortality rate declines, the weight of consumption of the old increases, which gives an incentive to an agent to take education to increase income of the middle, even if opportunity cost increases. Hence, the level of tertiary education voluntarily increases in an aging society.

In contrast, if the cost depends only on pecuniary cost, aging does not strength an incentive to take education. This is because agents take education if an increase in the income of the old is bigger than pecuniary costs. Hence, in the following analysis, we show that whether the level of recurrent education voluntarily increases in an aging society is unclear.

In the model, whether aging increases the level of recurrent education chosen by each agent depends on whether tertiary education and recurrent education are complementary or substitutes. In other words, if tertiary education and recurrent education are complements, aging increases the level of recurrent education, while if they are substitutes, aging decreases the level of recurrent education. This is because aging always increases higher education, aging increases the benefit of taking recurrent education in a complementary case. In contrast, aging may decrease the level of recurrent education since the benefit of taking recurrent education for agents who have already attained higher level of tertiary education is low.

In general, the relation between tertiary education and recurrent education is unclear, since it depends on a country and times. As stated above, OECD pointed out the importance of substitute type of recurrent education for workers who did not take enough higher education.

This is also true for such workers in Japan. However, such workers seem to be minority. It should be noted that if skills or knowledge acquired in the past depreciates over time, complementary recurrent education is important even for Japan.

4 Model

4.1 Lifetime utility

The following three period OLG model extends a consumption loan model of Samuelson (1958) to three periods. Time is discrete and goes from zero to infinity, but each agent lives for finite periods. Every period, N_t agents are born. Let the constant $n > -1$ denote the birth rate. As in Yaari (1965), agents die with probability q at the beginning of the old. The probability q is mortality rate, and a decline in q means aging accompanying longevity.

Life time expected utility function $U(c_t^y, c_{t+1}^m, c_{t+2}^o)$ born in period t is written as follows:

$$U(c_t^y, c_{t+1}^m, c_{t+2}^o) = \ln c_t^y + \delta \ln(1-\lambda_t) + \beta \ln c_{t+1}^m + \beta^2(1-q) \ln c_{t+2}^o. \tag{1}$$

where $c_t^y, c_{t+1}^m, c_{t+2}^o$ are period t consumption of the young born in period t, period $t+1$ consumption of the middle born in period t, and period $t+2$ consumption of the old born in period t, and β is discount factor. Besides, $\delta \ln(1-\lambda_t)$ means utility when agents born in period t allocate a constant proportion λ_t of working hours of the young. Since each agent survives with probability $1-q$ at the beginning of the old, the utility from consumption in period $t+2$ by the old born in period $t+2$, $\ln c_{t+2}^o$, is multiplied by $1-q$. Each agent maximizes this lifetime expected utility under the lifetime budget constraint.

4.2 the process of human capital accumulation

In the following analysis, we assume that goods are produced only from human capital each period. Human capital is accumulated through primary education of the young and recurrent education of the middle. We assume that the young succeed a fraction γ of human capital h_t of the middle (their parents) born in period $t-1$ through primary education without any cost. On the other hand, it takes some cost to take tertiary education and recurrent education.

Regarding higher education, agents need not to pay any pecuniary cost similar to primary education. However, the young born in period t allocate a fraction $\lambda_t (0 < \lambda_t < 1)$ of each total time normalized to 1 to tertiary education and the rest to labor. It follows that when the young take higher education, their income decreases by reducing working hours and disutility occurs. We call such disutility as "disutility from taking higher education."

On the other hand, regarding recurrent education, disutility from taking education does not occur, unlike higher education. However, when they take recurrent education$\epsilon_{t+1} > 0$, the young in period t have to pay pecuniary cost $\eta \epsilon_{t+1}$.

When the young take higher education, their human capital increases in the middle, which increases the income of the middle. In addition, when the middle take recurrent education, human capital immediately increases, which increases the income of the middle.

In the following analysis, we assume that human capital of the young born in period t, γh_t, increases to $\gamma \psi(\lambda_t, \epsilon_{t+1}) h_t$. We assume that ψ is increasing in λ_t and ϵ_{t+1} and concave, and satisfies $\psi(\lambda_t, \epsilon_{t+1}) h_t \geq 1$. Hence, the dynamics of human capital is written as follows.

$$h_{t+1} = \psi(\lambda_t, \epsilon_{t+1}) h_t. \tag{2}$$

4.3 Budget constraint

The following budget constraints include two types of costs accompanying human capital accumulation, i.e. disutility from taking higher education and pecuniary cost of recurrent education. Since disutility from taking tertiary education does not affect the budget, disutility from taking tertiary education does not appear in the budget constraints.

In period t, the young born in period $t$receive wages w_t per human capital, takes tertiary education λ_t, borrows b_t^y from the middle born in period $t-1$, and consume c_t^y. Hence, the budget constraint of the young born in period t is written as follows.

$$(1-\lambda_t)w_t\gamma h_t + b_t^y = c_t^y. \tag{3}$$

In period $t+1$, the middle born in period t receive w_{t+1} per human capital, pay pecuniary cost $\eta\epsilon_{t+1}$ to take recurrent education $\epsilon_{t+1} > 0$, repay the debt from the old born in period $t-1$, lend b_{t+1}^m to the young born in period $t+1$, and consume c_{t+1}^m. Let r_{t+1} denote the real interest rate in period $t+1$. Hence, the budget constraint of the middle born in period t is written as follows.

$$\gamma w_{t+1}\psi(\lambda_t, \epsilon_{t+1})h_t = c_{t+1}^m + (1+r_{t+1})b_t^y + b_{t+1}^m + \eta\epsilon_{t+1}h_t. \tag{4}$$

In period $t+2$, in period $t+2$, the old born in period t receives the repayment from the middle born in period $t+1$, and receives assets from the same generation as long as they survive. Hence, the budget constraint of the old born in period t is given as follows.

$$\frac{(1+r_{t+2})}{1-q}b_{t+1}^m = c_{t+2}^o. \tag{5}$$

Equations (3), (4), and (5) give the following lifetime budget constraint.

$$\begin{aligned} &c_t^y + \frac{c_{t+1}^m}{1+r_{t+1}} + \frac{(1-q)c_{t+2}^o}{(1+r_{t+1})(1+r_{t+2})} = \Omega_t, \\ &\text{where } \Omega_t \equiv (1-\lambda_t)w_t\gamma h_t + \frac{\gamma w_{t+1}\psi(\lambda_t, \epsilon_{t+1})h_t}{1+r_{t+1}} - \frac{\eta\epsilon_{t+1}h_t}{1+r_{t+1}}, \end{aligned} \tag{6}$$

Ω_t is the discounted present value of the expected lifetime income of the generation born in period t.

4.4 maximization concerning consumption and education

Each agent maximizes his expected lifetime utility (1) under the lifetime budget constraint (6). Its first-order condition gives the following equation concerning optimal consumption $(c_t^y, c_{t+1}^m, c_{t+2}^o)$ and optimal education level $(\lambda_t, \epsilon_{t+1})$ as follows.

$$
\begin{aligned}
c_t^y &= \frac{\Omega_t}{1+\beta+\beta^2(1-q)}, \\
c_{t+1}^m &= \frac{\beta(1+r_{t+1})\Omega_t}{1+\beta+\beta^2(1-q)}, \\
c_{t+2}^o &= \frac{\beta^2(1+r_{t+1})(1+r_{t+2})\Omega_t}{1+\beta+\beta^2(1-q)}, && (7) \\
\frac{\delta}{1-\lambda_t} &= \frac{1}{c_t^y}\{-w_t\gamma h_t + \frac{\gamma w_{t+1}\psi_\lambda h_t}{1+r_{t+1}}\}, && (8) \\
\gamma w_{t+1}\psi_\epsilon &= \eta, && (9)
\end{aligned}
$$

where each generation decides consumption level according to the life-cycle hypothesis.

Equation (7) shows that the generation born in period t chooses the level of consumption level to smooth their lifetime consumption, given the discount rate, the interest rate, and mortality rate.

Equation (8) describes a decision making concerning higher education of the young born in period t. The left side of (8) means that disutility from taking tertiary education occurs by reducing working hours by $1-\lambda_t$. On the other hand, the right side of (8) means that when the young born in period t takes higher education, the income of the young decreases, while human capital increases, and then, the income of the middle increases. Hence, equation (8) shows that the young born in period t choose the level of tertiary education λ_t to equalize the cost of tertiary education consisting of disutility from taking tertiary education and an opportunity cost of reducing the income of the young, and the benefit of an increases in the income of the middle born in period t.

Equation (9) describes a decision making concerning recurrent education of the middle born in period t. The left side of (9) means that as soon as the middle born in period t takes recurrent education, their human capital immediately increases and the income of the middle born in period t increases. On the other hand, the right side of (9) shows that the middle born in period t pays pecuniary cost to take recurrent education. Hence, (9) shows that the middle born in period t decides the level of recurrent education ϵ_{t+1} to equalize the benefit of an increase in the income of the middle born in period t and the cost of taking recurrent education.

4.5 Market clearing condition

Human capital accumulated through tertiary education and recurrent education is homogeneous, although tertiary education and recurrent education are complements or substitutes. Then, the total amount of human capital is the sum of per capita human capital of the young born in period t, $(1-\lambda_t)\delta\psi(\lambda_{t-1}, \epsilon_t)h_{t-1}$, and per capita human capital of the middle born in period $t-1$, $\psi(\lambda_{t-1}, \epsilon_t)h_{t-1}$. Hence, the total amount of human capital H_t supplied in labor market in period t is written as follows.

$$\begin{aligned} H_t &= N_t(1-\lambda_t)\delta\psi(\lambda_{t-1}, \epsilon_t)h_{t-1} + N_{t-1}\psi(\lambda_{t-1}, \epsilon_t)h_{t-1} \\ &= N_{t-1}\{(1+n)(1-\lambda_t)\delta + 1\}\psi(\lambda_{t-1}, \epsilon_t)h_{t-1} = N_{t-1}\{(1+n)(1-\lambda_t)\delta + 1\}h_t. \end{aligned} \tag{10}$$

Firms produce goods only from human capital. The production function per capita $f(h_t)$ is assumed to be written as follows.

$$f(h_t) = Ah_t. \tag{11}$$

As of this production function, marginal productivity per human capital is always constant at A. Hence, the optimal wage rate w_t to balance the labor market is determined by the labor demand, regardless of the total amount of human capital as follows.

$$w_t = A. \tag{12}$$

On the other hand, the goods market clearing condition is given as follows.

$$c_t^y + \frac{c_t^m}{1+n} + \frac{(1-q)c_t^o}{(1+n)^2} = (1-\lambda_t)A\gamma h_t + \frac{\gamma A\psi(\lambda_{t-1}, \epsilon_t)h_{t-1}}{1+n} - \frac{\eta\epsilon_t h_{t-1}}{1+n}, \tag{13}$$

where the left side of (13) is the sum of consumption of the young born in period t, the middle born in period $t-1$, and the old born in period $t-2$ normalized by population. On the other hand, the right side of (13) is the sum of income of the young born in period t, the middle born in period $t-1$, and the old born in period $t-2$ normalized by population.

It should be noted that the equilibrium interest rate r_t is determined to satisfy the goods market equilibrium condition each period, since savings do not appear in the model. Then, we investigate the equilibrium real interest rate to equalize the supply and demand of the goods market.

First, substituting the optimal consumption (7), the dynamics of human capital (2), and the equilibrium wage rate (12) into the demand of goods market (13), the left side if (13) is given as follows.

$$\text{Left side of (13)} = \frac{1}{1+\beta+\beta^2(1-q)}[(1-\lambda_t)A\gamma h_t + \frac{\gamma A\psi(\lambda_t, \epsilon_{t+1})h_t}{1+r_{t+1}} - \frac{\eta\epsilon_{t+1}h_t}{1+r_{t+1}}$$
$$+\frac{\beta(1+r_t)}{1+n}\{\frac{(1-\lambda_{t-1})A\gamma h_t}{\psi(\lambda_{t-1},\epsilon_t)} + \frac{\gamma A h_t}{1+r_t} - \frac{\eta\epsilon_t h_t}{(1+r_t)\psi(\lambda_{t-1},\epsilon_t)}\}$$
$$+\frac{(1-q)\beta^2(1+r_{t-1})(1+r_t)}{(1+n)^2}$$
$$\times\{\frac{(1-\lambda_{t-2})A\gamma h_t}{\psi(\lambda_{t-2},\epsilon_{t-1})\psi(\lambda_{t-1},\epsilon_t)} + \frac{\gamma A h_t}{(1+r_{t-1})\psi(\lambda_{t-1},\epsilon_t)} - \frac{\eta\epsilon_{t-1}h_t}{(1+r_{t-1})\psi(\lambda_{t-1},\epsilon_t)\psi(\lambda_{t-2},\epsilon_{t-1})}\}].$$

Next, substituting the dynamics of human capital (2) into the supply of goods market, the right side of (13) is given as follows.

$$\text{Right side of (13)} = (1-\lambda_t)A\gamma h_t + \frac{A\gamma h_t}{1+n} - \frac{\eta\epsilon_t h_t}{(1+n)\psi(\lambda_{t-1},\epsilon_t)}.$$

The equilibrium real interest rate r_t is determined to equalize the left side and the right side if equation (13). It should be noted that the derivation of r_t to equalize the left side and the right side is complicated. Then, in the following analysis, we focus on the steady-state equilibrium, in which human capital h_t increases at the constant rate $\psi(\lambda, epsilon)$, but educational level λ, ϵ, and the wage rate w, the real interest rate r is constant.

5 Equilibrium conditions of the steady-state equilibrium

In the following section, we restrict the steady-state equilibrium. In this equilibrium, the life-time income Ω_t and consumption $(c_t^y, c_{t+1}^m, c_{t+2}^o)$ are proportionately grow to human capital.

5.1 Definition of the steady-state equilibrium

The steady-state equilibrium is given as the sequence of endogenous variables $\{c_t^y, c_{t+1}^m, c_{t+2}^o, \lambda, \epsilon, h_t, w, r\}$ that satisfies each agent's optimization and market clearing conditions. In other words, each agent determines the optimal consumption $(c_t^y, c_{t+1}^m, c_{t+2}^o)$ according to (7), and the optimal level of tertiary education λ^* and recurrent education ϵ^* according to (8) and (9). Human capital is accumulated according to (2). The wage rate w satisfies (12). The real interest rate satisfies (14). The goods market clearing condition is given by (13). The labor market clearing condition is given by (10).

In the following analysis, we derive the equilibrium interest rate r in the steady state. In the steady state, the left side and the right side of (13) are given respectively as follows.

$$\text{Left side of (13)} = A\gamma h_t\{(1-\lambda) + \frac{\psi}{1+r} - \frac{\eta\epsilon}{(1+r)A\gamma}\}\frac{1+\frac{\beta(1+r)}{\psi(1+n)}+\frac{(1-q)\beta^2(1+r)^2}{\psi^2(1+n)^2}}{1+\beta+\beta^2(1-q)},$$
$$\text{Right side of (13)} = A\gamma h_t\{(1-\lambda) + \frac{1}{1+n} - \frac{\eta\epsilon}{(1+n)\psi A\gamma}\}.$$

The equilibrium interest rate equalizes the left side and the right side of equation (13). Hence, the steady-state real interest rate is given as follows.

$$1+r = (1+n)\psi. \tag{14}$$

Equation (14) shows that the interest rate depends on the population growth rate and the growth rate of human capital. The result that the interest rate depends on population growth rate is the same as that of Samuelson. It should be noted that the growth rate of human capital affects the interest rate in this model, since the growth rate of human capital has the same impact as population growth rate.

6 Effects of a decline in mortality rate on recurrent education

This section analyzes the effects of a decline in mortality rate on recurrent education. First, utilizing the equilibrium wage (12) and the equilibrium interest rate (14), the optimal condition of tertiary education (8) is written as follows.

$$\frac{\delta}{1-\lambda} = \frac{1}{\tilde{c}^y}\{-A\gamma + \frac{A\gamma\psi_\lambda}{(1+n)\psi}\},$$
$$\text{where } \tilde{c}^y = \frac{c_t^y}{h_t} = \frac{1}{1+\beta+\beta^2(1-q)}\{(1-\lambda)A\gamma + \frac{A\gamma}{1+n} - \frac{\eta\epsilon}{(1+n)\psi(\lambda,\epsilon)}\}.$$

Here, $\tilde{c}^y$ is constant in the steady state, since the optimal consumption $\tilde{c_t}^y$ is given by (7) and the optimal lifetime income Ω_t is given by (6).

We define the following implicit function given ϵ^* as follows by utilizing the above equation concerning the optimal condition of higher education to analyze the effects of a decline in mortality rate on higher education.

$$L(\lambda,\epsilon,q) \equiv \delta\tilde{c}^y + (1-\lambda)\{A\gamma - \frac{A\gamma\psi_\lambda}{(1+n)\psi}\} = 0. \tag{15}$$

On the other hand, by utilizing (12), the optimal condition of recurrent education in the steady state is given as follows.

$$\gamma A\psi_\epsilon = \eta.$$

We define the following implicit function given λ^* as follows by utilizing the above equation concerning the optimal condition of recurrent education to analyze the effects of a decline in mortality rate on recurrent education.

$$M(\lambda, \epsilon) \equiv A\psi_\epsilon - \eta = 0. \tag{16}$$

We analyze the effects of a decline in mortality rate on tertiary education and recurrent education by utilizing implicit functions concerning tertiary education and recurrent education, equations (15) and (16). By totally differentiating (15) and (16), the effects of a decline in mortality rate on tertiary education and recurrent education are given as follows.

$$\begin{bmatrix} \frac{d\lambda}{dq} \\ \frac{d\epsilon}{dq} \end{bmatrix} = -\begin{bmatrix} \frac{\partial L}{\partial \lambda} & \frac{\partial L}{\partial \epsilon} \\ \frac{\partial M}{\partial \lambda} & \frac{\partial M}{\partial \epsilon} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial q} \\ \frac{\partial M}{\partial q} \end{bmatrix}, \tag{17}$$

where $\frac{\partial M}{\partial q} = 0$ from (16). In other words, since mortality rate does not affect decision making concerning recurrent education, (17) is rewritten as follows.

$$\begin{bmatrix} \frac{d\lambda}{dq} \\ \frac{d\epsilon}{dq} \end{bmatrix} = -\frac{1}{\frac{\partial L}{\partial \lambda}\frac{\partial M}{\partial \epsilon} - \frac{\partial L}{\partial \epsilon}\frac{\partial M}{\partial \lambda}} \begin{bmatrix} \frac{\partial M}{\partial \epsilon} \\ -\frac{\partial M}{\partial \lambda} \end{bmatrix} \frac{\partial L}{\partial q}, \text{ where} \tag{18}$$

$$\frac{\partial M}{\partial \lambda} = A\psi_{\epsilon\lambda},$$

$$\frac{\partial M}{\partial \epsilon} = A\psi_{\epsilon\epsilon} < 0,$$

$$\frac{\partial L}{\partial q} = \delta\frac{\partial \tilde{c}^y}{\partial q} = \frac{\delta\beta^2\tilde{c}^y}{1+\beta+\beta^2(1-q)} > 0. \tag{19}$$

In the following analysis, we investigate the sign of the denominator of (18) to analyze the effects of a decline in mortality rate q on tertiary education and recurrent education, i.e. the signs of $d\lambda/dq$ and $d\epsilon/dq$. Each term in (19) is expanded as follows.

$$
\begin{aligned}
\frac{\partial L}{\partial \lambda} &= \delta\frac{\partial \tilde{c}^y}{\partial \lambda} + \frac{A\gamma\psi_\lambda}{(1+n)\psi} - \frac{(1-\lambda)A\gamma}{1+n}\frac{\partial(\frac{\psi_\lambda}{\psi})}{\partial \lambda} \\
&= [-\frac{\delta\eta}{\{1+\beta+\beta^2(1-q)\}(1+n)\psi}\frac{d\epsilon}{d\lambda} + \frac{\delta\eta}{\{1+\beta+\beta^2(1-q)\}(1+n)}\frac{\epsilon\psi_\lambda}{\psi^2}] \\
&+[\frac{A\gamma\psi_\lambda}{(1+n)\psi} - \frac{(1-\lambda)A\gamma}{1+n}\frac{\psi_{\lambda\lambda}\psi-\psi_\lambda^2}{\psi^2}], \\
\frac{\partial L}{\partial \epsilon} &= \delta\frac{\partial \tilde{c}^y}{\partial \epsilon} + \frac{d\lambda}{d\epsilon}\frac{A\gamma\psi_\lambda}{(1+n)\psi} - \frac{(1-\lambda)A\gamma}{1+n}\frac{\partial(\frac{\psi_\lambda}{\psi})}{\partial \epsilon} \\
&= [-\frac{\delta\eta}{\{1+\beta+\beta^2(1-q)\}(1+n)}\frac{\psi-\epsilon\psi_\epsilon}{\psi^2}] + [\frac{d\lambda}{d\epsilon}\frac{A\gamma\psi_\lambda}{(1+n)\psi} - \frac{(1-\lambda)A\gamma}{1+n}\frac{\psi_{\epsilon\lambda}\psi-\psi_\epsilon\psi_\lambda}{\psi^2}].
\end{aligned} \tag{20}
$$

The first term of the right side of (18), $\frac{1}{\frac{\partial L}{\partial \lambda}\frac{\partial M}{\partial \epsilon}-\frac{\partial L}{\partial \epsilon}\frac{\partial M}{\partial \lambda}}$, is given as follows.

$$
\begin{aligned}
\frac{1}{\frac{\partial L}{\partial \lambda}\frac{\partial M}{\partial \epsilon}-\frac{\partial L}{\partial \epsilon}\frac{\partial M}{\partial \lambda}} &= [\frac{A\delta\eta}{\{1+\beta+\beta^2(1-q)\}(1+n)\psi}(\psi_{\epsilon\lambda}-\psi_{\epsilon\epsilon}\frac{d\epsilon}{d\lambda}) \\
&+\frac{A\delta\eta\epsilon}{\{1+\beta+\beta^2(1-q)\}(1+n)\psi^2}(\psi_\lambda\psi_{\epsilon\epsilon}-\psi_\epsilon\psi_{\epsilon\lambda})] \\
&+[\frac{A^2\gamma\psi_\lambda}{(1+n)\psi}(\psi_{\epsilon\epsilon}-\psi_{\epsilon\lambda}\frac{d\lambda}{d\epsilon}) + \frac{(1-\lambda)A^2\gamma}{(1+n)\psi^2}(\psi_{\epsilon\lambda}^2\psi-\psi_\epsilon\psi_\lambda\psi_{\epsilon\lambda}-\psi_{\epsilon\epsilon}\psi_{\lambda\lambda}\psi+\psi_{\epsilon\epsilon}\psi_\lambda^2)] \\
&= [\frac{A\delta\eta\epsilon}{\{1+\beta+\beta^2(1-q)\}(1+n)\psi^2}(\psi_\lambda\psi_{\epsilon\epsilon}-\psi_\epsilon\psi_{\epsilon\lambda})] \\
&+[\frac{(1-\lambda)A^2\gamma}{(1+n)\psi^2}(\psi_{\epsilon\lambda}^2\psi-\psi_\epsilon\psi_\lambda\psi_{\epsilon\lambda}-\psi_{\epsilon\epsilon}\psi_{\lambda\lambda}\psi+\psi_{\epsilon\epsilon}\psi_\lambda^2)],
\end{aligned} \tag{21}
$$

where since $\psi_{\epsilon\lambda}^2 < \psi_{\epsilon\epsilon}\psi_{\lambda\lambda}$, as long as $\psi_\lambda \leq (\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})\psi_\epsilon$, $\frac{1}{\frac{\partial L}{\partial \lambda}\frac{\partial M}{\partial \epsilon}-\frac{\partial L}{\partial \epsilon}\frac{\partial M}{\partial \lambda}} < 0$ holds. Hence, except for the case where the absolute value of $\psi_{\epsilon\lambda}(< 0)$ is extremely large, i.e. the substitutability of tertiary education and recurrent education is extremely high, $\frac{1}{\frac{\partial L}{\partial \lambda}\frac{\partial M}{\partial \epsilon}-\frac{\partial L}{\partial \epsilon}\frac{\partial M}{\partial \lambda}} < 0$ holds.

It should be noted that the sufficient condition $\psi_\lambda \leq (\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})\psi_\epsilon$ means that the indirect effect of recurrent education on output $(\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})\psi_\epsilon$ is not smaller than the direct effect of tertiary education on output ψ_λ. It follows that unless the substitutability of tertiary education and recurrent education is extremely high, the indirect effect on output through recurrent education is equal to or bigger than the direct effect of tertiary education on output.

As a result, regarding the sign of $d\lambda/dq$, utilizing $\partial M/\partial\epsilon < 0$ and $\partial L/\partial q > 0$ of (20), except for the case where the absolute value of $\psi_{\epsilon\lambda} < 0$ is extremely large, $d\lambda/dq < 0$ always holds. Hence, in the case where recurrent education is introduced, as long as tertiary education and recurrent education are not strong substitutes, as mortality rate q declines, tertiary education is always promoted.

In contrast, regarding the sign of $d\epsilon/dq$, utilizing $\frac{\partial M}{\partial\lambda} = A\psi_{\lambda\epsilon}$ of (20), from (18), if tertiary education and recurrent education are complements, $d\epsilon/dq < 0$ always holds, while if tertiary education and recurrent education are substitutes, as long as the substitutability is not extremely high, recurrent education is unlikely to be accumulated in an aging society.

The above argument shows that a decline in mortality rate always promotes tertiary education in the case where recurrent education does not exist. Moreover, in the case where recurrent education exists, except for the case where tertiary education and recurrent education are strong substitutes, a decline in mortality rate promotes higher education. Given this result, if tertiary education and recurrent education are complementary, a decline in mortality rate always promote recurrent education, while if tertiary education and recurrent education are substitutes, a decline in mortality rate always decreases recurrent education.

7 Effect of a decline in mortality rate on human capital accumulation

This section analyzes the effect of a decline in mortality rate on human capital accumulation, based on the results in the last section. From the dynamics of human capital (2), the effect of mortality rate on the growth of human capital is written as follows.

$$\frac{d(h_{t+1}/h_t)}{dq} = \gamma\{\psi_\lambda(\lambda^*,\epsilon^*)\frac{d\lambda^*}{dq} + \psi_\epsilon(\lambda^*,\epsilon^*)\frac{d\epsilon^*}{dq}\} = \gamma\frac{d\lambda}{dq}(\psi_\lambda - \frac{\psi_\epsilon\psi_{\epsilon\lambda}}{\psi_{\epsilon\epsilon}}). \tag{22}$$

As showed in the last section, except for the case whether the absolute value of $\psi_{\epsilon\lambda} < 0$ is extremely large, $d\lambda/dq < 0$ always holds. Hence, the sign of $d(h_{t+1}/h_t)/dq$ depends on the sign of $\psi_\lambda - (\psi_\epsilon\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})$. Since the sign of $\psi_\lambda - (\psi_\epsilon\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})$ is always positive in a complementary case, i.e. $\psi_{\epsilon\lambda} > 0$, a decline in mortality rate always promotes human capital accumulation. On the other hand, in a strongly substitute case, i.e. $\psi_{\epsilon\lambda} < 0$ the sign of $\psi_\lambda - (\psi_\epsilon\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})$ is indeterminate. In particular, if the absolute value of $\psi_{\epsilon\lambda} < 0$ is huge, a decline in mortality rate hinders human capital accumulation.

In the following analysis, we investigate the effects of a decline in mortality rate q on tertiary education λ^*, recurrent education ϵ^*, and the growth of human capital H_{t+1}/H_t by taking a numerical example in cases where tertiary education and recurrent education are complements and substitutes.

The growth of human capital in the steady state is written as follows from the dynamics of human capital (2) and the total amount of human capital (10).

$$\frac{H_{t+1}}{H_t} = \frac{N_t\{(1+n)(1-\lambda^*)\delta+1\}h_{t-1}}{N_{t-1}\{(1+n)(1-\lambda^*)\delta+1\}h_t} = (1+n)\psi(\lambda^*,\epsilon^*). \tag{23}$$

Hence, the impact of a decline in mortality rate on the growth of total human capital $\frac{H_{t+1}}{H_t}$ is determined by its impact on $\psi(\lambda^*,\epsilon^*)$ through tertiary education λ^* and recurrent education ϵ^*.

7.1 Complementary case

This section analyzes the case where tertiary education and recurrent education are complementary and the production function is Cobb-Douglas function $\psi(\lambda^*,\epsilon^*) = P_1\lambda^{*\alpha}\epsilon^{*\alpha}$. In this case, ψ_λ, ψ_ϵ, $\psi_{\epsilon\epsilon}$, and $\psi_{\epsilon\lambda}$ are given as follows.

$$\begin{aligned}
&\psi_\lambda = P_1\alpha\lambda^{*\alpha-1}\epsilon^{*1-\alpha},\\
&\psi_\epsilon = P_1(1-\alpha)\lambda^{*\alpha}\epsilon^{*-\alpha},\\
&\psi_{\epsilon\epsilon} = -P_1\alpha(1-\alpha)\lambda^{*\alpha}\epsilon^{*-\alpha-1},\\
&\psi_{\epsilon\lambda} = P_1\alpha(1-\alpha)\lambda^{*\alpha-1}\epsilon^{*-\alpha}.
\end{aligned}$$

Since $\psi_\lambda \leq (\psi_\epsilon\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})$, $d\lambda/dq < 0$ holds. In other words, a decline in mortality rate increases higher education. Moreover, since tertiary education and recurrent education are complementary, $d\epsilon/dq < 0$ holds. In other words, a decline in mortality rate increases recurrent education.

The level of tertiary education and recurrent education in the steady state (λ^*,ϵ^*) satisfy the following equations (24) and (25) from the implicit functions regarding the optimization of tertiary education and recurrent education (15) and (16).

$$\frac{\delta}{1+\beta+\beta^2(1-q)}\{(1-\lambda)\gamma + \frac{\gamma}{1+n} - \frac{1-\alpha}{1+n}\} + (1-\lambda)\gamma\{1 - \frac{\alpha}{(1+n)\lambda}\} = 0, \tag{24}$$

$$AP_1(1-\alpha)\gamma(\frac{\lambda}{\epsilon})^\alpha - \eta = 0. \tag{25}$$

In the following numerical example, we assume that $n = 0.1, \alpha = 0.7, \beta = 0.8, \gamma = 0.9, \delta = 0.9, A = 0.2, P_1 = 1, \eta = 0.1, q = [0, 0.2]$. The effects of a decline in mortality rate q on tertiary education λ^*, recurrent education ϵ^*, and the growth rate of total human capital H_{t+1}/H_t that satisfy (24) and (25) are depicted in Figure 5, Figure 6, Figure 7, respectively.

Figure 5 depicts the level of higher education. This figure shows that a decline in mortality rate increases tertiary education and $d\lambda/dq < 0$ holds. Figure 6 depicts the level of recurrent education. This figure shows that a decline in mortality rate increases recurrent education and $d\epsilon/dq < 0$ holds in the complementary case. Figure 7 depicts the growth rate of total human capital. This figure shows that a decline in mortality rate increases the growth rate of total human capital. These results mean that a decline in mortality rate increases both tertiary education and recurrent education and hence total human capital in the complementary case. In such an economy, when the choice to take recurrent education is left to the private sector, total human capital is accumulated even in an aging society.

7.2 Perfectly substitute case

This section analyzes the case where tertiary education and recurrent education are perfectly substitutes and the production function is given by $\psi(\lambda^*, \epsilon^*) = P_2(\lambda^* + \epsilon^*)^\alpha$. In this case, ψ_λ, ψ_ϵ, $\psi_{\epsilon\epsilon}$, and $\psi_{\epsilon\lambda}$ are given as follows.

$$\begin{aligned}
\psi_\lambda &= P_2\alpha(\lambda^* + \epsilon^*)^{\alpha-1},\\
\psi_\epsilon &= P_2\alpha(\lambda^* + \epsilon^*)^{\alpha-1},\\
\psi_{\epsilon\epsilon} &= -P_2\alpha(1-\alpha)(\lambda^* + \epsilon^*)^{\alpha-2},\\
\psi_{\epsilon\lambda} &= -P_2\alpha(1-\alpha)(\lambda^* + \epsilon^*)^{\alpha-2}.
\end{aligned}$$

Since $\psi_\lambda = (\psi_\epsilon\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})$, $d\lambda/dq < 0$ holds. In other words, a decline in mortality rate increases higher education. Moreover, since tertiary education and recurrent education are perfectly substitutes, $d\epsilon/dq > 0$ holds. In other words, a decline in mortality rate decreases recurrent education.

The level of tertiary education and recurrent education in the steady state (λ^*, ϵ^*) satisfy the following equations (26) and (27) from the implicit functions regarding the optimization of tertiary education and recurrent education (15) and (16).

$$\begin{aligned}
&\frac{\delta}{1+\beta+\beta^2(1-q)}\{(1-\lambda)A\gamma + \frac{A\gamma}{1+n} - \frac{\eta\epsilon}{(1+n)P_2(\lambda+\epsilon)^\alpha}\}\\
&+(1-\lambda)A\gamma\{1 - \frac{\alpha}{(1+n)(\lambda+\epsilon)}\} = 0, \qquad (26)\\
&AP_2\alpha(\lambda+\epsilon)^{\alpha-1} - \eta = 0. \qquad (27)
\end{aligned}$$

In the following numerical example, we assume that $n = 0.1, \alpha = 0.7, \beta = 0.8, \gamma = 0.6, \delta = 0.7, A = 0.2, P_2 = 0.54, \eta = 0.1, q = [0, 0.2]$. The effects of a decline in mortality rate q on tertiary education λ^*, recurrent education ϵ^*, and the growth rate of total human capital H_{t+1}/H_t that satisfy (26) and (27) are depicted in Figure 8, Figure 9, Figure 10, respectively.

Figure 8 depicts the level of higher education. This figure shows that a decline in mortality rate increases tertiary education and $d\lambda/dq < 0$ holds. Figure 9 depicts the level of recurrent education. This figure shows that a decline in mortality rate increases recurrent education and $d\epsilon/dq > 0$ holds in the perfectly substitute case. Figure 10 depicts the growth rate of total human capital. This figure shows that a decline in mortality rate does not affect the growth rate of total human capital. These results mean that a decline in mortality rate increases higher education, but decreases recurrent education, which offsets the impact of higher education. Hence, total human capital does not increase in the perfectly substitute case. In such an economy, when the choice to take recurrent education is left to the private sector, the optimal level of recurrent education for economic growth cannot be achieved in an aging society. In this case, government should support recurrent education.

7.3 Substitute case

This section analyzes the case where tertiary education and recurrent education are substitutes and the production function is given by $\psi(\lambda^*, \epsilon^*) = P_3(\lambda^* + \epsilon^*)^\alpha + \chi\epsilon^*$. In this case, ψ_λ, ψ_ϵ, $\psi_{\epsilon\epsilon}$, and $\psi_{\epsilon\lambda}$ are given as follows.

$$
\begin{aligned}
\psi_\lambda &= P_3\alpha(\lambda^* + \epsilon^*)^{\alpha-1}, \\
\psi_\epsilon &= P_3\alpha(\lambda^* + \epsilon^*)^{\alpha-1} + \chi, \\
\psi_{\epsilon\epsilon} &= -P_3\alpha(1-\alpha)(\lambda^* + \epsilon^*)^{\alpha-2}, \\
\psi_{\epsilon\lambda} &= -P_3\alpha(1-\alpha)(\lambda^* + \epsilon^*)^{\alpha-2}.
\end{aligned}
$$

Since $\psi_\lambda = (\psi_\epsilon\psi_{\epsilon\lambda}/\psi_{\epsilon\epsilon})$, $d\lambda/dq < 0$ holds. In other words, a decline in mortality rate increases higher education. Moreover, since tertiary education and recurrent education are substitutes, $d\epsilon/dq > 0$ holds. In other words, a decline in mortality rate decreases recurrent education.

The level of tertiary education and recurrent education in the steady state (λ^*, ϵ^*) satisfy the following equations (28) and (29) from the implicit functions regarding the optimization of tertiary education and recurrent education (15) and (16).

$$
\begin{aligned}
&\frac{\delta}{1+\beta+\beta^2(1-q)}[(1-\lambda)A\gamma + \frac{A\gamma}{1+n} - \frac{\eta\epsilon}{(1+n)\{P_3(\lambda+\epsilon)^\alpha + \chi\epsilon\}}] \\
&+(1-\lambda)A\gamma + [1 - \frac{P_3\alpha(\lambda+\epsilon)^{\alpha-1}}{(1+n)\{P_3(\lambda+\epsilon)^\alpha + \chi\epsilon\}}] = 0, \qquad (28) \\
&A\{P_3\alpha(\lambda+\epsilon)^{\alpha-1} + \chi\} - \eta = 0. \qquad (29)
\end{aligned}
$$

In the following numerical example, we assume that $n = 0.1, \alpha = 0.5, \beta = 0.8, \gamma = 0.6, \delta = 0.7, A = 0.2, P_3 = 0.54, \eta = 0.1, \chi = 0.02, q = [0, 0.2]$. The effects of a decline in mortality rate q on tertiary education λ^*, recurrent education ϵ^*, and the growth rate of total human capital H_{t+1}/H_t that satisfy (28) and (29) are depicted in Figure 11, Figure 12, Figure 13, respectively.

Figure 11 depicts the level of higher education. This figure shows that a decline in mortality rate increases tertiary education and $d\lambda/dq < 0$ holds. Figure 12 depicts the level of recurrent education. This figure shows that a decline in mortality rate increases recurrent education and $d\epsilon/dq > 0$ holds in the substitute case. Figure 13 depicts the growth rate of total human capital. This figure shows that a decline in mortality rate decreases the growth rate of total human capital. These results mean that a decline in mortality rate increases higher education, but decreases recurrent education, which exceeds the impact of higher education. Hence, total human capital decreases in the substitute case. In such an economy, when the choice to take recurrent education is left to the private sector, the optimal level of recurrent education for economic growth cannot be achieved in an aging society. In this case, government should support recurrent education.

8 Conclusion

This paper studies whether the optimal level of human capital for economic growth is attained in an aging society, when the choice of taking recurrent education is left to the private sector. We showed that even in an aging society, whether human capital is automatically accumulated through recurrent education is unclear. In other words, if tertiary education and recurrent education are complements, aging promotes human capital accumulation thorough both tertiary education and recurrent education, while if tertiary education and recurrent education are substitutes, aging increases higher education, but decreases recurrent education, and hence hinders human capital accumulation.

In Japan, utilization of labor force of the retired elderly and retired female workers is more important in rapid aging and depopulation. To tackle this problem, promoting recurrent education, especially at formal educational institutions is becoming a critical policy issue, since the participation rate in recurrent education is relatively low from an international point of view.

In Japan, the level of tertiary education is already high. However, since human capital depreciates over time, skills and knowledge acquired in the past may not be useful. This suggests that we need not only tertiary education but also recurrent education for the elderly and retired female workers for economic growth.

When we examine this problem, it is important to consider whether tertiary education and recurrent education are complements or substitutes. It goes without saying that educational infrastructure is important for promoting recurrent education. However, even when the educational infrastructure is sufficiently established, if each worker has no incentive to take recurrent education, recurrent education will not be promoted. In this case, government should implement some policies to encourage people to take recurrent education.

This paper did not explicitly analyze the Japanese traditional employment system such as lifetime employment system and seniority wage system. These institutional factors may affect investment in human capital by firms. For example, traditional Japanese firms have invested in firm-specific human capital through OJT[3]. But, such firm-specific human capital disappears when workers retires. Hence, government should consider how to encourage retired workers who work part-time to take recurrent education such as reemployment after retirement age and tax deduction for firms to promote recurrent education.

Besides, in recent years, irregular workers and workers who changed their careers are increasing. Such workers may not to be given an opportunity to take recurrent education from firms. It would be important to investigate how recurrent education for irregular workers and workers who changed their careers should be by introducing heterogeneity of workers into the model.

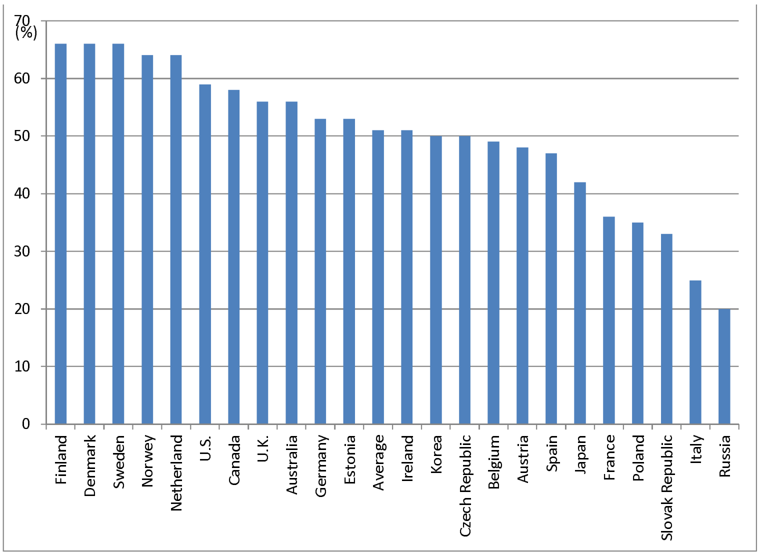


Figure 1: Participation in recurrent education of each country (OECD, 2014)

[3]A setting of the model that recurrent education of the middle incurs pecuniary cost, while tertiary education of the young does not incur pecuniary cost is interpreted to reflect the cost structure under which junior workers take OJT in exchange for lower wages than their productivity, while senior workers receive higher wages that their productivity under the seniority wage system.

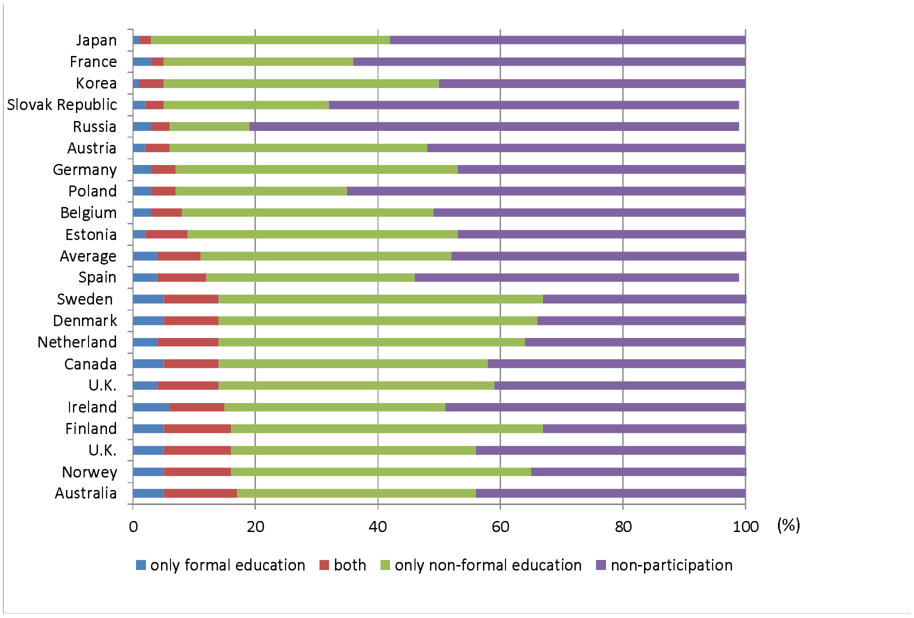
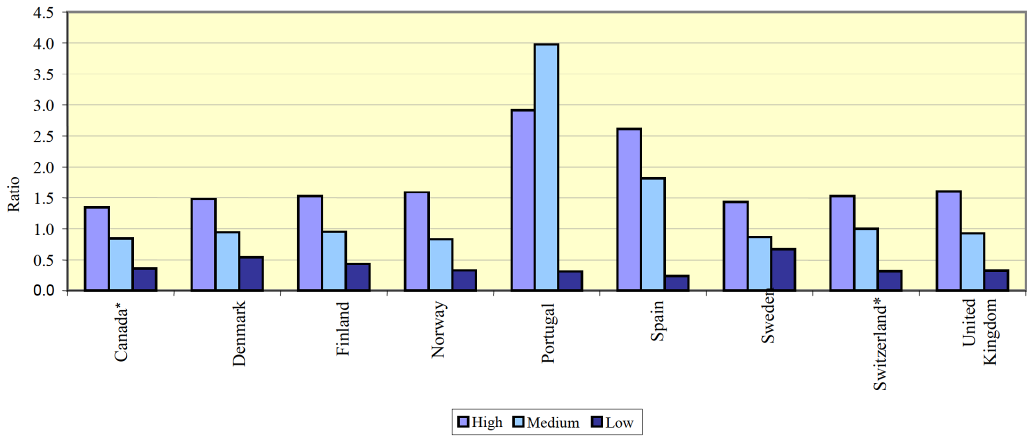


Figure 2: Participation rates of each country in 4 cases (OECD, 2014)



Note: A ratio superior to 1 implies that the proportion of persons in that category is above the average country participation rate; a ratio between 0 and 1 below the average rate.
* Period of reference is four weeks except for Canada and Switzerland, where it is one year.
Source: Eurostat, European Union Labour Force Survey data except for Canada (1997 AETS data).

Figure 3: Participation rates of three categories of educational level in each country (OECD, 2003)

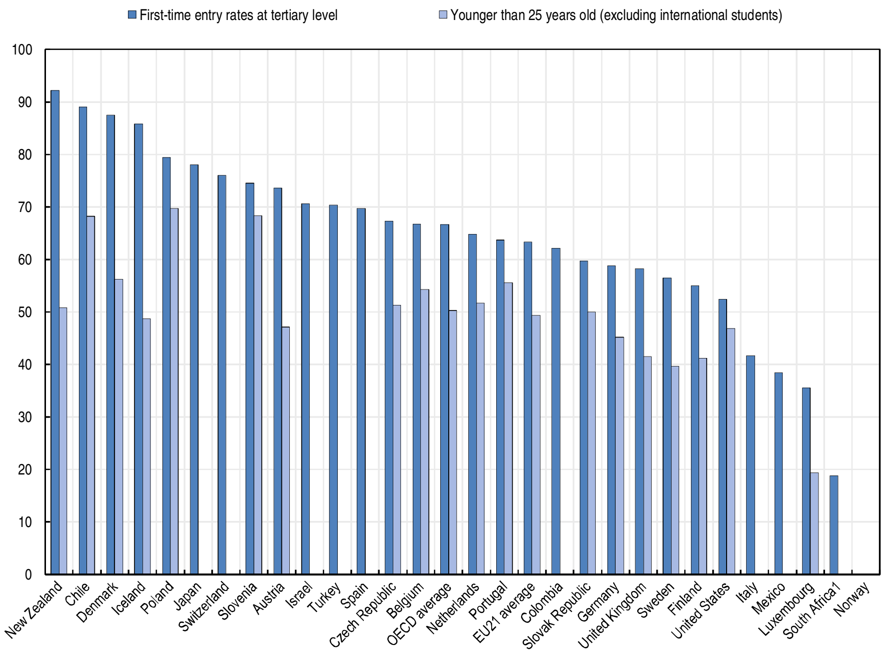


Figure 4: Participation rates in tertiary education of each country (OECD, 2015)

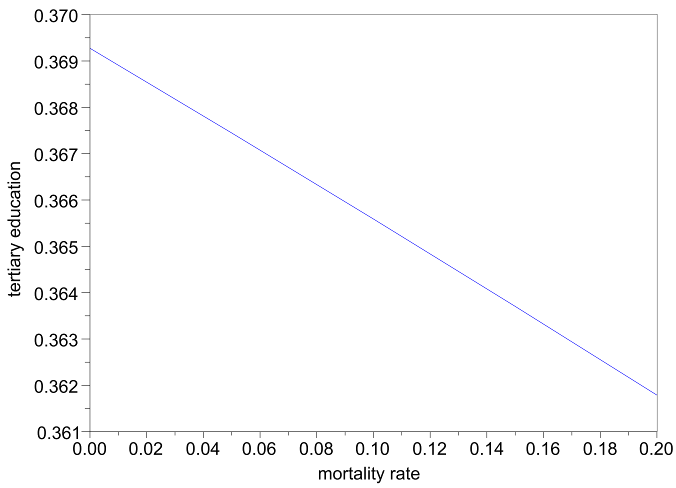


Figure 5: Effects of morality rate on higher education: Cobb-Douglas type complementary case

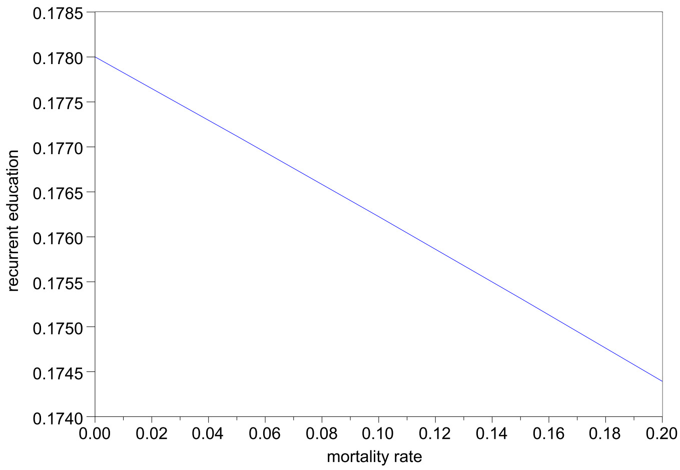


Figure 6: Effect of mortality rate on recurrent education: Cobb-Douglas type complementary case

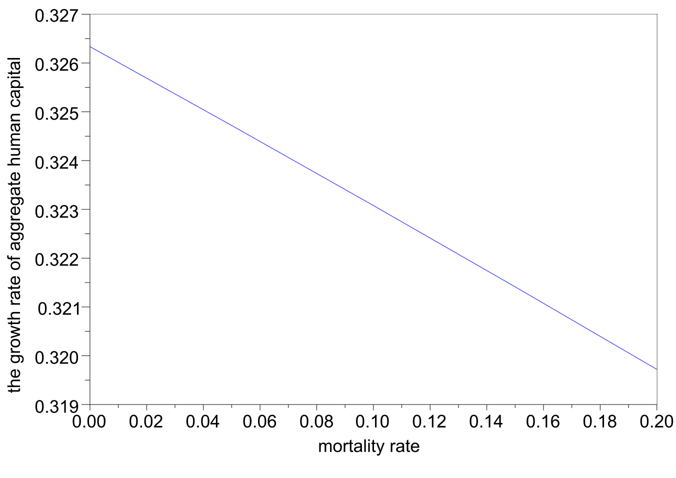


Figure 7: Effect of mortality rate on the growth of human capital: Cobb-Douglas type complementary case

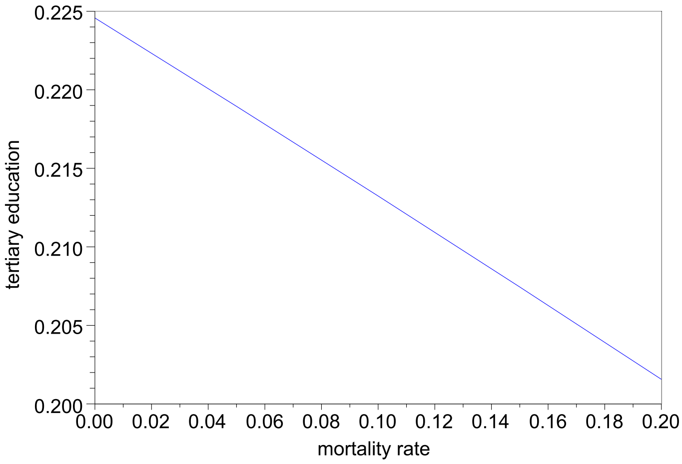


Figure 8: Effect of morality rate on higher education: perfectly substitute case

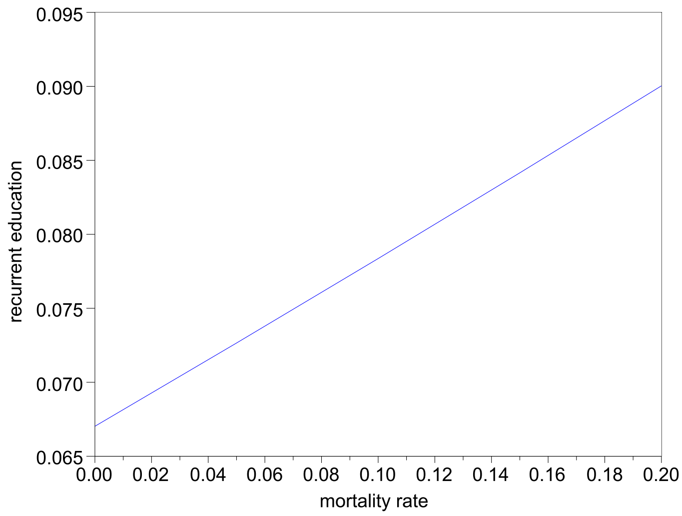


Figure 9: Effect of mortality rate on recurrent education: perfectly substitute case

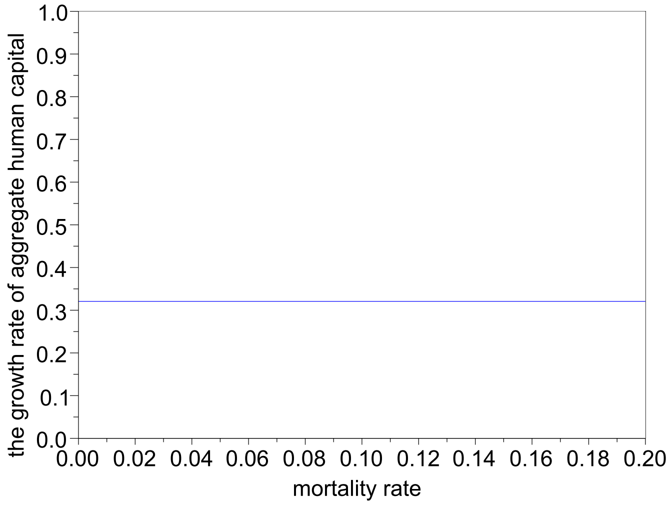


Figure 10: Effect of mortality rate on the growth of human capital: perfectly substitute case

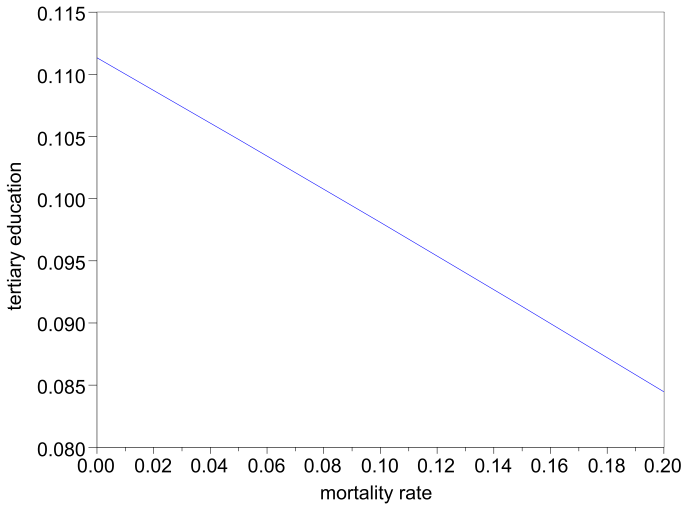


Figure 11: Effect of morality rate on higher education: substitute case

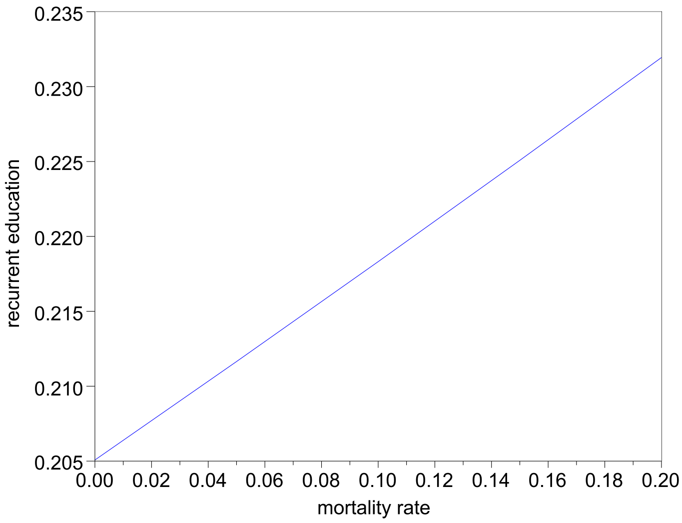


Figure 12: Effect of mortality rate on recurrent education: substitute case

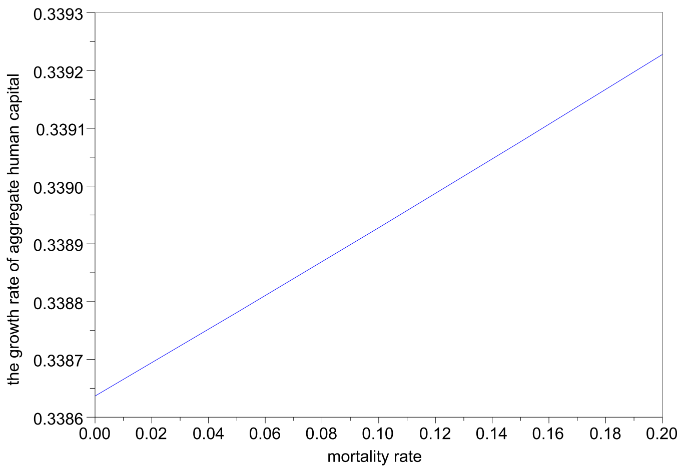


Figure 13: Effect of mortality rate on the growth of human capital: substitute case

Reference

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