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Abstract: We propose a job matching model of intermediary labor markets by developing the seminal work of Kelso and Crawford (1982, *Econometrica* 50:1483-1504). Using this model, we show that for an arbitrary fixed broker-fee rate, the salary-adjustment process converges to a core allocation in intermediary labor markets where high-skilled workers are matched to high-technology firms by the private middleman and low-skilled workers are matched to low-technology firms by the public middleman. This result means that the dual labor market is emerged as a stable outcome of job-matching promoted by the private and public middlemen. Finally, we discuss empirical relevance of our theoretical model by using the data of job placement services in Japan.

Keywords: Job matching; Private middleman; Public middleman; Dual labor market; Core

JEL classification: C78; J01

1 Introduction

In recent years, the economic design of labor markets is receiving increased attention among economists (for a survey, see Roth 2008). In particular, there is increasing the literature of this topic from the view of matching theory (for instance, see Hatfield, Kojima, and Kominers 2014). Although the matching theoretic literature provides various implications toward the design of bilateral labor markets (i.e. two sided labor markets between workers and firms), the economic design of intermediary labor markets is not fully analyzed. For instance, from 2007 to 2008, in Japan, the government did market testing of public employment services. In the market testing, public employment services showed higher performance for job seekers than intermediary agents. In those days, the government presumed that public employment services and intermediary agents play an alternative role. However, there may be a possibility that public employment services and intermediary agents are complements to each other. If so, coexistence of the private and public middlemen may support healthy labor markets. In order to design health labor markets with middlemen, we need understanding of the relation between roles of private/public middlemen and labor markets. Therefore, in the present study, we aim to demonstrate how private/public middlemen contribute to healthy labor markets from the view of matching theory.[1]

For the purpose, we develop the model proposed by Kelso and Crawford (1982), which is the seminal paper of a job matching model in bilateral labor markets. The job matching model proposed here describes the situation where (i) there is asymmetric information between workers and firms, and (ii) there are a private middleman and a public middleman. We explain our model briefly as follows.

There are high-skilled workers and low-skilled workers. Each worker knows his reservation value, but does not know marginal productivity of firms. We allow for heterogeneity of utility functions of each typed workers.

High-technology firms and low-technology firms exist in the market. Each firm does not know each worker's type as well as each worker's reservation value. Each high-technology firm does not know its marginal productivity. This is because each high-technology firm has a production function that depends on the number of high-skilled workers that it hires, and each high-

[1]The notion of healthy markets is developed from the view of "market quality economics", proposed by Yano (2009).

technology firm does not know who are high-skilled workers. On the other hand, each low-technology firm knows its marginal productivity. This is because each low-technology firm has a production function that depends on the number of workers that it hires. We allow for heterogeneity of production functions of all the firms.

Each middleman helps firms with their match to workers under asymmetric information mentioned above. In the existing literature, the role of middlemen is illuminated as an information expert or a matchmaker. Information experts are agents who smooth transaction by eliminating asymmetry of information between sellers and buyers (for instance, see Biglaiser 1993). Matchmakers are agents who match sellers to buyers by eliminating transaction costs between sellers and buyers (in the search theoretic literature, for instance, see Rubinstein and Wolinsky 1987; in the general equilibrium theoretic literature, for instance, see Oishi and Sakaue 2014, Oishi 2016). In the present study, we focus on the situation where the role of middlemen is both information experts and matchmakers.

In the present study, we introduce two middlemen: a private middleman (i.e. an intermediary agent) and a public middleman (i.e. a public employment service). The private middleman and the public middleman can identify each high-skilled worker as well as each high-technology firm. Each middleman incurs constant marginal costs for measuring types of them. We assume that transaction cost of the private middleman who matches firms that utilize him to workers is relatively small. We also assume that the private middleman's matching skill for measuring high types of firms/workers is relatively higher than the public middleman for measuring high types of them. Also, the private middleman and the public middleman can identify each low-skilled worker, and each low-technology firm, without any measuring cost of them. We assume that all the firms and all the workers know who is the private/public middleman and what is matching skill of the two middlemen. Under asymmetric information structure, each middleman matches a worker to a firm. By matching worker w to firm f, the private middleman gains his gross profit $r \cdot s_{wf}$ where s_{wf} is a salary that firm f pays to worker w and $r \in (0, 1)$ is a broker-fee rate. In the present model, the rate r is exogenously given by the government since this payment scheme is employed in intermediary labor markets in practice. On the other hand, the public middleman's gross profit is given by subsidy. We assume that the capacity of the number of workers that all the firms that utilize the public middleman hire depends on the net profit of the public middleman, which is the differ-

ence between the subsidy and the transaction cost. The government aims to attain no existence of involuntary unemployment with the minimal subsidy. In the present model, the end of the government is toward the second best, but it is often observed in labor market policy in practice.

Using the present model of intermediary labor markets, we demonstrate that the dual labor market is emerged as a stable outcome of job-matching promoted by the private and public middlemen. More strictly, under economic conditions of each firms' marginal productivity, each worker's reservation value, and gross substitutes, for an arbitrary fixed broker-fee rate, a salary-adjustment process converges to a core allocation in intermediary labor markets where high-skilled workers are matched to high-technology firms by the private middleman and low-skilled workers are matched to low-technology firms by the public middleman. In labor economics, the dual labor market is a market where high-skilled workers are matched to high-technology firms and low-skilled workers are matched to low-technology firms. Unfortunately, as far as we know, the existing literature of labor economics has not uncovered the relation between middlemen and emergence of the dual labor market, for instance, see Cahuc, Carcillo,and Zylberberg (2014). However, an economic view of emergence of the dual labor market is highly useful for designing intermediary labor market policy. For instance, whether the public employment service should be replaced by the intermediary agent is unclear without such an economic view as demonstrated in the present model. We shed a light on this interesting topic by modifying the salary-adjustment process (i.e. an algorithm yielding a core allocation between workers and firms), originally proposed by Kelso and Crawford (1982).

In order to consider empirical relevance of our theoretical model, section 4 of this paper gives a brief overview of the labor market and middlemen in Japan. We take up two indicators of the density of high-skilled workers in each region, namely the share of college graduates and the average wage in the region. We show first that the density of high-skilled workers correlates positively with the density of private, for-profit middlemen per population. This fact is consistent with our model in the sense that, in our model as well as in Japanese labor law, the private middlemen are required to derive their profits from the fee they charge proportionally to the wages of the workers whom they manage to match with firms. Thus, the higher the density of high-skilled workers in a region (such as Tokyo), the more profit opportunities for middlemen to exploit there. We also look at the performance measures of public middlemen, namely the matching rate and the vacancy-filling rate of

the public employment offices in each region. We show that the lower regional density of high-skilled workers is associated with the higher performance of public middlemen. From these findings, coupled with other existing evidence, we argue that private, fee-charging middlemen are primarily concerned with high-skilled workers in urban areas, whereas public middlemen offer important help for low-skilled workers, especially in non-urban, economically weak areas. To this extent, private and public middlemen are complements, not substitutes.

The paper is organized as follows. In Section 2, we explain the model of intermediary labor markets. In Section 3, we establish convergence of a core allocation in intermediary labor market. In Section 4, empirical relevance of our theoretical model is investigated. In Section 5, we discuss the relation between Myerson and Satterthwaite (1983) and the present study. Finally, in this section, we demonstrate a justification of a key assumption of our model by using theoretical/empirical evidence.

2 Model

Let W, M, and F be the set of workers, the set of middlemen, and the set of firms, respectively. Let w be an arbitrary index of W, i.e., $w \in W$. Similarly $m \in M$, and $f \in F$.

1: Workers

The set of workers consists of high-skilled workers and low-skilled workers. The set of **high-skilled workers** is W_H, and the set of **low-skilled workers** is W_L, that is, $W \equiv W_H \cup W_L$. Each worker knows his type $T_w \in \{L, H\}$, where L means low type, and H means high type.

Worker w's utility of working for firm f at salary s_{wf} is given by

$$u^w(f; s_{wf}),$$

where firm f pays $s_{wf} \in \mathbb{Z}_+$ to worker w. We allow for heterogeneity of workers' utilities. The utility function is continuous and strictly increasing in s_{wf}.

Let worker w's reservation value for firm f be given by σ_{wf} such that

$$u^w(f; \sigma_{wf}) = u^w(\emptyset; 0).$$

Notice that σ_{wf} may depend on f. We allow for heterogeneity of workers' reservation values.

Each worker does not know each firm's marginal productivity since he does not know each firm's technology.

2: Firms

The set of firms consists of high technology firms and low technology firms. The set of **high-technology firms** is F_H, and the set of **low-technology firms** is F_L, that is, $F \equiv F_H \cup F_L$. Let f_H be an arbitrary index of F_H, i.e., $f_H \in F_H$. Similarly, $f_L \in F_L$. Let C^f be the set of the workers that firm $f \in F$ hires.

Firm f pays $r_{m_1} s_{wf}$, where $r_{m_1} \in (0,1)$, to a **private middleman (i.e. intermediary agent)** m_1 if the middleman m_1 matches the firm f to a worker. This payment scheme is often observed in practical labor markets. The rate r_{m_1} is often determined by the government in practice. From the stylized fact, in our model, the rate $r_{m_1} \in (0,1)$ is fixed arbitrarily.

On the other hand, firm f pays $r_{m_2} s_{wf}$, where $r_{m_2} = 0$, to a **public middleman (i.e. public employment service)** m_2 if the middleman m_2 matches the firm f to a worker w. Let $r \in \{r_{m_1}, r_{m_2}\}$, that is, $r \in \{r_{m_1}, 0\}$.

Firm f_H's production function is $y^{f_H}(|\{w \in C^{f_H} : T_w = H\}|)$. This means that f_H's production depends on the number of high-skilled workers that firm f_H hires. If the members of C^{f_H} are only low-skilled workers, then $y^{f_H}(\cdot) \equiv 0$. We allow for heterogeneity of high-technology firms' production functions. We assume that y^{f_H} is continuous and strictly increasing in the number of high-skilled workers that firm f_H hires.

Firm f_L's production function is $y^{f_L}(|C^{f_L}|)$. This means that f_L's production depends on the number of workers that firm f_L hires. We allow for heterogeneity of low-technology firms' production functions. We assume that y^{f_L} is continuos and strictly increasing in the number of workers that firm f_L hires.

Notice that firm f's gross product is measured by the same unit as the s_{wf}.

Firm f_H's net profit, denoted π^{f_H}, is given by

$$\pi^{f_H}(C^{f_H}; (T_w)_{w \in C^{f_H}}; s_{f_H}) \equiv y^{f_H}(|\{w \in C^{f_H} : T_w = H\}|) - \sum_{w \in C^{f_H}} (1+r) s_{w f_H},$$

where $r \in \{r_{m_1}, 0\}$, and $s_{f_H} \equiv (s_{wf_H})_{w \in W}$ is the vector of salaries faced by firm f_H.

Firm f_L's net profit, denoted π^{f_L}, is given by

$$\pi^{f_L}(C^{f_L}; s_{f_L}) \equiv y^{f_L}(|\{w \in C^{f_L}\}|) - \sum_{w \in C^{f_L}} (1+r) s_{wf_L},$$

where $r \in \{r_{m_1}, 0\}$, and $s_{f_L} \equiv (s_{wf_L})_{w \in W}$ is the vector of salaries faced by firm f_L.

We assume that for each $f_H \in F_H$, and each $f_L \in F_L$,

$$y^{f_H}(1) \geq y^{f_L}(|C^{f_L}|).$$

This assumption means that high technology firm is more productive than low technology firm.

Let firm f_H's marginal productivity with respect to hiring worker $w \notin C^{f_H}$, denoted ρ^{f_H}, be given by

$$\rho^{f_H}(w; C^{f_H}) \equiv y^{f_H}(|i \in C^{f_H} \cup \{w\} : T_i = H|) - y^{f}(|i \in C^{f_H} : T_i = H|).$$

Let firm f_L's marginal productivity with respect to hiring worker $w \notin C^{f_L}$, denoted ρ^{f_L}, be given by

$$\rho^{f_L}(w; C^{f_L}) \equiv y^{f_L}(|C^{f_L} \cup \{w\}|) - y^{f_L}(|C^{f_L}|).$$

Each firm $f_H \in F_H$ does not know his marginal productivity with respect to hiring worker $w \notin C^{f_H}$ since he does not know each worker's type. On the other hand, each firm $f_L \in F_L$ knows his marginal productivity with respect to hiring worker $w \notin C^{f_L}$ since it is not necessary to know each worker's type.

Each firm does not know each worker's type as well as each worker's reservation value.

3: Middlemen

We consider two middlemen, that is $M \equiv \{m_1, m_2\}$. By m_1, we denote the "private middleman" (i.e. intermediary agent), and by m_2, we denote "public middleman" (i.e. public employment service). Let C^{m_1} be the set of

the firms that utilize middleman m_1. Similarly, C^{m_2} is the set of the firms that utilize middleman m_2.

Let middleman m_1's gross profit for helping firm $f \in C^{m_1}$ with its match to workers $w \in C^f$ be given by $v^{m_1}(wf; r_{m_1} s_{wf})$, which is continuous and strictly increasing in s_{wf}. We assume that $v^{m_1}(\emptyset; 0) \equiv 0$ if m_1 does not match any firm to any worker.

Each middleman $m_i \in M$ $(i = 1, 2)$ can identify each high-skilled worker, but he incurs constant marginal cost $K_i^H > 0$ per unit for measuring high skill of workers. Also, middleman $m_i \in M$ can identify each high-technology firm, but he incurs constant marginal cost $\tilde{K}_i^H > 0$ per unit for measuring high technology of firms. On the other hand, middleman $m_i \in M$ can identify each low-skilled worker, and each low-technology firm, without any transaction cost (i.e. measuring cost), that is, $K_i^L = \tilde{K}_i^L = 0$.

Middleman m_1's net profit, denoted π^{m_1}, is given by

$$\begin{aligned} & \pi^{m_1}(C^{m_1}; s_{wf}) \\ \equiv & \sum_{f \in C^{m_1}} \left[\sum_{w \in C^f} v^{m_1}(wf; r_{m_1} s_{wf}) - |C_f| K_1 - \tilde{K}_1 \right], \end{aligned}$$

where $K_1 \in \{K_1^H, K_1^L\}$ and $\tilde{K}_1 \in \{\tilde{K}_1^H, \tilde{K}_1^L\}$.

Middleman m_2's gross profit is denoted by $\tilde{\pi}^{m_2}$, which is the amount of the subsidy. Middleman m_2's net profit, denoted π^{m_2}, is given by

$$\begin{aligned} & \pi^{m_2}(C^{m_2}) \\ \equiv & \tilde{\pi}^{m_2} - \sum_{f \in C^{m_2}} \left(|C_f| K_2 + \tilde{K}_2 \right), \end{aligned}$$

where $K_2 \in \{K_2^H, K_2^L\}$ and $\tilde{K}_2 \in \{\tilde{K}_2^H, \tilde{K}_2^L\}$.

We denote by $G(\pi^{m_2})$ the capacity of the number of workers that all the firm $f \in C^{m_2}$ hire. The capacity function G is a strictly increasing and continuous function in π^{m_2}. We also assume that $G(0) \equiv 0$.

We assume that the government aims to attain no existence of involuntary unemployment with the minimal subsidy. Notice that the public middleman behaves unlike the profit maximizer.

We are interested in the situation where the private middleman's matching skill is relatively higher than the public middleman. For this view, we assume that the transaction cost of m_1 who matches a firm f to workers in

C^f is relatively small, that is, for an arbitrary fixed $r_{m_1} \in (0,1)$ and for each firm $f \in C^{m_1}$,

$$|W_H|K_1^H + \tilde{K}_1^H \leq v^{m_1}\left(w^* f^*; r_{m_1}\sigma_{w^* f^*}\right),$$

where $(w^*, f^*) \in \arg\min_{(w,f)\in W\times F}\sigma_{wf}$. We also put the following assumption of costs:

$$\max\{K_1^H, \tilde{K}_1^H\} < G^{-1}(\max\{|W_H|, |W_L|\}) < \min\{K_2^H, \tilde{K}_2^H\}.$$

This assumption says that $K_1^H < K_2^H$ and $\tilde{K}_1^H < \tilde{K}_2^H$, which implies that the private middleman's matching skill is relatively higher than the public middleman. It also says that transaction costs K_2^H and $\tilde{K}_2^H$ are relatively high.

We assume that each worker $w \in W$ and each firm $f \in F$ know the role of middlemen m_1 and m_2, respectively.

Furthermore, we assume the following conditions.

A1: High-technology firm's incentive

(i) For all $f_H \in F_H$, all $w_H \in W_H \backslash C^{f_H}$,[2]

$$\rho^{f_H}(w_H; C^{f_H}) \geq \sigma_{w_H f_H}.$$

(ii) For all $f_H \in F_H$, $w_L \in W_L \backslash C^{f_H}$,

$$\sigma_{w_L f_H} > 0.$$

The condition **A1-(i)** means that firm f_H hires high-skilled workers. The condition **A1-(ii)** implies that

$$\rho^{f_H}(w_L; C^{f_H}) = 0 < \sigma_{w_L f_H}.$$

Thus, firm f_H hires no low-skilled worker.

A2: Low-technology firm's incentive

(i) For all $f_L \in F_L$, $w_H \in W_H \backslash C^{f_L}$,

$$\rho^{f_L}(w_H; C^{f_L}) < \sigma_{w_H f_L}.$$

[2] $x \in A\backslash B$ means $x \in A$ and $x \notin B$.

(ii) For all $f_L \in F_L$, $w_L \in W_L \backslash C^{f_L}$,

$$\rho^{f_L}(w_L; C^{f_L}) \geq \sigma_{w_L f_L}.$$

The condition **A2-(i)** means firm f_L hires no high-skilled worker. The condition **A2-(ii)** means firm f_L hires low-skilled worker.

A3: No-free-lunch for firms

For all $f \in F$,

$$y^f(\emptyset) = 0.$$

The condition **A3** means that firm f produces nothing if it hires no worker.

A4: Gross substitutes

Let $M^{f_H}(s_{f_H}) \equiv \arg\max_C \pi^{f_H}(C; (T_w)_{w \in C}; s_{f_H})$ and $M^{f_L}(s_{f_L}) \equiv \arg\max_C \pi^{f_L}(C; s_{f_L})$. For all $f \in F$, if $C^f \in M^f(s_f)$ and $s'_f \geq s_f$, then there exists $\tilde{C}^f \in M^f(s'_f)$ such that

$$\{w \in C^f : \ s'_{wf} = s_{wf}\} \subseteq \tilde{C}^f.$$

The condition **A4** means that each firm never desires to fire a worker $w \in C^f$ such that $s'_{wf} = s_{wf}$ when the salaries of other workers rise.

A5: Lower bound of $\sigma_{w_H f_H}$

For all $f_H \in F_H$, all $f_L \in F_L$, all $w_H \in W_H$, and all $w_L \in W_L \backslash C^{f_L}$,

$$\sigma_{w_H f_H} > \rho^{f_L}(w_L; C^{f_L}).$$

The condition **A5** means that each high-skilled worker's reservation value is greater than firm f_L's marginal productivity with respect to hiring worker $w_L \notin C^{f_L}$. This is a key assumption not appearing in Kelso and Crawford (1982). A justification of the condition **A5** will be stated in Section 5.

Let $W \equiv \{1, 2, \cdots, n_W\}$ and $F = \{1, 2, \cdots, n_F\}$. We denote by $\varphi : W \to F$ a matching function: $\varphi(w)$ is the firm to which worker w is matched, and we allow for $\varphi(w) = \varphi(w')$ for some $w, w' \in W$ and $w \neq w'$. Let C^f be the set of workers hired by firm f, that is, $C^f \equiv \{w : f = \varphi(w)\}$.

An "individually rational allocation", denoted $(\varphi; (s_{1\varphi(1)}, \cdots, s_{n_W\varphi(n_W)}))$, is a pair of the matching function φ and the salary schedule $(s_{1\varphi(1)}, s_{2\varphi(2)}, \cdots, s_{n_W\varphi(n_W)})$ satisfying the following three conditions:

(a): For all $w \in W$,

$$s_{w\varphi(w)} \geq \sigma_{w\varphi(w)}.$$

(b): For all $f_H \in F_H$,

$$\pi^{f_H}(C^{f_H}; (T_w)_{w \in C^{f_H}}; s_{f_H}) \geq 0.$$

For all $f_L \in F_L$,

$$\pi^{f_L}(C^{f_L}; s_{f_L}) \geq 0.$$

(c): For all $w \in W$, and all $f \in F$,

$$\begin{cases} \pi^{m_1}(C^{m_1}; s_{wf}) \geq 0. \\ \pi^{m_2}(C^{m_2}) \geq 0. \end{cases}$$

An individually rational allocation $(\varphi; (s_{1\varphi(1)}, \cdots, s_{n_W\varphi(n_W)}))$ is *blocked* by a tuple $(f, \mathcal{C}; z^f)$, where $(f, \mathcal{C})$ is a pair of firm $f \in F$ and the set of workers that the firm $j \in F$ hires, and $z^f \equiv (z_{1f}, z_{2f}, \cdots, z_{n_W f}) \in \mathbb{Z}_+^{n_W}$ is a salary schedule of firm f, if the following conditions hold:

(i): For all $w \in \mathcal{C}$,

$$u^w(f; z_{wf}) \geq u^w(\varphi(w); s_{w\varphi(w)}),$$

and
(ii): For all $f \in F$,

$$\begin{cases} \pi^f(\mathcal{C}; (T_w)_{w \in \mathcal{C}}; z_f) \geq \pi^f(C^f; (T_w)_{w \in C^f}; s_f) & \text{if } f \in F_H, \\ \pi^f(\mathcal{C}; z_f) \geq \pi^f(C^f; s_f) & \text{if } f \in F_L, \end{cases}$$

with strict inequality holding for at least one member in the set $\{f\} \cup \mathcal{C}$.

Definition 1 *An individually rational allocation* $(\varphi; (s_{1\varphi(1)}, \cdots, s_{n_W\varphi(n_W)}))$ *is a core allocation in intermediary labor markets if it is not blocked by any tuple* $(f, \mathcal{C}; z^f)$.

Middlemen matches many worker to a firm by using the "salary-adjustment process". This process is initially proposed by Kelso and Crawford (1982) in the context of a bilateral labor market. We make a modification of the Kelso-Crawford process in two points. Firstly, the round 0 where each middleman sets a list of potential workers that all firms in C^m may hire is added. Secondary, in each round, firm $f \in C^m$ sets a list of permitted salaries for firm f's potential workers, and a list of no-salary for workers whom the firm f has no incentive to hire.

Formally, the salary-adjustment process in our model is as follows: Middleman m_1 and m_2 computes the following algorithm by using their information:

Round 0: Each middleman sets a list of potential workers that all firms in C^m may hire. Each list is denoted by $\mathcal{W}^{m_1}$ and $\mathcal{W}^{m_2}$, respectively, that is,

$$\begin{aligned} \mathcal{W}^{m_1} &\equiv \{w \in W : w \in \cup_{f \in C^{m_1}} C^f\}, \\ \mathcal{W}^{m_2} &\equiv \{w \in W : w \in \cup_{f \in C^{m_2}} C^f\}. \end{aligned}$$

Round 1: Each middleman consider the following process: Firm $f \in C^m$ sets a list of permitted salaries $(s_{wf}(0))_{w \in \mathcal{W}^m} = (\sigma_{wf})_{w \in \mathcal{W}^m}$ and $(s_{wf}(0))_{w \in W \setminus \mathcal{W}^m} = (0)_{w \in W \setminus \mathcal{W}^m}$, where 0 shows no incentive to hire a worker $w \notin \mathcal{W}^m$.

Each firm makes offers to all workers in $\mathcal{W}^m$. Each worker who receives offers rejects all but his favorite salary, which he tentatively accepts. Workers may break ties at any time however they like.

Round R ($R \geq 2$): Each middleman m compute the matching described below: On each round, each firm makes offers to the members of one of its favorite sets of workers in $\mathcal{W}^m$, given the list of permitted salaries $s^f(R) \equiv (s_{1f}(R), \cdots, s_{kf}(R))$, where $k = |\mathcal{W}^m|$. That is, firm $f \in C^m$ makes offers to the members of $C^f(s^f(R)) \subseteq \mathcal{W}^m$, where $C^f(s^f(R))$ maximizes $\pi_f(C; s^f(R))$. Firms may break ties between sets of workers however they like, with the following exception: Any offer made by firm $f \in C^m$ in round $R-1$ that was not rejected must be repeated in round R. On this round, the firm sacrifices no profits (by **A4**).

Each worker who receives one or more offers rejects all but his favorite salary, which he tentatively accepts. Workers may break tie at any time however they like.

If worker $w \in \mathcal{W}^m$ rejected an offer from firm $f \in C^m$ in round $R-1$, $s_{wf}(R) = s_{wf}(R-1) + 1$; otherwise $s_{wf}(R) = s_{wf}(R-1)$.

On the other hand, on each round, each firm makes offers to the members $w \notin \mathcal{W}^m$, given the list of permitted salaries $s^f(R) \equiv (s_{1f}(R), \cdots, s_{k'f}(R)) = (0)_{w \in W \backslash \mathcal{W}^m}$, where $k' = |W \backslash \mathcal{W}^m|$.

Stop: The process stops when no rejections in $\mathcal{W}^m$ are issued in some period. Workers in $\mathcal{W}^m$ then accept the offers that remain in force from the firm they have not rejected. Thus, each middleman finds the list of permitted salaries $(s_{wf}(t^*))_{w \in \mathcal{W}^m}$, where t^* is the number of the final round, for all workers in $\mathcal{W}^m$.

3 Convergence to a core allocation

We can now establish convergence to a core allocation in intermediary labor market by using the salary-adjustment process mentioned above.

Theorem 1 *For an arbitrary fixed $r_{m_1} \in (0,1)$, the salary-adjustment process converges to a core allocation in intermediary labor markets where high-skilled workers are matched to high-technology firms by the private middleman and low-skilled workers are matched to low-technology firms by the public middleman.*

Proof. We consider four steps for the proof.

Step 1: *At round 0, m_1 sets $\mathcal{W}^{m_1} = W_H$, and m_2 sets $\mathcal{W}^{m_2} = W_L$.*

By **A1** and **A2**, m_H matches f_H to w_H or f_L to w_L. If not, m_H can never gain a positive profit. By the algorithm, for each $f \in C^{m_2}$, $\pi^f(\cdot; s^f(t^*)) \geq y^f(\emptyset) = 0$, where t^* is the round at which the process stops. This implies the fact that all $f_L \in F_L$, and $w_L \in W_L \backslash C^{f_L}$, $\rho^{f_L}(w_L; C^{f_L}) - s_{w_L f_L} \geq 0$. By this fact together with **A5,** $s_{w_L f_L} < \sigma_{w_H f_H}$, which implies that $s_{w_L f_L} < s_{w_H f_H}$ by the algorithm. Thus, m_1 has incentive to match f_H to w_H since he obtains higher payoffs. Next, f_H must go to m_1 since m_2 does not match him to high-skilled workers. This is derived from the fact that for each $k = 1, 2, \cdots, |W_H|$ $G^{-1}(k) < \min\{K_2^H, \tilde{K}_2^H\}$. Thus m_2 makes the non-negative net-profits, which means that m_2 can avoid making existence of involuntary unemployment for high-skilled workers. On the other hand, f_L must go to m_2 since f_L can have no payment to m_2. By the fact that $k' = 1, 2, \cdots, |W_H|$

$G^{-1}(k') < K_2^H$, m_2 does not match f_L to w_H. Thus, m_2 can avoid making existence of involuntary unemployment for high-skilled workers. Since the government aims to attain no existence of involuntary unemployment for low-skilled workers, m_2 matches f_L to w_L. Therefore, at round 0, m_1 sets $\mathcal{W}^{m_1} = W_H$, and m_2 sets $\mathcal{W}^{m_2} = W_L$.

Step 2: *After a finite number of rounds, each worker in $\mathcal{W}^m$ has exactly one offer and the process stops.*

First, we can claim that each worker has at least one offer in each round by using the algorithm. By this observation together with the algorithm and the fact that $y^f(C)$ is finite, we can claim that after a finite number of rounds, each worker has exactly one offer and the process stops. These two claims are the same as in Lemmas 1and 2 in Kelso and Crawford (1982).

Step 3: *The process converges to an individually rational allocation.*

Let t^* be the round at which the process, and let ϕ and C_ϕ^f be the assignment to which it converges. By the algorithm, $s_{w\phi(w)}(t^*) \geq \sigma_{w\phi(w)}$.

Next, by the algorithm together with the assumption that $|W_H|K_1^H + \tilde{K}_1^H \leq v^{m_1}(w^*f^*; r_{m_1}\sigma_{w^*f^*})$, where $(w^*f^*) \in \arg\min_{(w,f)\in W\times F}\sigma_{wf}$, we have that for each $f \in C^{m_1}$ and each $w \in C^f \subset \mathcal{W}^{m_1}$,

$$\begin{aligned} v^{m_1}(wf; r_{m_1}s_{wf}(t^*)) - |C_f|K_1^H - \tilde{K}_1^H &\geq v^{m_1}(w^*f^*; r_{m_1}\sigma_{w^*f^*}) - |C_f|K_1^H - \tilde{K}_1^H \\ &\geq v^{m_1}(w^*f^*; r_{m_1}\sigma_{w^*f^*}) - |W_H|K_1^H - \tilde{K}_1^H \\ &\geq 0, \end{aligned}$$

which implies that $\pi^{m_1}(C^{m_1}; s_{wf}(t^*)) \geq 0$. By the assumption where the government aims to attain no expenditure of involuntary unemployment with the minimal subsidy, we can set $G(\pi^{m_2}) = \sum_{f\in C^{m_2}}|C^f|$ by Step 1. By the definition of G and Step 2, we have that

$$\pi^{m_2} = G^{-1}(\sum_{f\in C^{m_2}}|C^f|) \geq G^{-1}(0) = 0.$$

For each $f \in C^{m_1}$, by the algorithm and the fact that $y^f(\emptyset) = 0$ (by **A3**), $\pi^f(C_\phi^f; (T_w)_{w\in\mathcal{W}^{m_1}}; s^f(t^*)) \geq \pi^f(\emptyset; (T_w)_{w\in\mathcal{W}^m}; s^f(t^*)) = y^f(\emptyset) = 0$. Again, for each $f \in C^{m_2}$, $\pi^f(C_\phi^f; s^f(t^*)) \geq y^f(\emptyset) = 0$.

Step 4: *The process converges to a core allocation in the labor market.*

By Step 2, the process converges to a pair of the matching function φ and the salary schedule, denoted $(\phi; s_{1\phi(1)}, \cdots, s_{n\phi(n)})$. Let C^f_ϕ be the set of workers assigned to firm f by ϕ. Notice that the matching ϕ is composed of $\phi_H : W_H \to F_H$ and $\phi_L : W_L \to F_L$, which is the matching derived from the algorithm.

Case 1: $f \in F_H$.
Subcase 1-1: $\mathcal{C} \subseteq W_H$

Suppose not. That is, $(\phi; s_{1\phi(1)}, \cdots, s_{n\phi(n)})$ is not a *core allocation*. By the fact that $(\phi; s_{1\phi(1)}, \cdots, s_{n\phi(n)})$ is individually rational by Step 3 and the algorithm, there must exist a firm-set of workers combination $(f, \mathcal{C})$ and integer salaries z_f such that

$$u^w(f; z_{wf}) \geq u^w(\phi(w); s_{w\phi(w)}(t^*)) \text{ for each } w \in \mathcal{C},$$

and

$$\pi^f(\mathcal{C}; (T_w)_{w\in\mathcal{C}}; z_f) > \pi^f(C^f; (T_w)_{w\in C^f}; s_f).$$

This implies that worker $w \in \mathcal{C}$ must never have received an offer from firm f at a salary z_{wf} or greater. Since permitted salaries never fall, $s_f(t^*) \leq z_f$. Then,

$$\begin{aligned} \pi^f(\mathcal{C}; (T_w)_{w\in\mathcal{C}}; s_f(t^*)) &\geq \pi^f(\mathcal{C}; (T_w)_{w\in\mathcal{C}}; z_f) \\ &> \pi^f(C^f; (T_w)_{w\in C^f}; s_f(t^*)), \end{aligned}$$

which implies that C^f is never formed by the algorithm, a contradiction.

Subcase 1-2: $\mathcal{C} \subseteq W_H \cup W_L$ with $\mathcal{C} \cap W_L \neq \emptyset$.

Suppose not. There must exist a firm-set of workers combination $(f, \mathcal{C})$ and integer salaries z_f such that

$$u^w(f; z_{wf}) \geq u^w(\phi(w); s_{w\phi(w)}(t^*)) \text{ for each } w \in \mathcal{C},$$

and

$$\pi^f(\mathcal{C}; (T_w)_{w\in\mathcal{C}}; z_f) > \pi^f(C^f; (T_w)_{w\in C^f}; s_f).$$

It suffices to consider two possibilities:

(a) $C^f \cap \mathcal{C} \neq \emptyset$.

Since $\pi^f(\mathcal{C}; (T_w)_{w\in\mathcal{C}}; z_f) > \pi^f(C^f; (T_w)_{w\in C^f}; s_f)$, that is,

$$\begin{aligned} y^f(|\{w \in \mathcal{C} : T_w = H\}|) - \sum_{w\in\mathcal{C}}(1+r_{m_1})z_{wf} \\ > \; y^f(|\{w \in C^f : T_w = H\}|) - \sum_{w\in C^f}(1+r_{m_1})s_{wf}, \end{aligned}$$

for some $\bar{w} \in C^f \cap \mathcal{C}$ $z_{\bar{w}f} < s_{\bar{w}f}$, a contradiction, since permitted salaries never fall, $s_{\bar{w}f}(t^*) \leq z_{\bar{w}f}$.

(b) $C^f \cap \mathcal{C} = \emptyset$.

Let $w \in C^f$. Define a tuple $(f, \mathcal{C}'; \hat{z}^f)$, where $\mathcal{C}' = \mathcal{C} \cup \{w\}$, $\hat{z}_{wf} = s_{wf}$, and $\hat{z}_{w'f} = z_{w'f}$ for all $w' \in W \setminus \{w\}$. Since $\pi^f(\mathcal{C}'; (T_w)_{w\in\mathcal{C}'}; \hat{z}_f) > \pi^f(C^f; (T_w)_{w\in C^f}; s_f)$, that is,

$$\begin{aligned} y^f(|\{w \in \mathcal{C}' : T_w = H\}|) - \sum_{w\in\mathcal{C}}(1+r_{m_1})\hat{z}_{wf} \\ > \; y^f(|\{w \in C^f : T_w = H\}|) - \sum_{w\in C^f}(1+r_{m_1})s_{wf}, \end{aligned}$$

for some $\bar{w} \in C^f \cap \mathcal{C}'$ $\hat{z}_{\bar{w}f} < s_{\bar{w}f}$, a contradiction, since permitted salaries never fall, $s_{\bar{w}f}(t^*) \leq \hat{z}_{\bar{w}f}$.

Case 2: $f \in F_L$.
Subcase 2-1: $\mathcal{C} \subseteq W_L$

Suppose not. By the fact that $(\phi; s_{1\phi(1)}, \cdots, s_{n\phi(n)})$ is individually rational by Step 3 and the algorithm, there must exist a firm-set of workers combination $(f, \mathcal{C})$ and integer salaries z_f such that

$$u^w(f; z_{wf}) \geq u^w(\phi(w); s_{w\phi(w)}(t^*)) \text{ for each } w \in \mathcal{C},$$

and

$$\pi^f(\mathcal{C}; z_f) > \pi^f(C^f; s_f).$$

This implies that worker $w \in \mathcal{C}$ must never have received an offer from firm f at a salary z_{wf} or greater. Since permitted salaries never fall, $s_f(t^*) \leq z_f$. Then,

$$\pi^f(\mathcal{C}; s_f(t^*)) \geq \pi^f(\mathcal{C}; z_f) > \pi^f(C^f; s_f(t^*)),$$

which implies that C^f is never formed by the algorithm.

Subcase 2-2: $\mathcal{C} \subseteq W_L \cup W_H$ with $\mathcal{C} \cap W_H \neq \emptyset$.

Suppose not. There must exist a firm-set of workers combination $(f, \mathcal{C})$ and integer salaries z_f such that

$$u^w(f; z_{wf}) \geq u^w(\phi(w); s_{w\phi(w)}(t^*)) \text{ for each } w \in \mathcal{C},$$

and

$$\pi^f(\mathcal{C}; z_f) > \pi^f(C^f; s_f).$$

Since $\pi^f(\mathcal{C}; z_f) > \pi^f(C^f; s_f)$, that is,

$$y^f(|\{w \in \mathcal{C}\}|) - \sum_{w \in \mathcal{C}} z_{wf} > y^f(|\{w \in C^f\}|) - \sum_{w \in C^f} s_{wf},$$

it suffices to consider three possibilities:

(a) $y^f(|\{w \in \mathcal{C}\}|) > y^f(|\{w \in C^f\}|)$.

There must exist $w \in W_H$ such that

$$\rho^f(w; C^f) > z_{wf} \geq \sigma_{wf},$$

a contradiction, by **A2-(i)**.

(b) $y^f(|\{w \in \mathcal{C}\}|) = y^f(|\{w \in C^f\}|)$ and $C^f \cap \mathcal{C} \neq \emptyset$.

There must hold $z_{\tilde{w}f} < s_{\tilde{w}f}$ for some $\tilde{w} \in C^f \cap \mathcal{C}$, a contradiction, since permitted salaries never fall, $s_{\tilde{w}f}(t^*) \leq z_{\tilde{w}f}$.

(c) $y^f(|\{w \in \mathcal{C}\}|) = y^f(|\{w \in C^f\}|)$ and $C^f \cap \mathcal{C} = \emptyset$.

Let $w \in C^f$. Define a tuple $(f, \mathcal{C}'; \hat{z}^f)$, where $\mathcal{C}' = \mathcal{C} \cup \{w\}$, $\hat{z}_{wf} = s_{wf}$, and $\hat{z}_{w'f} = z_{w'f}$ for all $w' \in W \setminus \{w\}$. There must hold $\hat{z}_{\tilde{w}f} < s_{\tilde{w}f}$ for some $\tilde{w} \in C^f \cap \mathcal{C}'$, a contradiction, since permitted salaries never fall, $s_{\tilde{w}f}(t^*) \leq \hat{z}_{\tilde{w}f}$. *Q.E.D.*

4 Data relevance in Japanese labor markets

In order to consider empirical relevance of our theoretical model, in this section we briefly overview the labor market and middlemen in Japan.

Middlemen or intermediary organizations in labor markets are commonly called employment placement agencies. The main objective of employment

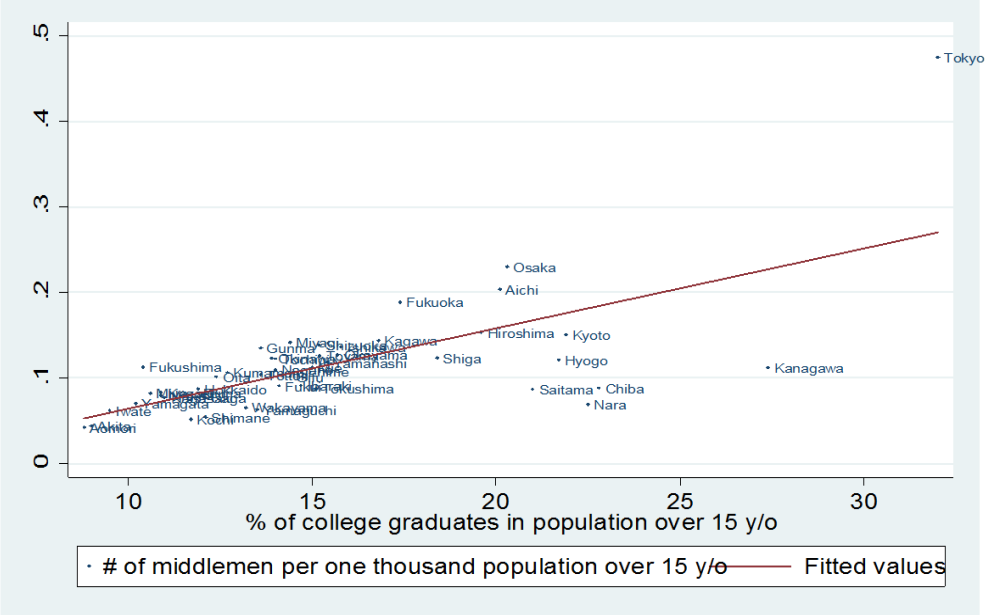

Figure 1: The number of private fee-charging job placement agencies per population and the share of college graduates in prefectures. **Source:** The data on regional college graduates share is taken from 2010 Census. The number of private, fee-charging placement agencies is taken from Report on the Employment Placement Business 2014, published by MHLW. Population over 15 years old is taken from 2014 Labor Force Survey, published by Ministry of Internal Affairs and Communications.

placement services, whether private or public, is to reduce job search costs and thereby promote matches between job vacancies and job seekers. In Japan, as in other industrialized countries, job placement service had long been a national monopoly since the early twentieth century, in accordance with the 1919 International Labor Organization (ILO) convention and recommendation, which prohibited the establishment and operation of for-profit employment placement agencies.

In 1997, ILO adopted the new convention concerning private employment agencies, and the corresponding changes in the Employment Security Act in Japan in the late 1990s and 2004 have lifted the ban on private agencies,

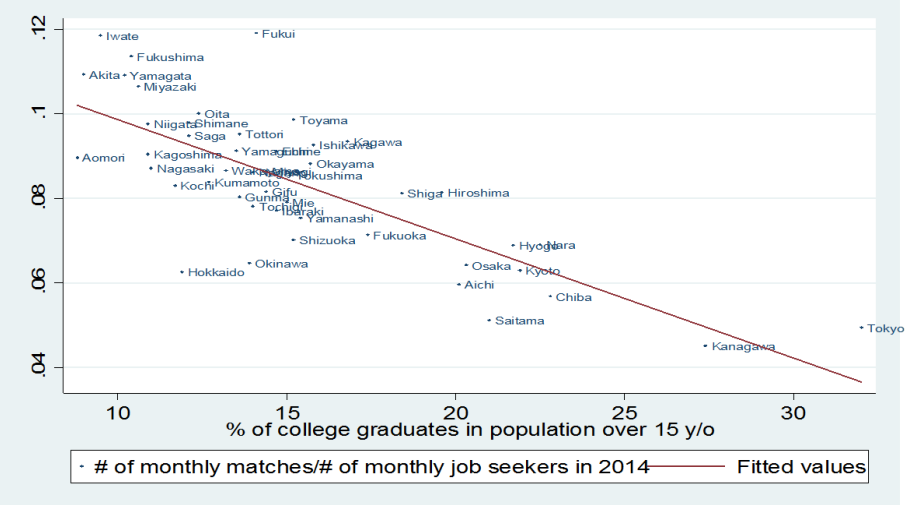

Figure 2: Matching rate of public placement agencies and the share of college graduates in prefectures, 2014. **Source:** Matching rate is the 2014 annual average and calculated from the data published in General Job Placement Situation by MHLW.

allowing them to coexist and compete with public agencies in the Japanese labor market.

Public placement agencies are government-funded and supply their services for free, for persons looking for work and firms with vacant jobs. They are legally not allowed to select their clients. Private placement agencies, in contrast, are basically free to choose which clients to serve. (Private placement agencies consist of non-profit ones and for-profit, fee-charging ones. Because non-profit private agencies comprise less than five percent of all private placement agencies in Japan, we do not discuss them hereafter.) Private placement agencies charge firms a fee for the job matching. The amount of a fee in many cases appears to be around thirty percent of the annual salary earned subsequently by the matched worker. (Charging workers the fee is prohibited by law in Japan, except for workers concerning managerial, professional or high-skilled jobs whose annual salary is above a threshold, as specified by the government.)

Placement agencies, public and private ones alike, offer a wide range of

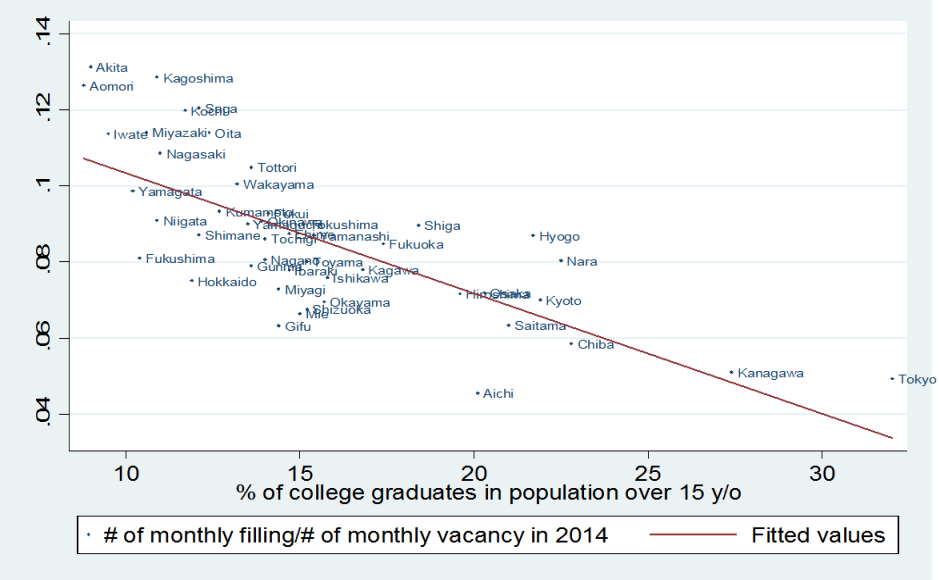

Figure 3: Filling rate of public placement agencies and the share of college graduates in prefectures, 2014. **Source:** Filling rate is the 2014 annual average and calculated from the data published in General Job Placement Situation by MHLW.

services. The simplest one is to offer free telephone calls for jobs listed by the agency. According to the cases, they also provide job seekers with assistance in drafting their resumes, in making personalized job search strategies, and in finding appropriate skill training.

Central to our interest is that, as shown below, closer look at the available evidence in Japan indicates that public and private agencies are largely complements to each other and deal with different clients. To be specific, and in the spirit of our theoretical model developed earlier, let us conceive labor market consisting of two tiers. The upper tire is primary labor market, in which workers are generally more educated, enjoy high wages, good benefits and employment security. The lower tire is called secondary labor market, in which workers are generally less educated, experience low wages, high turnover and job insecurity, often with fixed-term contract. In essence, this is traditionally known as dual labor markets model in labor economics

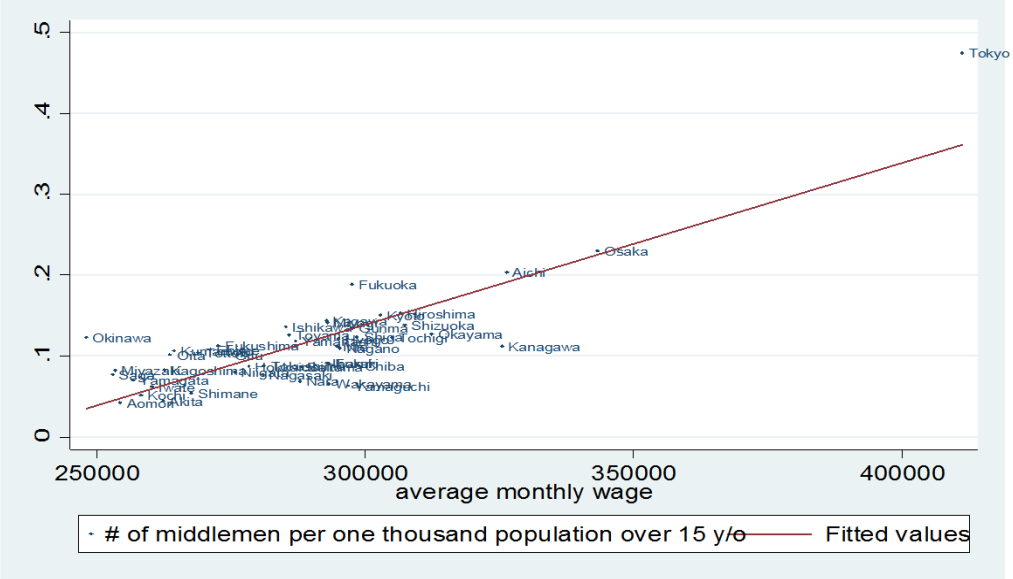

Figure 4: The density of for-profit placement agencies and the prefectural average wage, 2014. **Source:** The prefectural annual average wage is taken from Monthly Labour Survey published by MHLW.

literature.

Corresponding to this dual labor market, it is natural to suppose that job placement market also has a dual structure. Namely, private, fee-charging placement agencies play a central role in the high-wage, primary labor market, while they probably cannot attain sufficient profit in the low-wage, secondary market. On the other hand, public placement agencies cannot be a match for private placement agencies in primary labor market, but they supply vital services (for free) in the secondary labor market for those who cannot afford private placement services. Given this dual structure, one implication would be that it would not make much sense to evaluate public agencies and private ones just by comparing them in terms of some performance measures, such as the wage or skill level, since they deal with entirely different populations.

In what follows, we provide some evidence, concerning various indicators of primary/secondary labor markets and public/private placement agencies,

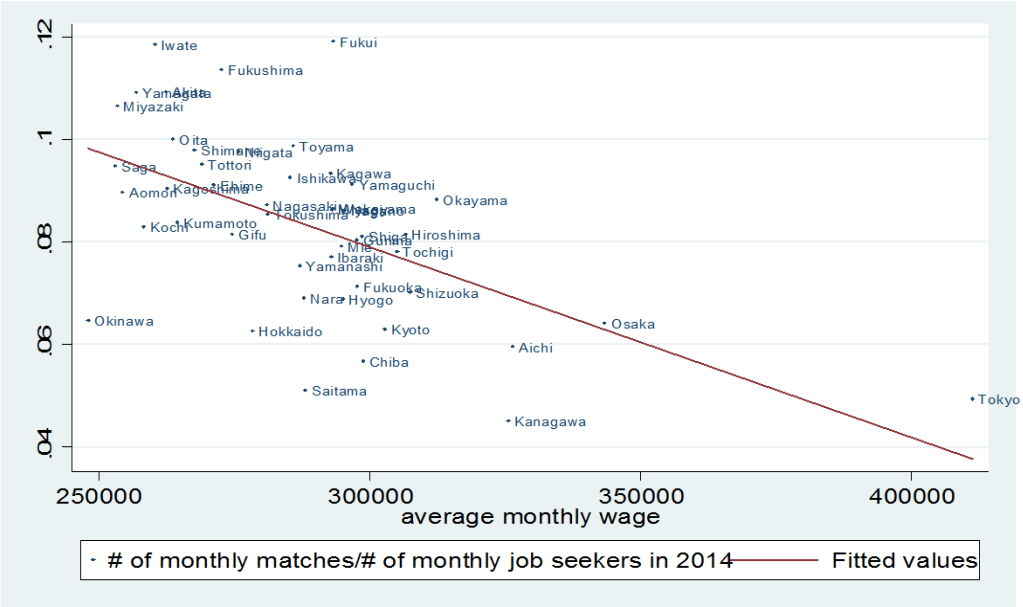

Figure 5: Matching rate at public placement agencies and the prefectural average wage, 2014.

which seem consistent with the dual market story.

First, the private placement agencies in Japan do tend to deal with high wage workers, compared to the public agencies. For instance, according to a survey conducted by RECRUIT Works Institute in 2010 (Working Persons Survey) for about ten thousand workers in the Tokyo metropolitan area, the average salary of previous jobs of job seekers who use private placement agencies is ¥5.6m a year, whereas that of those who use public agencies is ¥3.1m a year. The sizable difference in previous wages between two groups suggests that they belong to different populations and tiers of labor market.

Secondly, private job placement agencies appear to attract vacancies of better jobs, in terms of skill level, than public ones. Of all job vacancies that private placement agencies deal with, 54.6% are professional and technical jobs (which generally require high skills and pay high wages), whereas these jobs take only 22% of vacancies public agencies handle, according to the

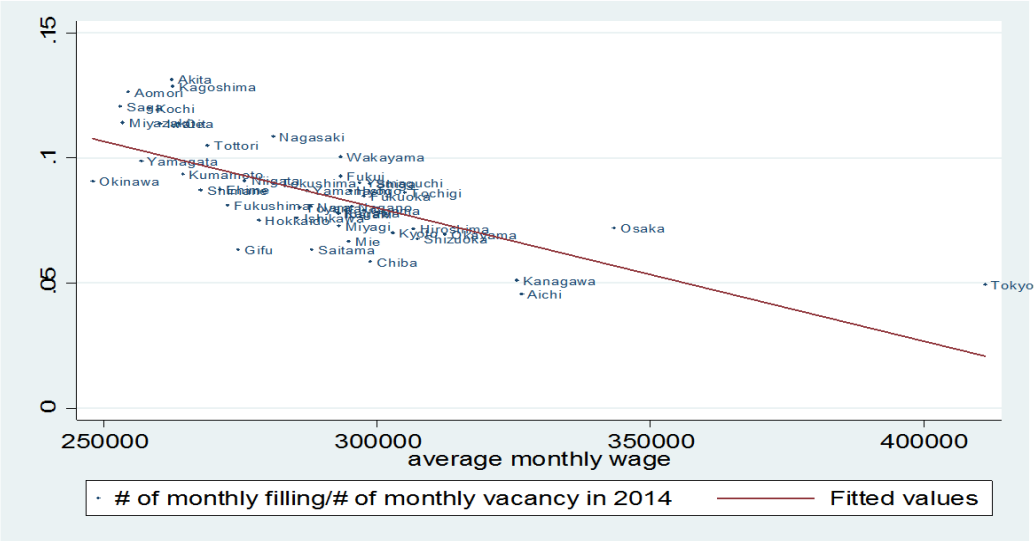

Figure 6: Filling rate at public placement agencies and the prefectural average wage.

surveys in 2014 by the Association of Job Information of Japan and the Ministry of Health, Labour and Welfare (MHLW hereafter). Several micro-econometric studies, of which most recent one being Kobayashi and Abe (2015), reached the similar conclusion (for a survey, see Watanabe, 2015, pp.19-23). Moreover, annual data from Survey on Employment Trends by MHLW reveal that the number and the share of professional or technical job vacancies listed in private agencies have increased steadily since the early 2000s (Watanabe, 2015, p.13).

Third, which of the two tires of labor market is dominant in a region turns out to vary widely and systematically across regions. Let us take the educational level of the population in a region as an obvious indicator of the extent of primary labor market. Figure 1 shows that prefectures with higher share of college graduates among population, such as Tokyo, Osaka and Aichi, tend to have more private placement agencies per population. In contrast, such rural areas as Aomori, Akita and Iwate, where less than one-tenth of population has a college degree, are concentrated on lower-left corner of the figure. The correlation coefficient is 0.68 and statistically significant at 1%

level. Consistently with this result, evidence from a nationally representative survey in 2007 revealed that highly educated workers are more likely to use private placement services than less educated workers (Watanabe, 2015, p.18).

Next, we found that in a region where the share of college graduates is high, the performance measures of public placement agencies tend to be low (Figures 2 and 3). An interpretation of this is that high skilled job seekers visit and register at public placement agencies only to collect their unemployment benefits, and find new jobs through other intermediaries. Hence the matching rate, which is defined as the ratio of the number of matchings to the number of registered job seekers at public placement agencies, is low in such regions (see Figure 2). A similar interpretation holds for the filling rate, which is defined as the ratio of the number of matchings to the number of registered job vacancies from firms (Figure 3).

Another indicator of the extent of primary labor market obviously is the regional average wage. Figure 4 indicates that average wage is associated positively with density of for-profit placement agencies. The correlation coefficient is 0.84. This is most likely because, for private job placement agencies, high wages imply high commission to earn for them, and this induces more entries to the market.

Additionally, Figures 5 and 6 show that higher average wage is associated with lower performance by public placement agencies, as measured by both matching rate and filling rate. An interpretation analogous to those given to Figures 2 and 3 applies naturally here.

Finally, although in our discussion we have apparently given equal weight in treating private and public intermediaries, it is worth bearing in mind that private job placement accounts for only tiny fraction of all job matchings in Japan. In 2000 it was 0.9 percent, according to Survey on Employment Trends by MHLW (Watanabe, 2015, p.3). Since then it has gradually and steadily increased, but in 2014 it still is a mere two and half percent. In contrast, job matchings which public placement offices mediated have accounted for about twenty percent, for the last two decades.

In summary, our quick overview of job placement services in Japan seems to justify the motivation and main formulations of our theoretical model. Empirically, it is noteworthy that, although private placement services have expanded recently and now play an important role in Japan, their scale is still rather limited. Also, their clients seem largely confined to the participants to primary labor market in urban areas, who are relatively well off. A much

greater number of workers and firms, who are less advantaged, appear to rely on free, public middlemen, especially in rural labor markets. It seems desirable then that any policy design to improve efficiency of job matchings and intermediaries need to take into accounts these (possibly inherent) dual structures of the labor market.

5 Concluding remarks

In this section, we point out concluding remarks related to Myerson and Satterthwaite (1983).

Myerson and Satterthwaite (1983) show the general impossibility of Bayesian incentive-compatible mechanisms that achieve (ex-post) efficient allocations in bilateral trading under asymmetric information. Immediately, this well-known impossibility theorem can be applied to the present model. That is, there is no (ex-post) efficient mechanisms in bilateral trading between a high-skilled worker and a high-technology firm under asymmetric information.[3] In contrast to this observation, in the present paper, Theorem 1 says that the private middleman can achieve efficient allocations in the sense of the core by eliminating asymmetry of information with transaction cost and by using the salary-adjustment process between high-skilled workers and high-technology firms.

On the other hand, Myerson and Satterthwaite (1983) also analyze a middleman (in their terminology, a "broker") in bilateral trading under asymmetric information. In their model, a middleman faces with asymmetric information in bilateral trading. Myerson and Satterthwaite (1983) show a possibility of the existence of Bayesian incentive-compatible mechanisms that achieve (ex-post) efficient allocations with the minimum expected subsidy required from the middleman when valuations do not have positive density over their respective intervals. This result implies that in general it is difficult for the middleman who is like a public middleman to achieve (ex-post) efficient allocations.[4] As demonstrated in the present model, thanks to

[3] More strictly, we assume a weaker version of information structure mentioned in Myerson and Satterthwaite (1983). Although they assume that a firm knows its marginal productivity, we assume that a (high-technology) firm does not know its marginal productivity.

[4] Although the purpose of the public middleman (or the government) described in the present study is to attain no existence of involuntary unemployment with the minimal

coexistence with the private middleman in intermediary labor markets, the public middleman can work for only low-skilled workers and low-technology firms. Therefore, in contrast to Myerson and Satterthwaite (1983), the public middleman can always achieve efficient allocations in the sense of the core by using the salary-adjustment process between low-skilled workers and low-technology firms.

Next, we demonstrate a justification of the condition **A5** by using theoretical/empirical evidence.

The standard proxy of the marginal productivity of labor is wage rate. This proxy is valid especially when the labor market is competitive. Thus, the fact that high-skilled workers earn more than low-skilled workers on average justifies the assumption that the high-skilled workers' wage rate is higher than the low-skilled workers' marginal productivity. On the other hand, that wage rate should be at least as large as reservation value of the worker is theoretically self-evident, because otherwise the worker would not accept the wage offer. Therefore, the condition **A5** seems to be quite natural.

There are other good reasons why the condition **A5** should hold. First, the standard model of job search predicts, and empirical studies have confirmed the prediction, that the higher unemployment benefits induce the longer unemployment duration, because the benefit defines the reservation value for the job seeker. And the amount of benefits is usually higher for high-skilled, high-wage workers. Secondly, more wealth (and hence higher unearned income flow) induces longer unemployment duration, again because the worker's reservation level is higher (cf. Card et al, 2007). And it is generally true that high-skilled workers tend to have greater wealth than low-skilled workers. Thirdly, according to both the standard human capital theory and the signaling model, highly educated person claims higher wage, so as to collect the return on their educational investment. Finally, a couple of recent empirical researches (e.g. Brown et al 2010; Krueger et al 2016) directly asked workers their reservation wages, and found that educational attainment correlates positively with reservation wage rate. Krueger et al (2016) also found that the reservation wage is initially about the same with the previous wage the workers used to earn (though it begins to decline as unemployment duration lengthens). This implies that high-skilled, high-wage job-seekers have high reservation wages.

subsidy, the purpose of the middleman described in Myerson and Satterthwaite (1983) is to maximize his expected profit.

Acknowledgements

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References

[1] G. Biglaiser. "Middlemen as Experts." Rand Journal of Economics, 24:212-223, 1993.

[2] S. Brown, J. Roberts, J, and K. Taylor. "Reservation wages, labour market participation and health." Journal of the Royal Statistical Society. Series A (Statistics in Society), 173(3), 501-529, 2010.

[3] P. Cahuc, S. Carcillo, and A. Zylberberg, Labor Economics (Second Edition), MIT Press, 2014.

[4] D. Card, R. Chetty, and A. Weber. "Cash-on-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market." Quarterly Journal of Economics, 122(4):1511–1560, 2007.

[5] J.H. Hatfield, F. Kojima, and S.D. Kominers. "Investment Incentives in Labor Market Matching." American Economic Review: Papers & Proceedings, 104:436-441, 2014.

[6] A.S. Kelso and V. P. Crawford. "Job Matching, Coalition Formation, and Gross Substitutes." Econometrica, 50:1483-1504, 1982.

[7] T. Kobayashi and M. Abe. "Differences of Job-Change between Public and Private Employment Service." Economic Analysis (Keizai Kenkyu), 188:93-118, 2015 (in Japanese).

[8] A.B. Krueger, and A.I. Mueller. "A Contribution to the Empirics of Reservation Wages." American Economic Journal: Economic Policy, 8(1):142-79, 2016.

[9] R. Myerson and M. Satterthwaite. "Efficient Mechanisms for Bilateral Trading." Journal of Economic Theory, 28:265-281, 1983.

[10] T. Oishi. "Economic Analysis of Intermediary Markets." in Chapter 7 of Market Quality and Modern Economy (M. Yano and Y. Furukawa eds.), 151-184, Keiso-Shobo, 2016 (in Japanese).

[11] T. Oishi and S. Sakaue. "Middlemen in the Shapley-Shubik Competitive Markets for Indivisible Goods." Mathematical Economics Letters, 2:19-26, 2014.

[12] A.E. Roth. "Deferred Acceptance Algorithms: History, Theory, Practice, and Open Questions." International Journal of Game Theory, 36:537-569, 2008.

[13] A. Rubinstein and A. Wolinsky. "Middlemen." Quarterly Journal of Economics 102:581-593, 1987.

[14] H. Watanabe. "Research into Private Employment Placement Services." JILPT Research Report 159, The Japan Institute for Labour Policy and Training, 2015 (in Japanese).

[15] M. Yano, "The Foundation of Market Quality Economics." Japanese Economic Review 60:1-31, 2009.