TCER Working Paper Series

Public good provision financed by nonlinear income tax under reduction of envy

Takuya Obara
Shuichi Tsugawa

February 2019

Working Paper E-130
http://tcer.or.jp/wp/pdf/e130.pdf

TOKYO CENTER FOR ECONOMIC RESEARCH
1-7-10-703 Iidabashi, Chiyoda-ku, Tokyo 102-0072, Japan

Abstract

We examine optimal taxation and public good provision by a government that considers reduction of envy as a constraint. We adopt the extended envy-freeness proposed by Diamantaras and Thomson (1990), called λ-equitability. We derive the modified Samuelson rule under an optimal nonlinear income tax and show, using a constant elasticity of substitution utility function, that the direction of distorting the original Samuelson rule to relax the λ envy-free constraint is crucially determined by the elasticity of substitution. Furthermore, we numerically show that the optimal level of provision increases (decreases) in the degree of envy-freeness when the original Samuelson rule is upwardly (downwardly) distorted.

Takuya Obara
Hitotsubashi University
Faculty of Economics
2-1 Naka, Kunitachi, Tokyo 186-8601, Japan
a141559z@r.hit-u.ac.jp

Shuichi Tsugawa
Kyushu University
Department of Urban and Environmental Engineering
744 Motooka Nishi-ku Fukuoka 819-0395, Japan
tsugawa@doc.kyushu-u.ac.jp

Public good provision financed by nonlinear income tax under reduction of envy

Takuya Obara* and Shuichi Tsugawa†

February 20, 2019

Abstract

We examine optimal taxation and public good provision by a government that considers reduction of envy as a constraint. We adopt the extended envy-freeness proposed by Diamantaras and Thomson (1990), called λ-equitability. We derive the modified Samuelson rule under an optimal nonlinear income tax and show, using a constant elasticity of substitution utility function, that the direction of distorting the original Samuelson rule to relax the λ envy-free constraint is crucially determined by the elasticity of substitution. Furthermore, we numerically show that the optimal level of provision increases (decreases) in the degree of envy-freeness when the original Samuelson rule is upwardly (downwardly) distorted.

JEL Classification: D63, H21, H41
Keywords: Income taxation, Public good provision, Envy free, Intensity of envy, Elasticity of substitution

*Faculty of Economics, Hitotsubashi University; a141559z@r.hit-u.ac.jp
†Department of Urban and Environmental Engineering, Kyushu University; tsugawa@doc.kyushu-u.ac.jp

1. Introduction

In an economy where agents have different skill levels, there are several ethical reasons to consider income redistribution. One such reason is the envy caused by income inequality. One agent envies another agent if he/she prefers the other's commodity bundle to his/her own. Complaints among citizens about income inequalities lead to collective decision making out of the way. Recently, World Economic Forum (2017) has been reported that income gap, one of the sources of envy, is a major driving force of polarized political outcomes. Further, as Bös and Tillmann (1985) noted,

> *the economic rationale for a minimization or reduction of envy by taxation is the following. Excessive envy in a society is an element of social disorder. Reducing envy in a society is a step towards increasing social harmony.* (p. 34)

Hence, reducing envy is not only a normative concept but also a relevant constraint for politicians concerned about harmony in society.

In the context of income taxation with endogenous labor supply, high-skilled agents cannot envy low-skilled ones because of the self-selection constraint. Conversely, low-skilled agents envy high-skilled ones. While it is difficult to apply the original envy-free constraint presented by Foley (1967), Kolm (1972), and Varian (1974), we replace the weaker and cardinal criterion proposed by Diamantaras and Thomson (1990) to evaluate the intensity of envy, called λ envy-free, and examine the optimal policy schedule under not only self-selection or incentive compatibility, which extracts true information on skill from each agent, but also constraints on the reduction of envy.

In this study, we investigate the optimal nonlinear income taxation with public good provision constrained by the reduction of envy, as well as the conventional constraints used by Boadway and Keen (1993). The objective of the government is to achieve a Pareto-efficient allocation, so that it maximizes low-class utility given the requirements for high-class utility, budget constraint, self-selection, and reduction of envy. In such situations, we derive the optimal provision rule of the public good, as well as the marginal income tax rate for each class. For the marginal income tax rate, we obtain the same results as Nishimura (2003b). Conversely, we derive the optimal provision rule following Boadway and Keen (1993), except for the distortion that arises from the λ envy-free constraint. This ethical constraint for the low class allows policymakers to compare the marginal rate of substitution (MRS) for the high class with that for the λ high class and use the difference to relax that constraint. In particular, because changing the amount of private consumption for the high class implies changing that for the λ high class λ times as much as for the original high class, the direction of the distortion is determined by whether the MRS is a step up or step down. To understand the provision rule, we use the constant elasticity of substitution (CES) utility function on private consumption and the public good and show that elasticity of substitution plays a key role in determining the sign. In addition, we conduct a numerical simulation to reveal the effect of λ on the public good provision. As extensions, we study the public good provision under mixed taxation, keeping the other settings constant.

The literature on optimal redistribution related to this paper can be categorized into optimal taxation under reduction of envy and taxation with public good provision. Regarding the optimal taxation for the reduction of envy, Nishimura (2003b) studies the optimal nonlinear income taxation under constraints on the reduction of envy, following the two-type model developed by Stern (1982) and Stiglitz (1982). He shows that the marginal income tax rate can increase only if leisure is a luxury. In addition, Nishimura (2003a) examines the optimal commodity taxation for the reduction of envy. Both these studies adopt a particular envy-free notion, namely, the λ envy-free of Diamantaras and Thomson (1990). While we adopt the same approach, our study introduces public good provision by the government. To the best of our knowledge, the present paper is the first to examine the optimal provision rule for a public good under reduction of envy when the government employs nonlinear income taxation.

Here, we describe the theoretical and conceptual differences between a weaker criterion of no-envy and the maximin criterion (Rawls (1971)) indicating that the allocation is one that maximizes the utility of the low-skilled individual. Theoretically, Nishimura (2003a) shows that the Diamantaras-Thomson allocation does not necessarily coincide with a Rawlsian type allocation. This implies that there may be a conflict between reduction of envy and compensation to the low-skilled individual. Therefore, utilitarian distributive concerns arising from income inequality may be different from envy-reduction concerns. Conceptually, an intuitive appeal of envy-free allocation as an equity criterion is that it does not require interpersonal comparability of utilities (e.g., Varian (1974)). Indeed, since the equity criterion allows individuals to judge fair allocations based on their own preferences, this notion is likely to be accepted as an equity criterion in economies where it is practically impossible to know the preferences of others.

On the optimal nonlinear income taxation with public good provision, Boadway and Keen (1993) show that a government provides a public good following the modified Samuelson rule, which embraces the self-selection term. For example, the term brings a downward pressure on the rule when the mimicker values the public good more than low-ability agents to redistribute more tax wealth.[1] Moreover, they demonstrate that the term disappears when the utility function is weakly separable between public and private goods (taken together) and leisure, which means that the original Samuelson rule is replicated. Edwards et al. (1994) and Nava et al. (1996) study the optimal nonlinear income and linear commodity taxation with public good provision, showing that the Samuelson rule is modified by two additional terms related to the self-selection constraint and revenue of indirect taxes, as well as that these terms disappear under the conventional conditions guaranteeing the original Samuelson rule. However, allowing for endogenously determined wages in line with Naito (1999), Pirttilä and Tuomala (2001) demonstrate that the public provision rule does not reduce to the first-best Samuelson principle due to general equilibrium effects, even under the weak separability conditions. In the present paper, when the government aims to reduce envy as equity considerations, we suggest novel cases where the result of Boadway and Keen (1993) does not hold without assuming the endogenous wage.

In the optimal tax literature, several theoretical studies have explored the effect of status or relative consumption (or income), that is, individuals' utilities depend not only on their own consumption of goods but also on their relative standing in society (e.g., Boskin and Sheshinski (1978), Oswald (1983), Seidman (1987), Persson (1995), Ireland (2001), Corneo (2002), Aronsson and Johansson-Stenman (2008), Balestrino (2009), Micheletto (2011), Kanbur and Tuomala (2013), Bruce and Peng (2018)). In particular, Aronsson and Johansson-Stenman (2008) and Micheletto (2011) examine the public good provision under the optimal nonlinear income tax in the presence of interdependence in individuals' utilities. Aronsson and Johansson-Stenman (2008) describe relative consumption as the difference between an individual's own consumption and the average consumption in an economy and show that the Samuelson rule should be upwardly distorted when leisure is weakly separable from private and public consumption. Additionally, Micheletto (2011) focuses only on the case where individuals care about the consumption of a richer group, according to evidence provided by Bowles and Park (2005). As the consumption of higher-income agents increases, it negatively affects the preferences of lower-income agents, which he refers to in his paper as the "Veblen effect". He also shows that the overprovision of the public good relative to the Samuelson rule is always optimal due to the Veblen effect if no self-selection constraints are binding. However, these studies on social comparisons that have been analyzed extensively in the optimal tax literature do not consider that the government must take equitable allocation into account, although individuals care about their relative positions owing to the Veblen effect. In other words, since the government does not care about fair distribution, the model allows the government to implement unfair distribution in the sense of violating an equity criterion for allocations. In contrast, this paper considers a situation in which the government is constrained by the fairness requirement for promoting harmony in society when agents do not have other-regarding preferences. Note that the fact the government cares about envy in the allocation does not stem from utility interdependence. Our standpoint is that the government's intervention is justified by equity concerns considering the notion of envy-free as an equity criterion for allocations.[2] Compared to the results of the optimal tax literature related to the Veblen effect, we show that the λ envy-free approach proposes cases wherein underprovision of the public good is optimal when the effect on self-selection constraints disappears, assuming a special form of the utility function.

Unlike the standard welfarist approach in which a government fully respects all aspects of individual preferences, there are several studies incorporating non-welfarist principles in policy evaluation. In a non-welfarist framework, the government is suspected to have a paternalistic motive for taxation/subsidization stemming from differences between social and private preference. There are various examples of public policies from the viewpoint of non-welfarism. First, poverty reduction is one of the non-welfarist concerns, and this point has been explored in some papers on this topic (e.g., Besley and Kanbur (1988), Besley and Coate (1992, 1995), Kanbur et al. (1994), Pirttilä and Tuomala (2004), Kanbur et al. (2018)). Instead of social welfare maximization, the government seeks to minimize poverty in society, which is defined as deprivation of individual consumption relative to some desired level and measured using the Gini-based index. Second, a strand of literature on merit goods and sin taxes is considered non-welfarist (e.g., Sandmo (1983), Besley (1988), Racionero (2001), Schroyen (2005), O'Donoghue and Rabin (2003, 2006)). In cases where individuals have self-control problems, they may disregard the beneficial impact of consumption of goods such as education and health or consume harmful goods such as alcohol and drugs in excess. If the government induces individuals to behave as if they had perfect self-control by employing tax and subsidy policies, these individuals might benefit. Thus, to correct these faulty choices, a paternalistic government reflects positive or negative effects that individuals do not care about into government's preferences. This leads to a subsidy on merit goods to encourage costly but beneficial consumption and a tax on sin goods to discourage harmful consumption. Third, relative consumption is related to not only welfarist literature but also non-welfarist literature.

Harsanyi (1982) argues that the government should not respect antisocial preferences such as envy. Following Harsanyi, the non-welfarist literature on relative consumption considers that the government does not include such preferences in the social objective function even if individuals care about social comparisons. For example, Micheletto (2011) and Aronsson and Johansson-Stenman (2018) investigate optimal nonlinear income tax policies under the welfarist case and the paternalist case. Finally, non-welfarist approaches have also been used in a framework with multi-dimensional heterogeneity. Boadway et al. (2002) consider that individuals differ in their ability and their preferences for leisure, and they examine the properties of the optimal nonlinear income tax when different weights can be assigned to individuals with different preferences for leisure. Fleurbaey and Maniquet (2006) derive the optimal income tax schedule in settings where the social planner maximizes the social index satisfying several axioms for fairness and inequality aversion. In their framework, weights are determined by fairness principles, a weak version of the Pigou-Dalton transfer principle and a condition precluding redistribution when all agents have the same skills. According to Kanbur et al. (2006), these papers are similar to the non-welfarist approach because weights decided by the government do not necessarily coincide with weights preferred by individuals. Moreover, Schokkaert et al. (2004) employ the concept of a reference preference for leisure through the advantage function. As a paternalistic criterion, the social planner evaluates individual preferences for leisure as social preferences reflecting socially desirable effort levels. Our paper may be related to the literature on non-welfarist public economics in the sense that the government cares about envy-free allocations despite individuals not having other-regarding preferences (or "envy"). However, this paper adopts the stance of introducing the concept of envy-free as an equity criterion for allocations when the government fully respects all aspects of individual preferences. This implies that our paper belongs to the strand of literature on welfarist public economics.

The remainder of this paper is organized as follows. Section 2 examines the optimal provision rule for pure public goods under the reduction of envy and section 3 presents simple numerical examples. Section 4 extends the model to the case of linear commodity taxation and section 5 offers concluding remarks.

2. Optimal income taxation with public good provision for reduction of envy

We consider a two-class economy in which each agent ($i = H, L$) possesses an exogenous skill level w_i, where $w_H > w_L > 0$. There is a continuum of individuals with unit mass. Let $n_H \in (0, 1)$ denote the proportion of high-skilled individuals and the remaining $n_L = 1 - n_H$ the proportion of low-skilled ones. They earn their income by supplying labor, and their earnings are the product of the unit wage (or skill level) and amount of labor supply. The government collects taxes on their income, which can be scheduled nonlinearly. In addition, it provides a public good by using the collected taxes.

First, we assume three types of goods: consumption (or after-tax income) $c \in \mathbb{R}_+$, labor supply l, and public good $G \in \mathbb{R}_+$. We also assume that each worker provides at most $\bar{l}$ labor, implying that he/she chooses the supply level l between 0 and $\bar{l}$. Every agent shares an identical utility function, $U(c_i, G, l_i)$, and U is twice continuously differentiable, strictly concave, strictly increasing in c and G, and strictly decreasing in l. Let Y be the labor income. If agent i, with skill w_i, earns labor income Y_i, we can replace this statement by the expression $U(c_i, G, \frac{Y_i}{w_i})$. To provide the public good, the government must incur production cost $\phi(G)$ with a strictly increasing, strictly convex, and twice continuously differentiable function. For all goods except the public good, a good with subscript i refers to one that agent i enjoys.

We assume that the government wants to achieve a constrained Pareto-efficient allocation. Specifically, we consider the problem of maximizing low-skilled utility subject to high-skilled agents having at least a given utility level, $\bar{u}$. The planner faces three other constraints. First, the government faces a resource constraint. Let $T : \mathbb{R} \to \mathbb{R}$ be the income tax function; agent i's budget constraint is written as $c_i = w_i l_i - T(w_i l_i)$. Therefore, the government's resource constraint is

$$n_L T(w_L l_L) + n_H T(w_H l_H) = n_L(w_L l_L - c_L) + n_H(w_H l_H - c_H) \geq \phi(G). \tag{1}$$

Second, the policymaker cannot observe agents' skill directly but does know their earned income. Hence, we require that he/she resolves the information asymmetry problem, called the *self-selection* constraint. We formulate this as follows:

$$U(c_i, G, l_i) \geq U(c_j, G, \frac{w_j}{w_i} l_j), \tag{2}$$

for any $i, j = H, L$ with $i \neq j$. Finally, we impose an ethical constraint for reducing envy. The equity concept of no-envy faces a difficulty in the second-best situation, since the low-skilled agent always envies the high-skilled one, whereas the high-skilled agent never envies the low-skilled agent.[3] As a less-demanding criterion of envy reduction, we adopt the λ envy-free introduced by Diamantaras and Thomson (1990) and used by Nishimura (2003a,b) as a cardinal measure of the intensity of envy. The reason for employing cardinal concepts is that, according to Bös and Tillmann (1985), ordinal concepts are not useful as there is an invariant hierarchy of envy in the second-best analysis. Also, note that the Lagrangian expression of the optimization problem with the λ envy-free constraint (equation (4)) is similar to the social objective of Varian (1976), who incorporates degrees of envy, not constraint, into the social objective. However, λ envy-free is better in the sense that it is independent of the comparability and cardinality of utility functions (see Nishimura (2003b) for details).

Let λ_{ij} be a nonnegative real number, such that $U(c_i, G, l_i) = U(\lambda_{ij} c_j, G, \bar{l} - \lambda_{ij}(\bar{l} - l_j))$ when $U(c_i, G, l_i) \leq U(c_j, G, l_j)$ and $\lambda_{ij} \equiv 1$ when $U(c_i, G, l_i) > U(c_j, G, l_j)$. If λ_{ij} is unity, it is the no-envy case. When agent i envies agent j, the value of λ_{ij} represents the amount by which one would have to decrease j's bundle to stop agent i from envying agent j. In other words, λ_{ij} indicates the intensity of envy. Assume that agent i compares his/her own bundle with the bundle containing the public good and a proportional contraction of agent j's consumption and leisure between points $(0, \bar{l})$ and (x_j, l_j). Let $\lambda \equiv \min_{ij} \lambda_{ij}$. Under the binding self-selection constraint, $\lambda = \lambda_{LH}$, since $\lambda_{LH} < 1$ and $\lambda_{HL} = 1$. An allocation is then λ envy-free if $U(c_i, G, l_i) \geq U(\lambda c_j, G, \bar{l} - \lambda(\bar{l} - l_j))$ for all i and j. We consider that the government is constrained by a given λ envy-free requirement:

$$U(c_i, G, l_i) \geq U(\lambda c_j, G, \bar{l} - \lambda(\bar{l} - l_j)), \tag{3}$$

for any $i, j = H, L$ with $i \neq j$.[4] Because a high-skilled agent never envies a low-skilled agent, we focus only on the λ envy-free constraint for the low-skilled agent.

Summarizing the above, the policymaker's optimization problem can be written as follows:

$$\max_{\{c_i,l_i\}_{i=L,H},G} U(c_L, G, l_L),$$

subject to

$$\begin{aligned}
&U(c_H, G, l_H) \geq \bar{u} \\
&n_L(w_L l_L - c_L) + n_H(w_H l_H - c_H) \geq \phi(G) \\
&U(c_i, G, l_i) \geq U(c_j, G, \frac{w_j}{w_i} l_j) \quad \text{where } i, j = H, L \text{ with } i \neq j \\
&U(c_L, G, l_L) \geq U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H)).
\end{aligned}$$

The Lagrangian is

$$\begin{aligned}
&\mathcal{L}(c_L, c_H, l_L, l_H, G; \gamma, \delta_r, \delta_{sH}, \delta_{sL}, \delta_e) = \\
&U(c_L, G, l_L) + \gamma\{U(c_H, G, l_H) - \bar{u}\} \\
&+\delta_r\{n_L(w_L l_L - c_L) + n_H(w_H l_H - c_H) - \phi(G)\} \\
&+\delta_{sH}\{U(c_H, G, l_H) - U(c_L, G, \frac{w_L}{w_H} l_L)\} + \delta_{sL}\{U(c_L, G, l_L) - U(c_H, G, \frac{w_H}{w_L} l_H)\} \\
&+\delta_e\{U(c_L, G, l_L) - U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))\},
\end{aligned} \tag{4}$$

where $\gamma, \delta_r, \delta_{sH}, \delta_{sL}$, and δ_e are the Lagrangian multipliers associated with the first, second, third, fourth, and fifth constraints, respectively.[5] Note that this problem is almost the same as that of Boadway and Keen (1993), but we incorporate the λ envy-free constraint. Appendix A shows the first-order conditions with respect to the Lagrangian. Hereafter, we focus only on the redistributive cases: $\delta_{sL} = 0$ and $\delta_{sH} > 0$.

2.1 Marginal income tax rate

We derive the marginal income tax rate for each type in the same way as Nishimura (2003b). Let $U_a^i \equiv \partial U(c_i, G, l_i)/\partial a_i$, $\hat{U}_c \equiv \partial U(c_L, G, \frac{w_L}{w_H} l_L)/\partial c_L$, $\hat{U}_l \equiv \partial U(c_L, G, \frac{w_L}{w_H} l_L)/\partial(\frac{w_L}{w_H} l_L)$, and $\bar{U}_a \equiv \partial U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))/\partial(\lambda a_H)$, where $i = H, L$ and $a = c, l$. The next lemma provides the marginal income tax rates.

Lemma 1. *Under the redistributive cases when $\delta_{sL} = 0$ and $\delta_{sH} > 0$,*

1. *The marginal income tax rate at the bottom is*

$$T'(w_L l_L) = \frac{\delta_{sH}\hat{U}_c}{\delta_r}\Big[MRS^L(y, c) - \hat{MRS}(y, c)\Big] > 0,$$

where $MRS^L(y, c) = -\frac{1}{w_L}\frac{U_l^L}{U_c^L}$ and $\hat{MRS}(y, c) = -\frac{1}{w_H}\frac{\hat{U}_l}{\hat{U}_c}$.

2. *The marginal income tax rate at the top is*

$$T'(w_H l_H) = \frac{\lambda\delta_e\bar{U}_c}{\delta_r w_H}\Big[MRS_{lc}^H - \bar{MRS}_{lc}\Big],$$

where $MRS^H_{lc} \equiv -\frac{U^H_l}{U^H_c}$ *is the MRS for* l_H *measured by* c_H *and* $\bar{MRS}_{lc} \equiv -\frac{\bar{U}_l}{\bar{U}_c}$ *is the MRS measured at* $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$.

This lemma is consistent with Nishimura (2003b).[6] Because of the self-selection constraint for agents with high skill, the marginal income tax rate for low-skilled agents must be positive, as shown by Stiglitz (1982). Conversely, the marginal income tax rate on the top is different from the standard result presented by Stiglitz (1982), since the term that represents the effect of the λ envy-free constraint appears.[7] Nishimura (2003b) shows that if the income elasticity of leisure is greater (less) than 1, MRS^H_{lc} is greater (less) than $\bar{MRS}_{lc}$, which means that the marginal income tax rate on the top must be positive (negative).[8] Of course, if $MRS^H_{lc} = \bar{MRS}_{lc}$, it must be zero. Moreover, if the equitability constraint is not binding (i.e., $\delta_e = 0$), then it must be zero.

2.2 Provision rule of the public good

This section presents the public good provision rule at the optimum. As per Boadway and Keen (1993), the optimal provision rule includes the self-selection term, which plays an important role in income redistribution. If the mimicker places more weight on the public good based on private consumption than the mimicked one with low skill, the government should reduce its production and transfer the tax revenue to low-class agents. In addition, to relax the λ envy-free constraint, the government must increase or decrease the amount. For instance, if the evaluation of the public good for the private good at the λ-scaled bundle $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$ is higher than that the high-skilled agent receives, then he/she must reduce the provision level to redistribute more income.

Let $U^i_G \equiv \partial U(c_i, G, l_i)/\partial G$, $\hat{U}_G \equiv \partial U(c_L, G, \frac{w_L}{w_H} l_L)/\partial G$, and $\bar{U}_G \equiv \partial U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))/\partial G$, where $i = H, L$. Formally, we can derive the optimal rule with respect to the public good provision in the next proposition.

Proposition 1. *Under the optimal nonlinear income taxation with the* λ *envy-free and self-selection constraint, the optimal provision rule is characterized by*

$$\sum_{i=H,L} n_i MRS^i_{Gc} + \frac{\delta_{sH}}{\delta_r} \hat{U}_c (MRS^L_{Gc} - \hat{MRS}_{Gc}) + \frac{\lambda \delta_e}{\delta_r} \bar{U}_c (MRS^H_{Gc} - \frac{1}{\lambda} \bar{MRS}_{Gc}) = \phi'(G), \quad (5)$$

where $MRS^i_{Gc} \equiv \frac{U^i_G}{U^i_c}$ *is type i's MRS for* G *measured by* c_i, $\hat{MRS}_{Gc} \equiv \frac{\hat{U}_G}{\hat{U}_c}$ *is the mimicker's MRS between c and G, and* $\bar{MRS}_{Gc} \equiv \frac{\bar{U}_G}{\bar{U}_c}$ *is the MRS measured at* $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$.

The first term is the sum of agent i's MRS for public good G measured by private consumption c_i and the second term the effect of the incentive constraint. The third term is a novel one, which reflects the effect on the λ envy-free constraint and whose implication is similar to that of the incentive constraint. Because λ distorts the consumption/leisure bundle for the envying agent, this term may not be zero. To relax the λ envy-free constraint, the government changes the provision level of the public good and makes room to improve welfare. We suggest an intuitive interpretation of the third term. Starting from the original Samuelson rule, consider the following redistribution. The government imposes an additional tax liability MRS^i_{Gc} on type-i individuals to increase G. The tax reform does not change the welfare of type-i individuals or the government's budget. The valuation of G of the envying agent is expressed by $\frac{1}{\lambda}M\bar{R}S_{Gc}$. If $MRS^H_{Gc} > \frac{1}{\lambda}M\bar{R}S_{Gc}$, the third term in equation (5) suggests that the original Samuelson rule should be upwardly shifted. This implies that an increase in G mitigates the intensity of envy for low-type agents because the tax liability of the envied agent is larger than that of the envying agent and, then, the difference between their utilities is reduced. Therefore, the upward distortion relaxes the λ envy-free constraint for low-type individuals.

Boadway and Keen (1993) show that the original Samuelson rule for the public good provision is replicated when each agent's preference is represented by $U(H(c, G), l)$, namely, c and G are weakly separable with l in the utility function. In this case, while the second bracket on the left-hand side is zero, it is ambiguous whether the third bracket is zero.

2.3 A special case: CES utility function

This subsection derives the direction of the distortion due to the binding λ envy-free constraint on the provision rule by using a concrete utility function. To examine the direction of the distortion, we assume that the utility function is expressed by $U(H(c, G), l)$ and $H(c, G)$ is the CES functional form: $H(c, G) = (\alpha c^\rho + \beta G^\rho)^{\frac{1}{\rho}}$, where $\rho \leq 1$. If ρ converges to zero, $H(c, G)$ converges to the Cobb–Douglas expression (i.e., $H(c, G) = c^\alpha G^\beta$). In this case, the round bracket can be represented by

$$MRS^H_{Gc} - \frac{1}{\lambda}M\bar{R}S_{Gc} = (1 - \lambda^{-\rho})\left(\frac{\beta}{\alpha}\right)\left(\frac{c_H}{G}\right)^{1-\rho}.$$

$1 - \lambda^{-\rho}$ determines the sign and the elasticity of substitution $\frac{1}{1-\rho}$ plays a crucial role since $\lambda < 1$. If $\frac{1}{1-\rho} \in (0, 1)$, then the direction of the distortion is positive; otherwise, the direction is negative except for $\frac{1}{1-\rho} = 1$. If $\frac{1}{1-\rho} = 1$, the bracket equals zero and, thus, the third and second terms disappear. To sum up, the next corollary describes the direction of the distortion on the provision rule.

Corollary 1. *Assume that all agents have the following utility function:* $H(c, G) = (\alpha c^\rho + \beta G^\rho)^{\frac{1}{\rho}}$. *The optimal provision rule distorts*

- *Downwardly if the elasticity of substitution* $\frac{1}{1-\rho} \in (1, +\infty)$ *or* $\rho = 1$*;*
- *Upwardly if the elasticity of substitution* $\frac{1}{1-\rho} \in (0, 1)$*.*

In addition, the rule coincides with the Samuelson rule if the elasticity of substitution equals one.

If the elasticity of substitution is above one, the government increases private consumption for the high type by decreasing the provision level of the public good. The elasticity of substitution refers to the variation of the ratio between private consumption and the public good ($\frac{c_i}{G}$) when MRS^i_{Gc} changes. Hence, when the elasticity of substitution is above one, a decrease in the ratio due to a proportional contraction of private consumption for the high type allows the MRS between private consumption and public good to decrease by less than the proportional decrease in the corresponding MRS. This means that $M\bar{R}S_{Gc} > \lambda MRS^{H}_{Gc}$, which is equivalent to stating that the corresponding MRS for envying agents (i.e., $\frac{1}{\lambda}M\bar{R}S_{Gc}$) is greater than the corresponding MRS for the envied agent (i.e., MRS^{H}_{Gc}). That is, envying agents value the public good more than the envied agent. Thus, it is desirable for the government that the amount of the public good decreases and private consumption for the high type increases. Also, the argument is symmetric for the opposite case where the elasticity of substitution is below one.

The Samuelson rule under $\rho = 0$ is valid because $H(\cdot)$ is homothetic in c. In this case, a proportional decrease of the envied agent's consumption implies that the MRS decreases proportionally as c decreases. Therefore, $MRS^{H}_{Gc} = \frac{1}{\lambda}M\bar{R}S_{Gc}$ holds.[9]

2.4 Remarks: λ envy-free constraint

We make three comments about the key constraint (3) in our model, the λ envy-free constraint: (i) the form of the λ envy-free constraint; (ii) whether the λ envy-free constraint for the low-type is binding at the optimum; and (iii) the interaction between the self-selection constraint for the high-type and the λ envy-free constraint for the low-type.

First, we explain why the intensity of envy λ is not applied to the amount of the public good in the λ envy-free constraint. In our model, we consider that agent i compares two bundles: his/her own bundle, (x_i, G, l_i), and the bundle containing the public good and a proportion λ of agent j's consumption and leisure, $(\lambda c_j, G, \bar{l} - \lambda(\bar{l} - l_j))$. Remember that the classic concept of no-envy is not useful here due to the self-selection constraint. If no agent envies any other agent's bundle, including the public good and the proportional contraction of consumption and leisure, the allocation satisfies the λ envy-free constraint. This implies that the utility of one agent does not increase, even if the government allocates the bundle of any other agent consisting of the public good and λ-scaled consumption and leisure to the agent. From this viewpoint, if one shrinks the amount of the public good in the λ envy-free constraint, the government needs to be able to implement the λ proportion of the public good to prevent one agent from envying another agent. However, since all individuals share the same amount of the public good (i.e., public goods are non-rivalrous) provided by the government in this economy, such an allocation is infeasible. Therefore, it is inconsistent to impose the intensity of envy λ on the amount of the public good. Hence, we employ the version of the λ envy-free constraint given by equation (3).

If we were to rewrite the constraint as:

$$U(c_i, G, l_i) \geq U(\lambda c_j, \lambda G, \bar{l} - \lambda(\bar{l} - l_j)),$$

then the optimal Samuelson rule is:

$$\sum_{i=H,L} n_i MRS^{i}_{Gc} + \frac{\delta_{sH}}{\delta_r}\hat{U}_c(MRS^{L}_{Gc} - M\hat{R}S_{Gc}) + \frac{\lambda\delta_e}{\delta_r}\bar{U}_c(MRS^{H}_{Gc} - M\tilde{R}S_{Gc}) = \phi'(G),$$

where $M\tilde{R}S_{Gc} \equiv \frac{\tilde{U}_G}{\tilde{U}_c}$, $\tilde{U}_G \equiv \partial U(\lambda c_H, \lambda G, \bar{l} - \lambda(\bar{l} - l_H))/\partial(\lambda G)$, and $\tilde{U}_c \equiv \partial U(\lambda c_H, \lambda G, \bar{l} - \lambda(\bar{l} - l_H))/\partial(\lambda c_H)$. If we assume that the utility function is described by $U(H(c, G), l)$, the second term disappears. Also, if the function $H(\cdot)$ is homothetic, the marginal rate of substitution between G and c is constant on the path of the λ-contraction of the envied agent's allocation, that is, $MRS^H_{Gc} = \tilde{MRS}_{Gc}$. Therefore, a sufficient condition for the original Samuelson rule is that $H(\cdot)$ is homothetic. For example, if $H(\cdot)$ is the CES functional form used in subsection 2.3, the original Samuelson rule is desirable.

Second, throughout the paper, we consider the Pareto frontier when the government imposes λ-equitability as a prerequisite, as with Nishimura (2003a,b). However, it is unclear whether the λ envy-free constraint for the low-type is binding at the optimum, although we can specify the interval of λ such that the constraint can be binding. For example, choosing any plausible utility function and profile of wages, let us consider a set Ω consisting of all allocations satisfying the three constraints (the constraint that high-type agents have at least a given level of utility, resource constraint, and self-selection constraint for the high-type) are binding and the λ envy-free constraint for the low-type is slack for some λ. Each element is denoted by $\omega \equiv \{(c^\omega_L, G^\omega, \ell^\omega_L), (c^\omega_H, G^\omega, \ell^\omega_H)\} \in \Omega$. For each $\omega \in \Omega$, a threshold λ^ω exists such that the λ^ω envy-free constraint for the low-type is binding in allocation ω, i.e., $U(c^\omega_L, G^\omega, l^\omega_L) = U(\lambda^\omega c^\omega_H, G^\omega, \bar{l} - \lambda^\omega(\bar{l} - l^\omega_H))$. This in turn implies that the λ envy-free constraint for the low-type is violated in allocation ω for all $\lambda > \lambda^\omega$. This is because, when we fix any $\lambda > \bar{\lambda}$, $U(c^\omega_L, G^\omega, l^\omega_L) < U(\lambda c^\omega_H, G^\omega, \bar{l} - \lambda(\bar{l} - l^\omega_H))$ holds in allocation ω from the fact that $U(\lambda c^\omega_H, G^\omega, \bar{l} - \lambda(\bar{l} - l^\omega_H))$ is an increasing function in λ. Additionally, λ^ω is well defined by the intermediate value theorem because, since $U(\lambda c^\omega_H, G^\omega, \bar{l} - \lambda(\bar{l} - l^\omega_H))$ is continuously increasing in λ, the utility of low-type agents is lower than that for the high-type ones because of the self-selection constraint for the high-type, and is greater than $U(0, G, \bar{l})$ due to the interior solutions of c_L and l_L. Here, let $\bar{\lambda} \equiv \sup_{\omega \in \Omega} \lambda^\omega$ be the supremum of all λ^ω and $\underline{\lambda} \equiv \inf_{\omega \in \Omega} \lambda^\omega$ be the infimum of all λ^ω. Note that, when $\lambda > \bar{\lambda}$ or $\lambda < \underline{\lambda}$, no allocation exists in which not only the above three constraints but also the λ envy-free constraint for the low-type is binding. This means that allocations such that all four constraints are binding exist for some $\lambda \in (\underline{\lambda}, \bar{\lambda})$. However, this does not imply that such an allocation is implemented at the optimum. This is the unresolved question for future research. In the present paper, instead of clarifying the question analytically, we provide numerical examples in Tables 1 and 2 indicating the desirability of allocations such that the λ envy-free constraint for the low-type is binding (see Section 3 for more details). Case I indicates results when the above three constraints are binding and the λ envy-free constraint for the low-type is slack. Cases II, III, and IV describe results when all four constraints are binding.

Finally, we mention the interaction between self-selection constraints and λ envy-free constraints. We have assumed that the self-selection constraint is always binding. However, if the government can implement the first-best Pareto efficient allocation, that is, if the self-selection constraint is slack, we cannot ignore the case where the self-selection constraint for the high-type is slack and the λ envy-free constraint for the low-type is binding. In other words, it is not necessary that the binding self-selection constraint for the high-type is a necessary condition for the binding λ envy-free constraint for the low-type. In such a situation, the original Samuelson rule should be modified only based on the effect of the third term on the left-hand side of equation (5).

3. Numerical examples

The previous section did not refer to the extent to which the λ envy-free requirement affects the amount of the public good. Therefore, this section presents a quantitative analysis of the amount of the public good. First, we examine the impact of the λ envy-free requirement on the utility level of a low-skilled agent and the amount of the public good. Next, we present the sensitivity of the amount of the public good with respect to the changes in the parameter value expressing the intensity of envy, namely λ.

In the simulation, we make the following assumptions. First, not only for the sake of simplicity but also for focusing on the effect of the λ envy-free requirement on the provision level of public good, we assume that the functional form of the utility is $U(c, G, l) = H(c, G) - v(l)$, where the sub-utility function $H(\cdot)$ takes the CES form with $\alpha = \beta = 0.5$ and the disutility of labor $v(\cdot)$ takes an isoelastic form: $v(\ell_i) = \ell_i^{1+1/e}/(1+1/e)$ with $e > 0$. According to the empirical estimates (e.g., Chetty et al. (2011)), we set $e = 2$. Second, Fang (2006) and Goldin and Katz (2007) estimate that the college wage premium is approximately 60%. We normalize low-type individuals' parameter w_L to equal one and, thus, that of high-type individuals is assumed to be $w_H = 1.6$. Third, according to an OECD (2010) report, approximately one-quarter of all adults have attained tertiary education. Therefore, we assume that 25% of individuals are high-skilled workers. In other words, we set $n_H = 0.25$ and $n_L = 0.75$. Finally, we assume that $\bar{u}$ is unity and the cost function takes the following form, strictly increasing and strictly convex: $\phi(\cdot) = G^2$.

We suggest numerical examples for two cases: $\rho = 1$ and $\rho = -1$. Table 1 presents four cases in which the original Samuelson rule is downwardly distorted. Notice that Case I corresponds to results when the three constraints (the constraint that high-type agents have at least a given level of utility, resource constraint, and self-selection constraint for the high-type) are binding and the λ envy-free constraint for the low-type is slack, and Cases II, III, and IV describe results when all the four constraints are binding. First, the utility level of the low-skilled agent decreases as λ increases, since the second-best frontier decreases. Second, the provision level decreases when the λ envy-free constraint is binding, which supports the results of Corollary 1. Third, the provision level decreases as λ increases. The intuition is that the government reinforces the income redistribution by distorting the provision level to alleviate the intensified envy. On the other hand, Table 2 describes four cases where the original Samuelson rule is upwardly distorted. Similar to Table 1, Case I exhibits the results when the above three constraints are binding and the λ envy-free constraint for the low-type is slack, and Cases II, III, and IV indicate results when all the four constraints are binding. As with the results in Table 1, low-skilled utility decreases as λ increases. However, in contrast to the results in Table 1, the provision level increases under the reduction of envy and increases much more as λ increases. That is, the government mitigates the intensified envy by the public good provision rather than by the income redistribution.

4. Mixed taxation

Here, we examine the optimal provision rule for public goods when the government employs not only labor income but also commodity taxes. We assume that the government can only levy linear commodity taxes, since it cannot observe individuals' consumption levels.

Again, we define the identical utility function of agent i as $U(c_i, x_i, G, l_i)$, where c_i is a numéraire commodity and x_i another commodity. The producer price of commodity x is constant and normalized to unity for simplicity. While the government cannot impose any taxes on the numéraire good, it imposes proportional commodity tax t on x_i. For simplicity, we assume that $n_H = n_L = 1$, which does not affect the tax schedule crucially. The other notations are the same as in the previous section.

Following Mirrlees (1976) and Jacobs and Boadway (2014), we decompose individual optimization into two stages. In the first stage, each agent chooses the amount of labor supply given nonlinear income taxes, which allows us to determine disposable income $R_i \equiv w_i l_i - T(w_i l_i)$. In the second stage, each agent expenses his/her disposable income to consume a numéraire and another commodity. We assume that individuals anticipate the outcome for the second stage in the first stage. Now, we formally analyze individuals' problem. In the second stage, given $\{p, R_i, G, l_i\}$, agent i chooses c_i and x_i to maximize utility $U(c_i, x_i, G, l_i)$ subject to budget constraint $c_i + p x_i = R_i$, where $p \equiv 1 + t$ is the consumer price with respect to another commodity. The first-order conditions with respect to c_i and x_i yield

$$\frac{U_x^i}{U_c^i} = p. \tag{6}$$

The maximization problem in the second stage yields conditional commodity demands with respect to a numéraire and another commodity denoted by $c_i^* \equiv c(p, R_i, G, l_i)$ and $x_i^* \equiv x(p, R_i, G, l_i)$, respectively. As a result, substituting these solutions into the utility function yields a conditional indirect utility function, $V_i \equiv V(p, R_i, G, l_i) \equiv U(c_i^*, x_i^*, G, l_i)$. Let V_p^i, V_R^i, V_G^i, and V_l^i be the partial derivatives of V_i with respect to p, R_i, G, and l, respectively. From Roy's identity and the Slutsky decomposition, we obtain the following relationship:

$$-\frac{V_p^i}{V_R^i} = x_i^*, \tag{7}$$

$$\frac{\partial x_i^*}{\partial p} = \frac{\partial \tilde{x}_i}{\partial p} - \frac{\partial x_i^*}{\partial R_i} \cdot x_i^*, \tag{8}$$

$$\frac{\partial x_i^*}{\partial G} = \frac{\partial \tilde{x}_i}{\partial G} + \frac{\partial x_i^*}{\partial R_i} \frac{V_G^i}{V_R^i}, \tag{9}$$

$$\frac{\partial c_i^*}{\partial p} = \frac{\partial \tilde{c}_i}{\partial p} - \frac{\partial c_i^*}{\partial R_i} \cdot x_i^*, \tag{10}$$

$$\frac{\partial c_i^*}{\partial G} = \frac{\partial \tilde{c}_i}{\partial G} + \frac{\partial c_i^*}{\partial R_i} \frac{V_G^i}{V_R^i}, \tag{11}$$

where $\tilde{c}_i$ and $\tilde{x}_i$ indicate the compensated conditional demands of individual i for the numéraire and the taxable good, respectively.

In the first stage, each agent chooses the amount of labor supply to maximize conditional indirect utility V_i subject to $R_i = w_i l_i - T(w_i l_i)$. The first-order condition is given by

$$-\frac{V_l^i}{w_i V_R^i} = -\frac{U_l^i}{w_i U_c^i} = 1 - T'(w_i l_i). \tag{12}$$

As above, the government faces budget constraint, self-selection constraint to prevent high-skilled workers from mimicking low-skilled ones, and λ-equitability constraint for reducing envy. We respectively formulate these as follows:

$$\sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] \geq \phi(G), \tag{13}$$

$$V(p, R_H, G, l_H) \geq V(p, R_L, G, \frac{w_L}{w_H} l_L) \equiv \hat{V}, \tag{14}$$

$$V(p, R_L, G, l_L) \geq U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H)) \equiv \bar{V}. \tag{15}$$

To sum up, the restricted Pareto optimization problem to the government is given by

$$\max_{\{R_i, l_i\}_{i=L,H}, p, G} V(p, R_L, G, l_L),$$

subject to

$$\begin{aligned}
& V(p, R_H, G, l_H) \geq \bar{u} \\
& \sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] \geq \phi(G) \\
& V(p, R_H, G, l_H) \geq V(p, R_L, G, \frac{w_L}{w_H} l_L) \\
& V(p, R_L, G, l_L) \geq U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H)).
\end{aligned}$$

The Lagrangian is

$$\begin{aligned}
\mathcal{L}(p, R_L, R_H, l_L, l_H, G; \mu, \gamma, \delta, \eta) =& V(p, R_L, G, l_L) + \mu\{V(p, R_H, G, l_H) - \bar{u}\} \\
& + \gamma\{\sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] - \phi(G)\} \\
& + \delta\{V(p, R_H, G, l_H) - V(p, R_L, G, \frac{w_L}{w_H} l_L)\} \\
& + \eta\{V(p, R_L, G, l_L) - U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))\},
\end{aligned} \tag{16}$$

where μ, γ, δ and η are the Lagrangian multipliers corresponding to the constraints, respectively. Appendix B shows the first-order conditions with respect to the Lagrangian.

Before analyzing the provision rule for public goods, it is useful to explore the optimal linear commodity tax rate. Let $\hat{V}_p$, $\hat{V}_R$, and $\hat{V}_G$ be the partial derivatives of $\hat{V}$ with respect to p, R, and G. The linear commodity tax rate is characterized by the following proposition.

Proposition 2. *Assume that the allocations are restricted by reduction of envy. The optimal commodity tax rate under the nonlinear labor income tax and public good provision is given by*

$$t \sum_{i=H,L} \frac{\partial \tilde{x}_i}{\partial p} = \frac{\delta}{\gamma} \hat{V}_R (x_L^* - \hat{x}) + \frac{\lambda \eta}{\gamma} \left[\bar{U}_c \frac{\partial \tilde{c}_H}{\partial p} + \bar{U}_x \frac{\partial \tilde{x}_H}{\partial p} \right], \tag{17}$$

where $\hat{x} \equiv x(p, R_L, G, \frac{w_L}{w_H} l_L)$ is the mimicker's demand for another commodity and $\bar{U}_r$ ($r = c, x$) the derivative of r at the λ-scaled bundle $(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))$.

The first term on the right-hand side is the self-selection effect; if the agent's utility is separable between the commodity part and labor supply term, then it must disappear. We see this effect frequently in existing studies on mixed taxation. However, the second term on the right-hand side is the original part for reducing envy, as seen in Nishimura (2003a,b). Each term between the brackets is the inner product of the marginal utility of the low-skilled agent and the substitution effect of the compensated demand, which reflects the reduction of envy by discouraging the high-skilled agent's consumption because of taxation. Moreover, the second term on the right-hand side can be rewritten as:[10]

$$\frac{\lambda\eta}{\gamma}\left[\bar{U}_c\frac{\partial\tilde{c}_H}{\partial p}+\bar{U}_x\frac{\partial\tilde{x}_H}{\partial p}\right]=\frac{\partial\tilde{x}_H}{\partial p}\bar{U}_c\left[\frac{\bar{U}_x}{\bar{U}_c}-\frac{U_x^H}{U_c^H}\right]\equiv\frac{\partial\tilde{x}_H}{\partial p}\bar{U}_c\left[M\bar{R}S_{cx}-MRS_{cx}\right]. \qquad (18)$$

If the envying agent prefers the taxable good to the numéraire more than the envied agent, namely $M\bar{R}S_{cx} > MRS_{cx}$, it is taxed more heavily. This term remains even if the utility function is weakly separable between the public and private goods (taken together) and leisure, namely $U(H(c_i, x_i, G), l_i)$, while the first term on the right-hand side of equation (17) disappears. To replicate the Atkinson and Stiglitz (1976) theorem (hereafter, A-S theorem), we assume the following functional form: $H(f(c_i, x_i), G)$, where $f(\cdot)$ is homothetic. In this case, the second term on the right-hand side of equation (18) disappears, which means that commodity taxation is superfluous. The sufficient condition to hold the A-S theorem is slightly different from that in Nishimura (2003a,b), since we impose an additional restriction, namely weak separability between all types of private consumption and the public good.

We now characterize the optimal provision rule for the public good. The optimal rule with respect to public good provision can be derived as in the next proposition.

Proposition 3. *Under linear commodity tax in addition to nonlinear income tax, the optimal provision rule considering reduction of envy is characterized by*

$$\sum_{i=H,L}\frac{V_G^i}{V_R^i}+\frac{\delta}{\gamma}\hat{V}_R\left[\frac{V_G^L}{V_R^L}-\frac{\hat{V}_G}{\hat{V}_R}\right]-\frac{\eta}{\gamma}\left[\bar{U}_c\frac{\partial\lambda\tilde{c}_H}{\partial G}+\bar{U}_x\frac{\partial\lambda\tilde{x}_H}{\partial G}+\bar{U}_G\right]=\phi'(G)-t\sum_{i=H,L}\frac{\partial\tilde{x}_i}{\partial G}, \qquad (19)$$

where $\bar{V}_k$ is the derivative of $\bar{V}=U(\lambda c_H^*, \lambda x_H^*, G, \bar{l}-\lambda(\bar{l}-l_H))$ with respect to $k = G, R$.

On the left-hand side, the first term amounts to the sum of the evaluation for public good G based on marginal utility for disposable income R and the second term is the self-selection effect. The remaining part corresponds to the λ-equitability effect, which is different from that of Nava et al. (1996). This part consists of two effects. The first is the indirect effect, which is the inner product of the marginal utility of the low-skilled agent and the substitution effect of the compensated demand; this reflects a reduction of envy by discouraging consumption by the high-skilled agent because of the provision of the public good. The second is the direct effect, which reduces envy by decreasing the amount of the public good. On the right-hand side, the first term is the marginal cost of the public good and the second term is analogous to Nava et al. (1996), which means that the impact on indirect tax revenue increases the provision level through the compensated effects on consumption for a change in the level.

When can we apply the original Samuelson rule in this case? The third term on the left-hand side of equation (19) can be manipulated to yield

$$-\frac{\eta}{\gamma}\left[\bar{U}_c\frac{\partial\lambda\tilde{c}_H}{\partial G}+\bar{U}_x\frac{\partial\lambda\tilde{x}_H}{\partial G}+\bar{U}_G\right]=\frac{\lambda\eta}{\gamma}\bar{U}_c\left[\frac{U_G^H}{U_c^H}-\frac{1}{\lambda}\frac{\bar{U}_G}{\bar{U}_c}\right]+\frac{\lambda\eta}{\gamma}\frac{\partial\tilde{x}_H}{\partial G}\bar{U}_c\left[\frac{U_x^H}{U_c^H}-\frac{\bar{U}_x}{\bar{U}_c}\right]$$
$$\equiv\frac{\lambda\eta}{\gamma}\bar{U}_c\left[MRS_{Gc}-\frac{1}{\lambda}M\bar{R}S_{Gc}\right]+\frac{\lambda\eta}{\gamma}\frac{\partial\tilde{x}_H}{\partial G}\bar{U}_c\left[MRS_{cx}-M\bar{R}S_{cx}\right]. \quad (20)$$

Following the analysis above, if the agent's utility is expressed by $U(H(c_i, x_i, G), l_i)$, then the second term on the left-hand side of equation (19), which is the self-selection term, disappears. In addition, if function H meets the following functional form: $H(f(c_i, x_i), G)$, where $f(\cdot)$ is homothetic, the second term on the right-hand side of equation (20) must disappear, since $MRS_{cx} = M\bar{R}S_{cx}$ holds. At the same time, the second term on the right-hand side of equation (19) also disappears since t is zero, as shown above. Therefore, as in the analysis without linear commodity tax, whether to deviate from the original Samuelson rule depends on the first term on the right-hand side of equation (20).

As in subsection 2.3, we investigate the direction of the distortions when the utility function takes the CES form and has weak separability between labor and the other variables. Let the utility function be $H = (\alpha f(\cdot)^\rho + \beta G^\rho)^{\frac{1}{\rho}}$, where $\rho \leq 1$ and $f(\cdot)$ is homothetic. In this setting, the first term on the right-hand side of equation (20) can be rewritten as

$$MRS_{Gc}-\frac{1}{\lambda}M\bar{R}S_{Gc}=(1-\lambda^{-\rho})\frac{\beta G^{\rho-1}}{\alpha f(\cdot)^{\rho-1}f_c(\cdot)}.$$

Therefore, whether the original Samuelson condition is valid depends crucially on the elasticity of substitution. As with the result of Corollary 1, if the elasticity of substitution is above (below) one, the optimal provision rule is downwardly (upwardly) distorted, although the original Samuelson condition holds when the elasticity of substitution equals one.[11]

5. Conclusion

In this study, we analyze the optimal policy for income taxation with public good provision by a government concerned with ethical constraint, namely, the reduction of envy. As the new constraint, we use the λ-equitability by Diamantaras and Thomson (1990). To provide the public good, we then derive the optimal provision rule as well as the marginal income tax rate in the optimal policy. Although the income tax part is the same as the results of Nishimura (2003a,b), the modified provision rule includes the effect of reducing envy, which is different from the modified Samuelson rule in Boadway and Keen (1993). To relax the ethical constraint, we adjust the amount of the provided public good to compare the evaluation of low-skilled agents with that of the referred commodity bundle. For instance, if an agent with the envied bundle places more weight on the public good than the low-skilled agent, he/she must decrease the provision level to use more tax income for redistribution. Furthermore, by using CES utility for the public good and private consumption, we show that if the elasticity of substitution is above (below) one, the original Samuelson condition is downwardly (upwardly) distorted. However, the original rule is valid if the elasticity of substitution is one. As an extension, we add a taxable consumption good and linear commodity tax and study both the optimal tax rate and the provision rule of the public good.

There are two policy implications from our model. First, when paying attention to the reduction of envy, the government must deal with the envied λ-scale bundle relative to the original bundle. Consequently, the government decreases the public good provision when the elasticity of substitution between private consumption and public good is below one. In other words, a change in the ratio of their marginal utility is sensitive to variations in the ratio of these volumes. The second implication is that the public good provision increases much more or decreases much less as the intensity of envy increases. Since increasing the degree tightens the envy-free constraint, policymakers cannot use the other redistribution scheme; instead, they must reinforce the distorted direction of public good provision.

In Scandinavian countries, citizens place more weight on egalitarianism. Hence, governments or tax authorities should pay attention to the overall coverage of social security to reduce inequalities. They rely on the services provided by national systems, and the shares of social security are always relatively high in their budgets. According to OECD (2015), the 2015 Gini indexes of these countries (e.g., Sweden, Norway, Finland, and Denmark) where people place more weight on egalitarianism such as social justice were lower than the OECD average, while the government spending in 2015 on social protection in these countries was higher than the OECD average (see OECD (2017)). On the other hand, residents in the United States believe that most services must be accessible on the market and, thus, the government's budget should decrease; they do not think it is necessary to make access to social security universal. In reality, despite such fiscal policies being framed, President Trump has cut the budgets for social security and Medicare, breaking the major promises to his target voters (i.e., the poor).

Several tasks are left to future research. First, because we derive only the modified Samuelson rule, there is room to derive the provision level in general cases, divided into types of taxpayers' utility functions. Second, the plan of future studies is to seek intensity λ guaranteeing the binding envy-free constraint at the optimum. Finally, related to the exogenous index λ, it would be interesting to conduct comparative statics of public good provision for λ analytically.

Acknowledgments

We would like to thank our supervisors, Helmuth Cremer, Jean-Marie Lozachmeur, and Motohiro Sato, for their invaluable comments and discussions. We are also grateful to Francesca Barigozzi, Takashi Kunimoto, Francçis Maniquet, Kaname Miyagishima, Pierre Pestieau, Takuro Yamashita, and the anonymous referees for their helpful comments. In addition, we thank the seminar participants at 18th meeting of the Association for Public Economic Theory in July 2017, Economic Theory and Policy Workshop at the Tokyo Metropolitan University in September 2017, the 23rd Decentralization Conference and Japanese Economic Association Autumn Meeting at Aoyama Gakuin University in September 2017, and the 14th Meeting of the Society for Social Choice and Welfare in June 2018.

Appendix A

Assume that $\delta_{sH} > 0$ and $\delta_{sL} = 0$. Differentiating Lagrangian (4) with respect to c_L, c_H, l_L, l_H and G,

$$\frac{\partial \mathcal{L}}{\partial c_H} = (\gamma + \delta_{sH})U_c^H - \delta_r n_H - \delta_e \lambda \bar{U}_c = 0, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial c_L} = (1 + \delta_e)U_c^L - \delta_r n_L - \delta_{sH} \hat{U}_c = 0, \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial l_H} = (\gamma + \delta_{sH})U_l^H + \delta_r n_H w_H - \delta_e \lambda \bar{U}_l = 0, \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial l_L} = (1 + \delta_e)U_l^L + \delta_r n_L w_L - \delta_{sH} \frac{w_L}{w_H} \hat{U}_l = 0, \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial G} = (\gamma + \delta_{sH})U_G^H + (1 + \delta_e)U_G^L - \delta_r \phi'(G) - \delta_e \bar{U}_G - \delta_{sH} \hat{U}_G = 0. \tag{A.5}$$

Rearranging (A.1) and (A.3) yields the optimal marginal income tax rate at the top. Conversely, we can derive the marginal income tax rate at the bottom by combining equations (A.2) and (A.4). The provision rule for the public good is obtained by substituting equations (A.1) and (A.2) into (A.5). □

Appendix B

Differentiating Lagrangian (20) with respect to p, R_L, R_H, and G,

$$\frac{\partial \mathcal{L}}{\partial p} = (1 + \eta)V_p^L + (\mu + \delta)V_p^H - \delta \hat{V}_p - \eta \bar{V}_p + \gamma \sum_{i=H,L} [x_i^* + (p-1)\frac{\partial x_i^*}{\partial p}] = 0, \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial R_H} = (\mu + \delta)V_R^H - \eta \bar{V}_R - \gamma + \gamma(p-1)\frac{\partial x_H^*}{\partial R_H} = 0, \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial R_L} = (1 + \eta)V_R^L - \delta \hat{V}_R - \gamma + \gamma(p-1)\frac{\partial x_L^*}{\partial R_L} = 0, \tag{B.3}$$

$$\frac{\partial \mathcal{L}}{\partial G} = (1 + \eta)V_G^L + (\mu + \delta)V_G^H - \delta \hat{V}_G - \eta \bar{V}_G + \gamma \sum_{i=H,L} (p-1)\frac{\partial x_i^*}{\partial G} - \gamma \phi'(G) = 0. \tag{B.4}$$

Equations (B.1), (B.2), and (B.3) give

$$\frac{\partial \mathcal{L}}{\partial p} + \sum_i \frac{\partial \mathcal{L}}{\partial R_i} x_i^* = 0. \tag{B.5}$$

By using equations (7), (8), (10), and $\hat{x} = -\frac{\hat{V}_p}{\hat{V}_R}$, equation (B.5) can be transformed into equation (17). In addition, by substituting equations (B.2) and (B.3) into (B.4) and using equations (9) and (11), we can derive equation (19).

□

Notes

[1]Boadway and Keen (1993) do not clarify how the amount of public good is distorted in the second best compared to the first best. Gaube (2005) provides a sufficient condition for both a lower and higher level of public expenditure in the second best than in the first best.

[2]To clarify how status effects should be reflected in the optimal provision rule of public goods expressed by equation (5), we will characterize the second-best Samuelson rule under reduction of envy when agents have other-regarding preferences in future research. Velez (2016) and Nakada (2018) explore the equitable allocation of multiple indivisible goods and money among agents with other-regarding preferences. However, they do not analyze the second-best provision rule for public goods under the setting.

[3]From the self-selection constraint for high-skilled agents, the following inequality holds: $U(c_H, G, l_H) \geq U(c_L, G, \frac{w_L}{w_H} l_L) > U(c_L, G, l_L)$. Therefore, the high-skilled agent never envies the low-skilled agent, which means that the envy-free constraint for the low-skilled agent is not satisfied.

[4]Nishimura (2000) presents the tax policy implications under the Pareto-efficient allocations that maximize λ as in Diamantaras and Thomson (1990). He also shows that envy is minimized at the leximin allocation that maximizes the utility of the low-skilled agent. By contrast, this study examines the second-best Pareto-efficient allocations corresponding to various λ, as in Nishimura (2003a,b).

[5]Nishimura (2003b) demonstrates that the second-best frontier with the λ envy-free constraint gradually shrinks as λ increases. Indeed, so long as δ_e is positive, $\mathcal{L}$ decreases λ.

[6]Nishimura (2003b) also examines these marginal income tax rates when the self-selection constraint for low-skilled workers is binding.

[7]Note that the difference in the MRS between consumption and labor, not the efficiency-unit labor, between the envying and the envied agent is useful information for the government, since the λ envy-free constraint allows it to consider a proportional decrease of the envied agent's bundle.

[8]According to the definition of Nishimura (2003b), if the income elasticity of leisure is greater (less) than 1, leisure is called a luxury (necessity).

[9]Consider that $H(\cdot)$ is homogeneous of degree k in c. The third term can be rewritten as follows: $\frac{\delta_e}{\delta_r} \lambda \bar{U}_c \left(\frac{H_G(G, c_H)}{H_{c_H}(G, c_H)} - \frac{H_G(G, \lambda c_H)}{\lambda H_{c_H}(G, \lambda c_H)} \right) = \frac{\delta_e}{\delta_r} \lambda \bar{u}_c^H \left(\frac{H_G(G, c_H)}{H_{c_H}(G, c_H)} - \frac{\lambda^k H_G(G, c_H)}{\lambda \times \lambda^{k-1} H_{c_H}(G, c_H)} \right) = 0$. As such, we obtain $MRS_{Gc}^H = \frac{1}{\lambda} M\bar{R}S_{Gc}$.

[10]By using individuals' budget constraint $c_i + px_i = R_i$, the following relationship holds: $\frac{\partial \tilde{c}_H}{\partial p} = -p \frac{\partial \tilde{x}_H}{\partial p}$.

[11]In general, if H is homogeneous of degree j in f on $H = H(f(c_i, x_i), G)$ and $f(\cdot)$ is homothetic under weak separability between labor and the other variables, the original Samuelson rule holds. The first term on the right-hand side of equation (20) can be rewritten as $\frac{\lambda \eta}{\gamma} \bar{U}_c \left(\frac{H_G(f(c_H, x_H), G)}{H_c(f(c_H, x_H), G) f_c(c_H, x_H)} - \frac{H_G(f(\lambda c_H, \lambda x_H), G)}{\lambda H_c(f(\lambda c_H, \lambda x_H), G) f_c(\lambda c_H, \lambda x_H)} \right) = \frac{\lambda \eta}{\gamma} \bar{U}_c \left(\frac{H_G(f(c_H, x_H), G)}{H_c(f(c_H, x_H), G) f_c(c_H, x_H)} - \frac{\lambda^{kj} H_G(f(c_H, x_H), G)}{\lambda \times \lambda^{kj-1} H_c(f(c_H, x_H), G) f_c(c_H, x_H)} \right) = 0$. Therefore, the λ-equitability term disappears and the original Samuelson rule is replicated.

References

Aronsson, T., Johansson-Stenman, O., 2008. When the Joneses' consumption hurts: optimal public good provision and nonlinear income taxation. *Journal of Public Economics* 92, 986–997.

Aronsson, T., Johansson-Stenman, O., 2018. Paternalism against Veblen: optimal taxation and non-respected preferences for social comparisons. *American Economic Journal: Economic Policy* 10, 39–76.

Atkinson, A. B., Stiglitz, J. E., 1976. The design of tax structure: direct versus indirect taxation. *Journal of Public Economics* 6, 55–75.

Balestrino, A., 2009. Tax avoidance, endogenous social norms, and the comparison income effect, CHILD Working Paper No.15.

Besley, T., 1988. A simple model for merit good arguments. *Journal of Public economics* 35, 371–383.

Besley, T., Coate, S., 1992. Workfare versus welfare: incentive arguments for work requirements in poverty-alleviation programs. *American Economic Review* 82, 249–261.

Besley, T., Coate, S., 1995. The design of income maintenance programmes. *Review of Economic Studies* 62, 187–221.

Besley, T., Kanbur, R., 1988. Food subsidies and poverty alleviation. *Economic Journal* 98, 701–719.

Boadway, R., Keen, M., 1993. Public goods, self-selection and optimal income taxation. *International Economic Review* 34, 463–478.

Boadway, R., Marchand, M., Pestieau, P., Racionero, M., 2002. Optimal redistribution with heterogenous preference for leisure. *Journal of Public Economic Theory* 4, 475–498.

Bös, D., Tillmann, G., 1985. An envy tax: theoretical principles and applications to the German surcharge on the rich. *Public Finance* 40, 35–63.

Boskin, M. J., Sheshinski, E., 1978. Optimal redistributive taxation when individual welfare depends upon relative income. *Quarterly Journal of Economics* 92, 589–601.

Bowles, S., Park, Y., 2005. Emulation, inequality, and work hours: was Thorsten Veblen right? *Economic Journal* 115, 397–412.

Bruce, D., Peng, L., 2018. Optimal taxation in the presence of income-dependent relative income effects. *Social Choice and Welfare* 51, 1–23.

Chetty, R., Guren, A., Manoli, D., Weber, A., 2011. Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. *American Economic Review* 101, 471–475.

Corneo, G., 2002. The efficient side of progressive income taxation. *European Economic Review* 46, 1359–1368.

Diamantaras, D., Thomson, W., 1990. A refinement and extension of the no-envy concept. *Economics Letters* 33, 217–222.

Edwards, J., Keen, M., Tuomala, M., 1994. Income tax, commodity taxes and public good provision: A brief guide. *FinanzArchiv* 51, 472–497.

Fang, H., 2006. Disentangling the college wage premium: estimating a model with endogenous education choices. *International Economic Review* 47, 1151–1185.

Fleurbaey, M., Maniquet, F., 2006. Fair income tax. *Review of Economic Studies* 73, 55–83.

Foley, D. K., 1967. Resource allocation and the public sector. *Yale Economic Essays* 7.

Gaube, T., 2005. Financing public goods with income taxation: provision rules vs. provision level. *International Tax and Public Finance* 12, 319–334.

Goldin, C., Katz, L. F., 2007. The race between education and technology: the evolution of U.S. educational wage differentials, NBER Working Paper, No. 12984.

Harsanyi, J., 1982. Morality and the theory of rational behavior. In A. Sen and B. Williams (eds.), *Utilitarianism and Beyond*. Cambridge: Cambridge University Press.

Ireland, N. J., 2001. Optimal income tax in the presence of status effects. *Journal of Public Economics* 81, 193–212.

Jacobs, B., Boadway, R., 2014. Optimal linear commodity taxation under optimal non-linear income taxation. *Journal of Public Economics* 117, 201–210.

Kanbur, R., Keen, M., Tuomala, M., 1994. Optimal non-linear income taxation for the alleviation of income-poverty. *European Economic Review* 38, 1613–1632.

Kanbur, R., Paukkeri, T., Pirttilä, J., Tuomala, M., 2018. Optimal taxation and public provision for poverty reduction. *International Tax and Public Finance* 25, 64–98.

Kanbur, R., Pirttilä, J., Tuomala, M., 2006. Non-welfarist optimal taxation and behavioral public economics. *Journal of Economic Surveys* 20, 849–868.

Kanbur, R., Tuomala, M., 2013. Relativity, inequality, and optimal nonlinear income taxation. *International Economic Review* 54, 1199–1217.

Kolm, S., 1972. Justice et équité, Editions du centre national de la recherche scientifique. Paris.

Micheletto, L., 2011. Optimal nonlinear redistributive taxation and public good provision in an economy with Veblen effects. *Journal of Public Economic Theory* 13, 71–96.

Mirrlees, J., 1976. Optimal tax theory: a synthesis. *Journal of Public Economics* 6, 327–358.

Naito, H., 1999. Re-examination of uniform commodity taxes under a non-linear income tax system and its implication for production efficiency. *Journal of Public Economics* 71, 165–188.

Nakada, S., 2018. Envy-free allocation of indivisible goods with money and externalities. *Economics Bulletin* 38, 52–59.

Nava, M., Schroyen, F., Marchand, M., 1996. Optimal fiscal and public expenditure policy in a two-class economy. *Journal of Public Economics* 61, 119–137.

Nishimura, Y., 2000. Envy minimization in the optimal tax context, Paper presented at the Fifth International Conference of Society of Social Choice and Welfare, University of Alicante, Spain.

Nishimura, Y., 2003a. Optimal commodity taxation for reduction of envy. *Social Choice and Welfare* 21, 501–527.

Nishimura, Y., 2003b. Optimal non-linear income taxation for reduction of envy. *Journal of Public Economics* 87, 363–386.

O'Donoghue, T., Rabin, M., 2003. Studying optimal paternalism, illustrated by a model of sin taxes. *American Economic Review* 93, 186–191.

O'Donoghue, T., Rabin, M., 2006. Optimal sin taxes. *Journal of Public Economics* 90, 1825–1849.

OECD, 2015. How's life? 2015 measuring well-being.

OECD, 2017. General government spending.

Oswald, A. J., 1983. Altruism, jealousy and the theory of optimal non-linear taxation. *Journal of Public Economics* 20, 77–87.

Persson, M., 1995. Why are taxes so high in egalitarian societies? *Scandinavian Journal of Economics* 97, 569–580.

Pirttilä, J., Tuomala, M., 2001. On optimal non-linear taxation and public good provision in an overlapping generations economy. *Journal of Public Economics* 79, 485–501.

Pirttilä, J., Tuomala, M., 2004. Poverty alleviation and tax policy. *European Economic Review* 48, 1075–1090.

Racionero, M., 2001. Optimal tax mix with merit goods. *Oxford Economic Papers* 53, 628–641.

Rawls, J., 1971. A theory of justice. Cambridge, Harvard University Press.

Sandmo, A., 1983. Ex post welfare economics and the theory of merit goods. *Economica* 50, 19–33.

Schokkaert, E., Van de gaer, D., Vandenbroucke, F., Luttens, R. I., 2004. Responsibility sensitive egalitarianism and optimal linear income taxation. *Mathematical social sciences* 48, 151–182.

Schroyen, F., 2005. An alternative way to model merit good arguments. *Journal of Public Economics* 89, 957–966.

Seidman, L. S., 1987. Relativity and efficient taxation. *Southern Economic Journal* 54, 463–474.

Stern, N., 1982. Optimum taxation with errors in administration. *Journal of Public Economics* 17, 181–211.

Stiglitz, J. E., 1982. Self-selection and Pareto efficient taxation. *Journal of Public Economics* 17, 213–240.

Varian, H., 1974. Equity, envy, and efficiency. *Journal of Economic Theory* 9, 63–91.

Varian, H., 1976. Two problems in the theory of fairness. *Journal of Public Economics* 5, 249–260.

Velez, R. A., 2016. Fairness and externalities. *Theoretical Economics* 11, 381–410.

World Economic Forum, 2017. The global risks report 2017 12th edition.

Table 1: Numerical examples under $\rho = 1$

	Low-skill utility	G
Case I Second best without the λ envy-free constraint	0.228045	0.499994
Case II Second best with the λ envy-free constraint (λ=0.89)	0.225014	0.497525
Case III Second best with the λ envy-free constraint (λ=0.91)	0.182929	0.489892
Case IV Second best with the λ envy-free constraint (λ=0.92)	0.0894965	0.450079

Table 2: Numerical examples under $\rho = -1$

	Low-skill utility	G
Case I Second best without the λ envy-free constraint	-0.406546	0.781823
Case II Second best with the λ envy-free constraint (λ=0.785)	-0.406572	0.78252
Case III Second best with the λ envy-free constraint (λ=0.79)	-0.428375	0.802776
Case IV Second best with the λ envy-free constraint (λ=0.792)	-0.481632	0.8229