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Can the optimal tariff be zero for a growing large country?*

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Abstract

Can the optimal tariff be zero for a growing large country? To pursue the possibility, we extend the Rivera-Batiz–Romer lab-equipment model of endogenous technological change to include asymmetric countries, import tariffs, and either homogeneous or heterogeneous firms. Each country's domestic revenue share is a sufficient statistic for its long-run growth rate, but it is not for its long-run welfare. A unilateral tariff reduction by either country always increases the balanced growth rate. A zero tariff is locally optimal for a country under a mild condition, which is automatically satisfied at a symmetric balanced growth path with the zero tariff.

JEL classification: F13; F43

Keywords: Optimal tariff; Large country; Lab-equipment model; Endogenous technological change; Heterogeneous firms

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1 Introduction

It has been taken for granted among trade economists that the optimal tariff for a large country is positive.[1] Suppose that a large country increases a tariff on its imported good. On the one hand, this incurs distortions in consumption and production, which harm the country's welfare. On the other hand, the tariff-induced decrease in the country's import demand for the good drives down its world price (otherwise, the country would be a small country), and the resulting improvement in the country's terms of trade benefits its welfare. Since the former effect is negligible around a free trade equilibrium, increasing the tariff from zero necessarily increases the country's welfare. The optimal tariff, if exists, must balance out the negative distortionary effect and the positive terms of trade effect. The positivity of the optimal tariff for a large country is widely confirmed in general equilibrium trade models, from the two-good standard trade model (e.g., Kaldor, 1940)[2] to the one-sector Melitz (2003) model (e.g., Felbermayr et al., 2013; Demidova, 2017).[3]

A theoretical and practical argument against the implementation of the optimal tariff theory is that it results in the prisoner's dilemma. A tariff of country 1 harms country 2 through the latter's terms of trade deterioration, and vice versa. Due to the negative externalities, each country's welfare at a Nash equilibrium of a "tariff war" is typically lower than that at a free trade equilibrium, that is, the Nash equilibrium is Pareto inferior to the free trade equilibrium.[4] A solution recommended by trade economists is for countries to commit to reciprocity in tariff reductions, one of the founding principles of the GATT/WTO (e.g., Bagwell and Staiger, 1999). However, the reciprocity argument is weak in that each country has an incentive to gain by deviating unilaterally from the free trade regime, as the recent U.S.-China trade disputes suggest. Can there be a model in which the optimal tariff is zero for a large country? If so, then the model will contribute to supporting global free trade more strongly. The purpose of this paper is to explore such a possibility.

We consider the role of economic growth in pursuing the possibility of a zero optimal tariff for a large country. While Rodriguez and Rodrik (2000) question the robustness of some major empirical studies reporting the positive relationship between trade liberalization and economic growth, more recent well-designed empirical papers such as Wacziarg and Welch (2008) and Estevadeordal and Taylor (2013) do find that the positive liberalization-growth relationship is robust, thereby overcoming Rodriguez and Rodrik's (2000) criticism. If it is true, then an increase in a tariff of a large country generates an additional welfare loss through slower growth, which might pull the country's optimal tariff down to zero. To characterize a large country's optimal tariff with the other country's tariff given, we have to allow for asymmetric countries in a two-country endogenous growth model.

We extend the lab-equipment version of Rivera-Batiz and Romer (1991a) (RBR hereafter), the first and simplest two-country model of endogenous technological change, to include asymmetric countries and import tariffs. RBR consider two alternative specifications of R&D, namely the knowledge-driven specification (i.e., labor and public knowledge are used in R&D) and the lab-equipment specification (i.e., a composite final good is used in R&D). For the knowledge-driven specification, Devereux and Lapham (1994) and Feenstra (1996) point out that a balanced growth path (BGP) of an asymmetric RBR model is either stable or unstable, depending on whether there are international knowledge spillovers or not.[5] However, despite being regarded as the starting point of endogenous technological change models (e.g., Acemoglu, 2009), it is

[1] A large country is defined as a country whose behavior affects the world prices of its traded goods.
[2] Horwell and Pearce (1970), Bond (1990), and Ogawa (2007) characterize the optimal tariff structure in multi-good settings. A general consensus is that there exists at least one good whose trade is taxed at the optimum.
[3] Felbermayr et al. (2013) assume CES preferences, whereas Demidova (2017) allows for variable markups.
[4] Johnson (1953) and Kennan and Riezman (1988) point out that, when a country is substantially larger than the other country, the former could have a higher welfare at the Nash equilibrium than at the free trade equilibrium.
[5] A BGP is a path on which all variables grow at constant (including zero) rates.

unknown whether an asymmetric BGP is stable under the lab-equipment specification without international knowledge spillovers. We first have to check if our asymmetric lab-equipment model is well-behaved so that it is applied to the optimal tariff problem.

Our RBR lab-equipment model consists of two asymmetric countries, each of which has a nontradable final good sector, a tradable intermediate good sector, and a nontradable R&D (i.e., knowledge good) sector. The core of the model is the intermediate good sector, where each potential entrant uses the knowledge good (produced one-to-one from the final good in the R&D sector) as the fixed input, and the final good as the variable input, to produce a differentiated variety under monopolistic competition and homogeneous technologies. The most difficult part of the asymmetric lab-equipment model is to determine the relative price of the final goods of the two countries: it affects the two countries' price indices of the intermediate goods, which in turn affects the relative final good price. We have to take such general equilibrium interactions into account. We later replace homogeneous intermediate good firms with heterogeneous ones following Melitz (2003), but even with the increased complexity, our main results regarding the long-run growth and welfare effects of a tariff change are qualitatively robust.

Before studying the full general equilibrium effects of a tariff change, we derive long-run growth and welfare formulas in line with Arkolakis et al. (2012). We find that country j's long-run growth rate of the number of varieties depends only and negatively on its revenue share of domestic varieties just like the ACR (Arkolakis–Costinot–Rodríguez-Clare) welfare formula. This implies a stabilization mechanism of the model. For example, when country j's relative number of varieties increases, the country imports relatively less varieties. This makes that country more closed, thereby leading to its slower growth. However, country j's long-run welfare depends not only on its domestic revenue share, but also directly on its import tariff through its tariff revenue. Unlike Arkolakis et al. (2012) and the related studies considering iceberg trade costs as the only variable trade barriers, a country's domestic share, or sometimes called "autarkiness", is not a sufficient statistic for its welfare in our model with import tariffs. It is this property that leaves open the possibility that the optimal tariff for a large country is positive.

Based on the derived long-run growth and welfare formulas, we obtain the following main results. First, an increase in the import tariff of either country decreases the balanced growth rate. An increase in country 1's import tariff directly causes it to import less. For country 1's trade surplus to be cleared in equilibrium, its relative price of the final good increases so that it exports less. This increases country 2's domestic revenue share, which slows down its growth. Also, both the increase in country 1's import tariff and the resulting increase in its domestic expenditure share increase its domestic revenue share, thereby lowering its growth. Therefore, after the adjustment of the relative number of varieties, the new balanced growth rate is lower than the old one. This is true even if reallocations across heterogeneous firms are added to the model. In the literature on trade and endogenous growth with homogeneous firms (e.g., Rivera-Batiz and Romer, 1991b; Baldwin and Forslid, 1999) and heterogeneous firms (e.g., Baldwin and Robert-Nicoud, 2008; Dinopoulos and Unel, 2011; Sampson, 2016; Ourens, 2016; Naito, 2017, 2019; Impullitti and Licandro, 2018; Akcigit et al., 2018; Fukuda, forthcoming; Perla et al., 2019), only a few papers allow for asymmetric trade liberalization (e.g., Baldwin and Forslid, 1999; Naito, 2017, 2019; Akcigit et al., 2018), and only Akcigit et al. (2018) study the long-run effects of a unilateral tariff change numerically. This paper analytically shows that even a unilateral tariff reduction always raises long-run growth.

Second, a zero tariff is locally optimal for country j if its export revenue share relative to country $k(\neq j)$'s is smaller than an upper bound, which is larger than one at a BGP with the zero tariff. An increase in country j's import tariff creates a welfare trade-off between gains from tariff revenue and losses from autarkiness.

The losses from autarkiness is relatively larger, the more patient countries are, and/or the more autarkic country j is before the tariff increase. If the above sufficient condition holds, then country j cannot gain by deviating unilaterally from its zero tariff. Moreover, the sufficient condition is automatically satisfied if the countries are similar, and/or the common subjective discount rate approaches zero. Therefore, zero optimal tariffs for large countries occur quite naturally in our model. We also supplement the above local analytical result with numerical experiments in a wider domain of ad valorem tariff rates from 0 to 100%. Numerical results allowing for different labor supplies indicate that: (i) in the homogeneous firm model, a zero tariff is optimal for both larger and smaller countries; and (ii) in the heterogeneous firm model, a zero tariff is optimal for a larger or equally large country, and also for a smaller country as long as its trading partner sets a zero optimal tariff. This implies that global free trade can be supported as a Nash equilibrium, whether the two countries are symmetric or asymmetric, and whether the intermediate good firms are homogeneous or heterogeneous.

A few recent papers find the possibility of a zero optimal tariff for a large country. Caliendo et al. (2017) extend the static Melitz–Pareto model to include two nontradable final good sectors, only one of which uses a variety of tradable intermediate goods. They show that a country's optimal tariff can be zero because a tariff reduction increases the number of entrants into the country's intermediate good industry, which was suboptimally low before the tariff reduction in their two-sector setup. In contrast, even without such intersectoral distortions, a large country's optimal tariff can be zero in our dynamic model. Akcigit et al. (2018) find the case of a zero optimal tariff based on the dynamic gains from trade liberalization in their two-country Schumpeterian growth model with heterogeneous firms. While allowing for transitional dynamics, their model is so complicated that most results are obtained numerically. By using a simpler RBR lab-equipment model and focusing on a BGP, we analytically show that the optimal tariff is quite likely to be zero for a growing large country.

The rest of this paper is organized as follows. Section 2 formulates a homogeneous firm lab-equipment model. Section 3 characterizes a BGP, and derives long-run growth and welfare formulas. Section 4 studies the long-run growth and welfare effects of a tariff change. Section 5 develops a heterogeneous firm lab-equipment model to do robustness checks. Section 6 numerically examines the global optimality of a zero tariff. Section 7 concludes.

2 A homogeneous firm lab-equipment model

To explore the possibility that the optimal tariff for a large country is zero in the simplest possible dynamic setting, we first build a RBR lab-equipment model with homogeneous firms, asymmetric countries, and import tariffs.[6] In country $j(=1,2)$, there are a nontradable final good sector, a tradable intermediate good sector, and a nontradable R&D (i.e., knowledge good) sector. The final good is produced from a variety of differentiated intermediate goods and labor under constant returns to scale and perfect competition. Each intermediate good is produced using the knowledge good as the fixed input, and the final good as the variable input. The knowledge good is produced from the final good under constant returns to scale and perfect competition.

2.1 Households

The utility maximization problem of the representative household in country j is given by:

[6] We omit non-tariff trade costs because adding them does not affect qualitative results.

$$
\begin{aligned}
\max: U_j &= \int_0^\infty \ln C_{jt} \exp(-\rho t) dt, j = 1, 2, \\
\text{s.t.}: \dot{W}_{jt} &= r_{jt} W_{jt} + w_{jt} L_j + T_{jt} - E_{jt}; \dot{W}_{jt} \equiv dW_{jt}/dt, E_{jt} \equiv p_{jt}^Y C_{jt}, \\
\text{given}: &\{r_{jt}, w_{jt}, T_{jt}, p_{jt}^Y\}_{t=0}^\infty, W_{j0},
\end{aligned}
$$

where $t (\in [0, \infty))$ is time (omitted whenever no confusion arises), U_j is the overall utility, C_j is consumption, ρ is the subjective discount rate, W_j is the asset, r_j is the interest rate, w_j is the wage rate, L_j is the exogenous supply of labor, T_j is the lump-sum transfer from country j's government, E_j is the consumption expenditure, and p_j^Y is the price of the final good. The second line represents the budget constraint. Parameters without country subscripts (e.g., ρ) are assumed to be the same across countries. It is straightforward to derive the Euler equation for E_j:

$$
\dot{E}_{jt}/E_{jt} = r_{jt} - \rho.
$$

2.2 Final good firms

The representative final good firm in country j solves the following problem:

$$
\begin{aligned}
\max: \pi_j^Y &= p_j^Y Y_j - \int_{\Theta_j} p_j(i) x_j(i) di - w_j L_j^Y, \\
\text{s.t.}: Y_j &= (X_j/\alpha_j)^{\alpha_j} [L_j^Y/(1-\alpha_j)]^{1-\alpha_j}; \alpha_j \in (0, 1), \\
: X_j &= (\int_{\Theta_j} x_j(i)^{(\sigma-1)/\sigma} di)^{\sigma/(\sigma-1)}; \sigma > 1, \\
\text{given}: &\, p_j^Y, \{p_j(i)\}_{i \in \Theta_j}, w_j,
\end{aligned}
$$

where π_j^Y is the profit of the representative final good firm, Y_j is the supply of the final good, Θ_j is the set of available varieties of intermediate goods, $p_j(i)$ is the demand price of variety i, $x_j(i)$ is the demand for variety i, L_j^Y is the demand for labor, X_j is the index of the intermediate goods, α_j is the Cobb-Douglas cost share of the intermediate goods, and σ is the elasticity of substitution across varieties. The second and third lines represent the production function for the final good and the intermediate good index function, respectively. Profit maximization is characterized by:

$$
p_j^Y = c_j^Y(P_j, w_j); c_j^Y(P_j, w_j) \equiv P_j^{\alpha_j} w_j^{1-\alpha_j}, P_j \equiv (\int_{\Theta_j} p_j(i)^{1-\sigma} di)^{1/(1-\sigma)} \Leftrightarrow p_j^Y Y_j = \int_{\Theta_j} p_j(i) x_j(i) di + w_j L_j^Y,
$$

where c_j^Y and P_j are the minimized cost to produce one unit of Y_j and the price index of the intermediate goods (i.e., the minimized cost to produce one unit of X_j), respectively. Because of constant returns to scale, profit maximization implies a zero profit.

2.3 Intermediate good firms

In country j, both the fixed entry cost $P_j^K \kappa_j^e$ and the unit final good requirement of one are the same for all intermediate good firms, where P_j^K is the price of the knowledge good, and κ_j^e is country j's one-time fixed entry cost in terms of the knowledge good. This allows us to omit the variety index i from now on. The profit maximization problem of a firm producing its differentiated variety in country j and selling it to market k is given by:

$$
\begin{aligned}
\max : \pi_{jk} &= p_{jk}^f y_{jk} - p_j^Y y_{jk}, j, k = 1, 2, \\
\text{s.t.} : y_{jk} &= x_{jk}, \\
: x_{jk} &= p_{jk}^{-\sigma} P_k^\sigma X_k = (\tau_{jk} p_{jk}^f)^{-\sigma} P_k^\sigma X_k; \tau_{jk} \geq 1, \tau_{jj} = 1, \\
\text{given} : p_j^Y&, P_k, X_k,
\end{aligned}
$$

where π_{jk} is the firm's profit, p_{jk}^f is the supply price of the firm's variety, y_{jk} is the supply of the firm's variety, x_{jk} is country k's demand for the firm's variety, p_{jk} is country k's demand price of the firm's variety, and $\tau_{jk}(\geq 1)$ is one plus country k's uniform and permanent ad valorem tariff rate on imports from country j (with $\tau_{jj} = 1$), the only policy variable in this paper.[7] The second and third lines represent the market-clearing condition and the conditional demand function for the firm's variety, respectively. The profit-maximizing supply price and the corresponding revenue, profit, and firm value are given by, respectively:

$$
\begin{aligned}
(p_{jk}^f - p_j^Y)/p_{jk}^f = 1/\sigma \Leftrightarrow p_{jk}^f &= p_j^Y/(1 - 1/\sigma), \\
e_{jk} \equiv p_{jk}^f y_{jk} &= \tau_{jk}^{-\sigma} [p_j^Y/(1 - 1/\sigma)]^{1-\sigma} P_k^\sigma X_k, \\
\pi_{jk} = e_{jk}/\sigma &= \tau_{jk}^{-\sigma} [p_j^Y/(1 - 1/\sigma)]^{1-\sigma} P_k^\sigma X_k/\sigma, \\
v_{jkt} &\equiv \int_t^\infty \pi_{jks} \exp(-\int_t^s (r_{ju} + \delta) du) ds,
\end{aligned}
$$

where δ is the exogenous rate of a bad shock forcing a firm to exit (e.g., Melitz, 2003).

Free entry requires that the fixed entry cost be equal to the sum of the firm values over all markets:

$$\textstyle\sum_k v_{jk} = P_j^K \kappa_j^e.$$

Finally, let n_j^e be the number of entrants in country j. Since all homogeneous firms sell their own unique products to all markets, n_j^e is also the number of varieties sold from country j to country k.

2.4 R&D firms

The representative R&D firm in country j solves the following problem:

[7] Applying Shephard's lemma to $\int_{\Theta_j} p_j(i) x_j(i) di = P_j X_j$ with $j = k$ gives $x_k(i) = (\partial P_k / \partial p_k(i)) X_k = p_k(i)^{-\sigma} P_k^\sigma X_k$.

$$\max : \pi_j^K = P_j^K Q_j^K - p_j^Y D_j,$$
$$\text{s.t.} : Q_j^K = D_j,$$
$$\text{given} : P_j^K, p_j^Y,$$

where π_j^K is the profit of the representative R&D firm, Q_j^K is the supply of the knowledge good, and D_j is the demand for the final good from the R&D sector. The second line represents the production function for the knowledge good. The first-order condition for profit maximization, which is equivalent to the zero profit condition, is given by:

$$P_j^K = p_j^Y \Leftrightarrow P_j^K Q_j^K = p_j^Y D_j.$$

2.5 Government

Country j's government budget constraint is given by:

$$T_j = \sum_k (\tau_{kj} - 1) n_k^e p_{kj}^f x_{kj}.$$

As usual, the government in country j collects its revenue only from its import tariff, and then transfers the revenue to country j's representative household.

2.6 Markets

The market-clearing conditions for the asset, labor, knowledge good, and final good are given by, respectively:

$$\begin{aligned}
W_j &= \sum_k n_j^e v_{jk}, j = 1, 2, \\
L_j &= L_j^Y, j = 1, 2, \\
Q_j^K &= \kappa_j^e (\dot{n}_j^e + \delta n_j^e), j = 1, 2, \\
Y_j &= C_j + D_j + F_j; F_j \equiv \sum_k n_j^e y_{jk}, j = 1, 2,
\end{aligned}$$

where F_j is the demand for the final good from the intermediate good sector.

Country j's Walras' law and its market-clearing conditions imply that:[8]

$$\begin{aligned}
\sum_k E_{jk} &= \sum_k E_{kj}; E_{jk} \equiv n_j^e e_{jk}, \\
E_{jk} &= E_{kj}, k \neq j,
\end{aligned}$$

where E_{jk} is country j's revenue of selling the intermediate goods to country k, or country k's expenditure for buying the intermediate goods from country j net of tariff. The first line shows country j's national budget constraint, saying that its total revenue of selling the intermediate goods to all destinations is equal to its

[8]Time differentiating country j's asset market-clearing condition, and using its no-arbitrage condition $\dot{v}_{jk} = (r_j + \delta) v_{jk} - \pi_{jk}$, household budget constraint, zero profit and free entry conditions for all sectors, and government budget constraint, we obtain country j's Walras' law: the sum of the values of excess demands for all markets is zero.

total expenditure for buying the intermediate goods from all sources net of tariff. Subtracting country j's domestic revenue and expenditure from the first line, we obtain the second line, country j's balance of trade.

We introduce revenue and expenditure shares, which play vital roles in modern trade models:

$$\lambda_{jk} \equiv n_j^e p_{jk}^f y_{jk} / \sum_l n_j^e p_{jl}^f y_{jl} = E_{jk} / \sum_l E_{jl}; \sum_k \lambda_{jk} = 1,$$
$$\zeta_{kj} \equiv n_k^e \tau_{kj} p_{kj}^f x_{kj} / \sum_l n_l^e \tau_{lj} p_{lj}^f x_{lj} = \tau_{kj} E_{kj} / \sum_l \tau_{lj} E_{lj}; \sum_k \zeta_{kj} = 1,$$

where λ_{jk} is the revenue share of varieties country j sells to country k, and ζ_{kj} is the expenditure share of varieties country j buys from country k. Due to import tariffs, country j's domestic revenue and expenditure shares are not necessarily equal. Using countries' import expenditure shares, country j's balance of trade is rewritten as:

$$(\zeta_{jk}/\tau_{jk}) P_k X_k = (\zeta_{kj}/\tau_{kj}) P_j X_j, k \neq j.$$

In the next section, we characterize a BGP, where all variables grow at constant (including zero) rates.

3 Balanced growth path

Let labor in country 2 be the numeraire: $w_2 \equiv 1$. Suppose that the world economy is on a BGP for $t \geq 0$. Following RBR, we impose the following restriction on α_j for the existence of a BGP:

$$\alpha_j = (\sigma - 1)/\sigma.$$

To understand the meaning of this restriction, suppose that n_j^e grows at the rate γ^* on a BGP, where an asterisk means that the economy is on a BGP. Then technologies imply that Y_j grows at the rate $\alpha_j[\sigma/(\sigma-1)]\gamma^*$. However, to meet the demands for the final good from the intermediate good and R&D sectors, Y_j must grow at the same rate as γ^*. Consequently, we require that $\alpha_j[\sigma/(\sigma-1)]\gamma^* = \gamma^*$, or $\alpha_j = (\sigma-1)/\sigma$.

One of the most difficult parts of the lab-equipment model with asymmetric countries is that the relative price of the final good of country 1 to country 2 p_1^Y/p_2^Y is determined in general equilibrium. To do this, we start from rewriting country j's intermediate good price index as:

$$P_j = \{\sum_k n_k^e [\tau_{kj} p_k^Y/(1-1/\sigma)]^{1-\sigma}\}^{1/(1-\sigma)} = (n_j^e)^{1/(1-\sigma)} p_j^Y \overline{m}_j/(1-1/\sigma); \tag{1}$$
$$\overline{m}_j \equiv [\sum_k (n_k^e/n_j^e)(\tau_{kj} p_k^Y/p_j^Y)^{1-\sigma}]^{1/(1-\sigma)}.$$

In Eq. (1), P_j is decreasing in n_j^e, whereas it is proportional to $p_j^Y \overline{m}_j$, a weighted average of marginal costs of selling to market j for domestic (i.e., country j's) and foreign (i.e., country $k(\neq j)$'s) intermediate good firms. Eq. (1) implies that $\overline{m}_j$ is increasing in country j's import tariff times foreign's relative price of the final good $\tau_{jk} p_k^Y/p_j^Y$, but decreasing in the relative number of foreign varieties n_k^e/n_j^e. This is simply because country j's representative final good firm benefits more from relatively cheaper, and/or relatively greater number of, foreign varieties.

Whereas p_k^Y/p_j^Y affects $\overline{m}_j$ in Eq. (1), $\overline{m}_j$ in turn affects p_k^Y/p_j^Y as follows. Substituting Eq. (1)

into $p_j^Y = c_j^Y(P_j, w_j) = P_j^{\alpha_j} w_j^{1-\alpha_j}$, solving it for p_j^Y with $n_j^e, \overline{m}_j$, and w_j given, and considering that $\alpha_j = (\sigma-1)/\sigma$, we obtain $p_j^Y = (n_j^e)^{-1}[\overline{m}_j/(1-1/\sigma)]^{\sigma-1} w_j$. This implies that p_1^Y/p_2^Y is given by:

$$(p_1^Y/p_2^Y)^* = (w_1^*/\chi^*)(\overline{m}_1^*/\overline{m}_2^*)^{\sigma-1}; \chi^* \equiv (n_1^e/n_2^e)^*, \tag{2}$$

where χ is the relative number of entrants in country 1 to country 2. Eq. (2) shows that $(p_1^Y/p_2^Y)^*$ is increasing in w_1^*, decreasing in χ^*, and increasing in $\overline{m}_1^*/\overline{m}_2^*$, as expected from Eq. (1) and $p_j^Y = c_j^Y(P_j, w_j)$. Eqs. (1) and (2) determine $\overline{m}_1^*, \overline{m}_2^*$, and $(p_1^Y/p_2^Y)^*$ as functions of w_1^*, χ^*, and tariffs.

Once $\overline{m}_j^*$ is expressed in terms of w_1^*, χ^*, and tariffs, it is related to country j's domestic expenditure and revenue shares as (see Appendix A for derivations):

$$\zeta_{jj} = \overline{m}_j^{\sigma-1}, \tag{3}$$

$$\lambda_{jj} = [1 + (\tau_{kj} - 1)\lambda_{jk}]\zeta_{jj} = \tau_{kj}\zeta_{jj}/[1 + (\tau_{kj} - 1)\zeta_{jj}], k \neq j. \tag{4}$$

Eqs. (3) and (4) imply that ζ_{jj}^* and λ_{jj}^* are also functions of w_1^*, χ^*, and tariffs. In particular, noting that $\partial \ln \lambda_{jj}/\partial \ln \zeta_{jj} = 1/[1 + (\tau_{kj} - 1)\zeta_{jj}] > 0$, $\overline{m}_j, \zeta_{jj}$, and λ_{jj} move in the same direction, with τ_{kj} given.

Country j's balance of trade is rewritten as:[9]

$$(\zeta_{12}^*/\tau_{12})L_2 = (\zeta_{21}^*/\tau_{21})w_1^* L_1. \tag{5}$$

Considering that $\zeta_{kj}^* = 1 - \zeta_{jj}^*, k \neq j$, is a function of w_1^*, χ^*, and tariffs, Eq. (5) determines w_1^* as a function of χ^* and tariffs. This is in line with Krugman (1980).

Finally, χ^* is determined by the balanced growth condition (see Appendix A for derivations):

$$\gamma_1^* = \gamma_2^* \equiv \gamma^*; \tag{6}$$

$$\gamma_j^* \equiv (\dot{n}_j^e/n_j^e)^* = (1 - 1/\sigma)L_j/\{\lambda_{jj}^*[1/(1-1/\sigma)]^{\sigma-1}\kappa_j^e\} - \rho - \delta. \tag{7}$$

Although we already assume that the economy is on a BGP to derive Eq. (7), it suggests stability force at work. Suppose that country 1's relative number of varieties χ increases. This directly increases $\overline{m}_1$, but it also indirectly increases $\overline{m}_1$ through an increase in country 2's relative price of the final good p_2^Y/p_1^Y from (the inverse of) Eq. (2). The resulting increase in $\overline{m}_1$ makes country 1 more closed in terms of both its domestic expenditure and revenue shares, which is bad for its long-run growth. Therefore, an increase in χ decreases γ_1.[10] Similarly, an increase in χ increases γ_2. This ensures that χ^* satisfying Eq. (6) is unique and stable if it exists: whenever $\chi < \chi^*$, we have $\gamma_1 > \gamma_2$, which increases χ to χ^*.

To sum up, the balanced trade condition (5) the balanced growth condition (6), together with Eqs. (1) to (4) and (7), determine a BGP: (w_1^*, χ^*).

Country j's long-run welfare (expressed in flow terms) is given by (see Appendix A for derivation):[11]

[9] Applying Shephard's lemma to $P_j X_j + w_j L_j^Y = c_j^Y(P_j, w_j)Y_j$ gives $X_j = (\partial c_j^Y(P_j, w_j)/\partial P_j)Y_j = \alpha_j(c_j^Y/P_j)Y_j$ and $L_j^Y = (\partial c_j^Y(P_j, w_j)/\partial w_j)Y_j = (1-\alpha_j)(c_j^Y/w_j)Y_j$. Combining them with $L_j = L_j^Y$ and $\alpha_j = (\sigma-1)/\sigma$, we obtain $P_j X_j = (\sigma-1)w_j L_j$.

[10] Calculations show that this is valid even if general equilibrium effects through w_1 are considered.

[11] Supposing that the representative household receives a constant utility flow $\ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^* = \rho U_j$ discounted by a factor $\exp(-\rho t)$ over an infinite horizon, its present discounted value is $\int_0^\infty \rho U_j \exp(-\rho t)dt = \rho U_j(1/\rho) = U_j$.

$$\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^* = \ln L_j + \ln K_j - \ln \lambda_{jj}^* + \ln \eta_j^* + (1/\rho)\gamma^*; \tag{8}$$
$$p_j^{Y*} \equiv p_{j0}^Y, K_j \equiv n_{j0}^e/[1/(1-1/\sigma)]^{\sigma-1},$$
$$\eta_j^* \equiv (1-1/\sigma)\rho/(\rho+\delta+\gamma^*) + 1 + \sigma(\tau_{kj}-1)\lambda_{jk}^* > 1, k \neq j.$$

Country j's long-run welfare is increasing in its consumption in the initial period of a BGP $C_j^* = E_j^*/p_j^{Y*}$ and the rate of decrease in p_{jt}^Y on a BGP. The latter is equal to γ^* because $p_{jt}^Y = (n_{jt}^e)^{-1}[\overline{m}_j^*/(1-1/\sigma)]^{\sigma-1}w_j^*$. The former is expressed as $E_j^*/p_j^{Y*} = L_j(K_j/\lambda_{jj}^*)\eta_j^*$. The part K_j/λ_{jj}^* basically comes from country j's real wage in terms of the final good. The last part η_j^* indicates the composition of country j's total income: interest income, wage income, and tariff revenue.

Eqs. (7) and (8) imply country j's long-run growth and welfare formulas:

$$d\gamma_j^* = -(\rho+\delta+\gamma^*)\widehat{\lambda}_{jj}^*; \widehat{\lambda}_{jj}^* \equiv d\ln\lambda_{jj}^* \equiv d\lambda_{jj}^*/\lambda_{jj}^*. \tag{9}$$

$$\begin{aligned}
\rho dU_j &= (\sigma/\eta_j^*)\lambda_{jk}^*\tau_{kj}\widehat{\tau}_{kj} - [1+(\sigma/\eta_j^*)(\tau_{kj}-1)(1-\lambda_{jk}^*)]\widehat{\lambda}_{jj}^* + \Gamma_j^* d\gamma^* \\
&= (\sigma/\eta_j^*)\lambda_{jk}^*\tau_{kj}\widehat{\tau}_{kj} - [1+(\sigma/\eta_j^*)(\tau_{kj}-1)(1-\lambda_{jk}^*) + \Omega_j^*]\widehat{\lambda}_{jj}^*; \qquad (10)\\
\Gamma_j^* &\equiv -[(1-1/\sigma)\rho/(\rho+\delta+\gamma^*)^2]/\eta_j^* + 1/\rho \\
&= [1/(\rho\eta_j^*)][(1-1/\sigma)\rho(\delta+\gamma^*)/(\rho+\delta+\gamma^*)^2 + 1 + \sigma(\tau_{kj}-1)\lambda_{jk}^*] > 0, \\
\Omega_j^* &\equiv \Gamma_j^*(\rho+\delta+\gamma^*), k \neq j.
\end{aligned}$$

Eq. (9) is the ACR (Arkolakis–Costinot–Rodríguez-Clare) formula (e.g., Arkolakis et al., 2012) for long-run growth: country j grows faster if and only if it becomes more open (i.e., λ_{jj}^* decreases). Arkolakis et al., (2012) show that, in a large class of new trade models with only iceberg trade costs, a country's welfare is a decreasing function of only one endogenous variable, that is, the country's domestic expenditure share. In the present model, a country's long-run growth is a decreasing function of its domestic revenue (not expenditure) share as the only one endogenous variable. The difference comes from the presence of revenue-generating import tariffs.

In Eq. (10), the ACR formula for long-run welfare, there are two terms in the far right-hand side. The first term represents the direct effect of a change in country j's import tariff τ_{kj} on its long-run welfare through a change in its tariff revenue. The second term summarizes the effects of a change in country j's domestic revenue share λ_{jj}^*, which is equal to its domestic expenditure share in a large class of new trade models without import tariffs as shown by Arkolakis et al. (2012). Specifically, suppose that country j becomes more closed (i.e., λ_{jj}^* increases). On the one hand, this decreases its long-run welfare by decreasing its real wage, and also its tariff revenue indirectly through a decrease in its revenue share of exported varieties. On the other hand, it decreases the balanced growth rate. This directly decreases the welfare by decreasing future consumption, but it indirectly increases the welfare by increasing the interest income from the asset. Since the direct growth effect is always stronger than the counteracting indirect growth effect as long as $Q_j^K = \kappa_j^e n_j^e(\gamma^*+\delta) \geq 0$, the decrease in the balanced growth rate necessarily decreases the welfare. Overall, more autarkiness (i.e., an increase in λ_{jj}^*) is bad for country j's long-run welfare.

Our results so far are summarized in the following proposition:

Proposition 1 *An increase in a country's domestic revenue share implies a decrease in the balanced growth rate, but it does not imply a decrease in its long-run welfare.*

As Eqs. (9) and (10) show, an increase in country j's domestic revenue share λ_{jj}^* necessarily decreases the balanced growth rate, and also partly decreases its long-run welfare. However, if the increase in λ_{jj}^* is caused by an increase in country j's import tariff τ_{kj}, which sounds quite natural, its long-run welfare partly increases through the increased tariff revenue. It is the last effect that usually causes a large country's optimal tariff to be positive. In the next section, we solve for general equilibrium effects of a tariff change to see how much the optimal tariff is for a growing large country.

4 Long-run effects of a tariff change

4.1 Long-run growth effect

Throughout this section, we omit asterisks just for notational simplicity. The long-run growth effects of tariff changes are derived in five steps: (i) from Eqs. (1) and (2), we solve for $\widehat{p_1^Y/p_2^Y} = \widehat{p_1^Y/p_2^Y}(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$ and $\widehat{\overline{m}}_j = \widehat{\overline{m}}_j(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (ii) substituting $\widehat{\overline{m}}_j = \widehat{\overline{m}}_j(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$ from step (i) into the logarithmically differentiated form of Eq. (3), and substituting it into the logarithmically differentiated form of Eq. (5), we solve for $\widehat{w}_1 = \widehat{w}_1(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iii) substituting the result from step (ii) back into $\widehat{\overline{m}}_j = \widehat{\overline{m}}_j(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$, and substituting it into the logarithmically differentiated forms of Eqs. (3) and (4), and then Eq. (9), we solve for $d\gamma_j = d\gamma_j(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iv) substituting the result from step (iii) into the differentiated form of Eq. (6), we solve for $\widehat{\chi} = \widehat{\chi}(\widehat{\tau}_{21}, \widehat{\tau}_{12})$; and (v) substituting the result from step (iv) back into $d\gamma_2 = d\gamma_2(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$, we solve for $d\gamma = d\gamma_2(\widehat{\tau}_{21}, \widehat{\tau}_{12})$. Following these steps, we finally obtain (see Appendix A for derivation):

$$d\gamma = -\sigma(\rho + \delta + \gamma)[\lambda_{jk}\lambda_{kj}/(\lambda_{jk} + \lambda_{kj})](\widehat{\tau}_{kj} + \widehat{\tau}_{jk}), k \neq j. \tag{11}$$

Eq. (11) immediately implies that:

$$\partial\gamma/\partial \ln \tau_{kj} = \partial\gamma/\partial \ln \tau_{jk} = -\sigma(\rho + \delta + \gamma)\lambda_{jk}\lambda_{kj}/(\lambda_{jk} + \lambda_{kj}) < 0 \forall j, k, k \neq j.$$

Proposition 2 *An increase in the import tariff of either country by the same rate decreases the balanced growth rate by the same amount.*

Suppose that country 1 increases its import tariff τ_{21}, with χ given. This directly increases $\overline{m}_1$ and hence ζ_{11}. However, the increase in ζ_{11} means that country 1 imports less, which causes country 1 to run a trade surplus. For country 1's balance of trade to be restored, w_1 and hence p_1^Y/p_2^Y increase so that country 1 exports less and import more. This makes country 2 more closed (i.e., increases $\overline{m}_2, \zeta_{22}$, and λ_{22}), causing it to grow more slowly (i.e., decreasing γ_2). For country 1, the direct effect dominates so that $\overline{m}_1$ and ζ_{11} increase. Both the increase in τ_{21} and the resulting increase in ζ_{11} make country 1 more closed in terms of its domestic revenue share (i.e., increase λ_{11}), thereby slowing down its growth (i.e., decreasing γ_1). Finally, since both γ_1 and γ_2 decrease with χ given, even after χ is adjusted, the new balanced growth rate is lower than the old one.

Proposition 2 has both qualitative and quantitative implications. Qualitatively, even a unilateral tariff reduction always raises long-run growth. In the RBR knowledge-driven model with homogeneous firms, symmetric countries, and international knowledge spillovers, Rivera-Batiz and Romer (1991b) and Baldwin and Forslid (1999) show that the relationship between a common tariff and the balanced growth rate is not monotonic.[12] However, introducing asymmetric countries in RBR is so difficult that there has been little attempt to study how a unilateral trade liberalization affects long-run growth.[13] We detect a robust negative relationship between a country's tariff and the balanced growth rate in the RBR lab-equipment model. Quantitatively, a 1% tariff reduction in either a larger or a smaller country has the same long-run growth effect. As trade theories tell us that a smaller country has a smaller terms of trade impact than a larger country, we might guess that a smaller country affects the balanced growth rate by less than a larger country. Our result demonstrates that this conjecture is not true.

Armed with Proposition 2, we characterize the optimal tariff of a large country in the next subsection.

4.2 Can the optimal tariff be zero for a growing large country?

Substituting Eqs. (9) and (11) into Eq. (10), the amount of change in country j's long-run welfare is expressed only in terms of the rates of changes in tariffs as:

$$\rho dU_j = \sigma\lambda_{jk}\{(\tau_{kj}/\eta_j)\widehat{\tau}_{kj} - [1 + (\sigma/\eta_j)(\tau_{kj} - 1)(1 - \lambda_{jk}) + \Omega_j][\lambda_{kj}/(\lambda_{jk} + \lambda_{kj})](\widehat{\tau}_{kj} + \widehat{\tau}_{jk})\}, k \neq j. \quad (12)$$

Eq. (12) immediately implies that:

$$\begin{aligned} &\rho\partial U_j/\partial \ln \tau_{jk} = -\sigma\lambda_{jk}[1 + (\sigma/\eta_j)(\tau_{kj} - 1)(1 - \lambda_{jk}) + \Omega_j]\lambda_{kj}/(\lambda_{jk} + \lambda_{kj}) < 0, \\ &\rho\partial U_j/\partial \ln \tau_{kj} = \sigma\lambda_{jk}\Psi_j; \Psi_j \equiv \tau_{kj}/\eta_j - [1 + (\sigma/\eta_j)(\tau_{kj} - 1)(1 - \lambda_{jk}) + \Omega_j]\lambda_{kj}/(\lambda_{jk} + \lambda_{kj}), k \neq j. \end{aligned} \quad (13)$$

An increase in either τ_{kj} or τ_{jk} decreases the balanced growth rate from Eq. (11). This implies from Eq. (9) that country j becomes more closed in terms of its domestic revenue share, which is bad for its long-run welfare. Since the increase in the other country's tariff τ_{jk} does not provide the tariff revenue to country j, it necessarily decreases country j's long-run welfare. However, the increase in country j's own tariff τ_{kj} creates a trade-off between gains from tariff revenue and losses from autarkiness, as represented by the first and second terms, respectively, in the definition of Ψ_j in Eq. (13). If country j's optimal tariff is positive, then it must satisfy $\Psi_j = 0$. Alternatively, if $\Psi_j < 0$ at $\tau_{kj} = 1$, then the status quo of zero tariff is locally optimal (and globally optimal if $\rho\partial^2 U_j/\partial(\ln \tau_{kj})^2 < 0 \forall \tau_{kj} \geq 1$). The condition is rewritten as:

$$\Psi_j|_{\tau_{kj}=1} = 1/\eta_j - (1 + \Omega_j)\lambda_{kj}/(\lambda_{jk} + \lambda_{kj}) < 0 \Leftrightarrow \lambda_{jk}/\lambda_{kj} < 1 - 1/\sigma + (\rho + \delta + \gamma)/\rho, k \neq j. \quad (14)$$

Proposition 3 *A zero tariff is locally optimal for country j if $\lambda_{jk}/\lambda_{kj} < 1 - 1/\sigma + (\rho + \delta + \gamma)/\rho, k \neq j$, at a BGP with $\tau_{kj} = 1$. In particular, it is true if the two countries are symmetric at the BGP.*

The sufficient condition for the zero optimal tariff (14) states that the export revenue share of country j relative to country k is smaller than the upper bound $1 - 1/\sigma + (\rho + \delta + \gamma)/\rho$, which is larger than one. This

[12] Rivera-Batiz and Romer (1991b) find a U-shaped relationship between a common ad valorem tariff and the balanced growth rate. Baldwin and Forslid (1999) even point out that the tariff-growth relationship can be either U- or inverted U-shaped, depending on whether the tariff is ad valorem or specific.

[13] Starting from a symmetric BGP, Baldwin and Forslid (1999) numerically find a U-shaped relationship between a country's iceberg trade cost and the balanced growth rate. However, whether it is true starting from an asymmetric BGP is left unknown.

means that the condition is automatically satisfied if $\lambda_{jk}/\lambda_{kj} = 1$, that is, the countries are symmetric. By continuity, Eq. (14) is true as long as the countries are similar. Moreover, the upper bound is monotonically decreasing in ρ.[14] As ρ becomes smaller and smaller, the permissible range of $\lambda_{jk}/\lambda_{kj}$ becomes larger and larger. In the limit, as ρ approaches zero from above, Eq. (14) is satisfied for all positive export revenue shares. Therefore, zero optimal tariffs for large countries are quite common in our model.

What would happen if there were no growth effect? Considering that $d\gamma^* = 0$ in the first line of Eq. (10), we would have $\Omega_j^* = 0$ in the second line of Eq. (10). Then the necessary and sufficient condition for $\Psi_j|_{\tau_{kj}=1} < 0$ would be $\lambda_{jk}/\lambda_{kj} < (1-1/\sigma)\rho/(\rho+\delta+\gamma)(<1), k \neq j$. This could not be satisfied if the two countries were symmetric; then we would rather have $\Psi_j|_{\tau_{kj}=1} > 0$, implying that country j's optimal tariff must be positive. This highlights the necessity of considering economic growth for our innovative result.

In the next section, we extend our lab-equipment model to include heterogeneous firms a la Melitz (2003) to see how robust our results are.

5 A heterogeneous firm lab-equipment model

5.1 Setup

We just add two things to the monopolistically competitive intermediate good sector of the homogeneous firm lab-equipment model. First, the unit final good requirement is not one for all firms, but denoted by a random variable a following a Pareto distribution:

$$G_j(a) \equiv (a/a_{j0})^\theta = a_{j0}^{-\theta} a^\theta; \theta > \sigma - 1,$$

where a_{j0} is a scale parameter representing the upper bound of a in country j, and θ is a shape parameter that is common across countries. Second, in addition to the fixed entry cost $P_j^K \kappa_j^e$, a firm producing its differentiated variety in country j incurs the fixed overhead cost $P_j^K \kappa_{jk}$ to sell its product to market k if and only if it is profitable, where κ_{jk} is country j's one-time fixed overhead cost in market k in terms of the knowledge good.

The profit-maximizing supply price, revenue, profit, and gross value of a firm indexed by a selling from country j to country k are given by, respectively:

$$\begin{aligned}
(p_{jk}^f(a) - p_j^Y a)/p_{jk}^f(a) = 1/\sigma \Leftrightarrow p_{jk}^f(a) &= p_j^Y a/(1-1/\sigma), \\
e_{jk}(a) \equiv p_{jk}^f(a) y_{jk}(a) &= \tau_{jk}^{-\sigma} [p_j^Y a/(1-1/\sigma)]^{1-\sigma} P_k^\sigma X_k, \\
\pi_{jk}(a) = p_{jk}^f(a) y_{jk}(a) - p_j^Y a y_{jk}(a) &= e_{jk}(a)/\sigma = \tau_{jk}^{-\sigma} [p_j^Y a/(1-1/\sigma)]^{1-\sigma} P_k^\sigma X_k/\sigma, \\
v_{jkt}(a) &\equiv \int_t^\infty \pi_{jks}(a) \exp(-\int_t^s (r_{ju}+\delta) du) ds.
\end{aligned}$$

The fixed overhead costs pin down country j's cutoff unit final good requirement in market k as:

$$v_{jkt}(a_{jkt}) = P_{jt}^K \kappa_{jk}, j, k = 1, 2. \tag{15}$$

[14]Eqs. (1) to (7) imply that w_1^* and χ^* depend on σ, but not on ρ or δ. Then, from Eq. (7), $\rho+\delta+\gamma = (1-1/\sigma)L_j/\{\lambda_{jj}[1/(1-1/\sigma)]^{\sigma-1}\kappa_j^e\}$ is independent of ρ and δ: an increase in ρ and/or δ decreases γ by the same amount so that both sides of this equation are unchanged.

Eq. (15) is called the zero cutoff profit condition, meaning that the gross value of the cutoff firm just covers the fixed overhead cost. It is assumed that firms have to pay a larger fixed overhead cost for exports than domestic sales: $\kappa_{jk} > \kappa_{jj}, k \neq j$. Using Eq. (15) and $e_{jks}(a)/e_{jks}(a_{jkt}) = (a/a_{jkt})^{1-\sigma} = \pi_{jks}(a)/\pi_{jks}(a_{jkt})$, $v_{jkt}(a)$ is rewritten as $v_{jkt}(a) = (a/a_{jkt})^{1-\sigma} P_{jt}^K \kappa_{jk} \geq P_{jt}^K \kappa_{jk} \Leftrightarrow a \leq a_{jkt}$. This verifies that a firm with a in country j profitably enters market k if and only if $a \leq a_{jk}$. An increase in a_{jk} means more entry into, whereas a decrease in a_{jk} means more exit from, market k. We assume that $a_{jk} < a_{jj} \forall j, k = 1, 2, k \neq j$, that is, only a fraction $G_j(a_{jk})/G_j(a_{jj})$ of country j's domestic surviving firms with $a \leq a_{jk} (< a_{jj})$ can also survive in their export market k.

Now that a is uncertain at the time of entry, free entry requires that the fixed entry cost be equal to the sum of the "expected" net firm values over all markets:

$$\textstyle\sum_k \displaystyle\int_0^{a_{jk}} (v_{jk}(a) - P_j^K \kappa_{jk}) g_j(a) da = P_j^K \kappa_j^e \Leftrightarrow \textstyle\sum_k \kappa_{jk} H_{jk}(a_{jk}) = \kappa_j^e; \tag{16}$$

$$\begin{aligned} H_{jk}(a_{jk}) &\equiv G_j(a_{jk}) h_{jk}(a_{jk}) = G_j(a_{jk})/(\beta - 1), \beta \equiv \theta/(\sigma - 1) > 1, \\ h_{jk}(a_{jk}) &\equiv (\overline{a}_{jk}(a_{jk})/a_{jk})^{1-\sigma} - 1 = 1/(\beta - 1), \\ \overline{a}_{jk}(a_{jk}) &\equiv (\int_0^{a_{jk}} a^{1-\sigma} \mu_{jk}(a|a_{jk}) da)^{1/(1-\sigma)} = [\beta/(\beta - 1)]^{1/(1-\sigma)} a_{jk}, \\ \mu_{jk}(a|a_{jk}) &\equiv g_j(a)/G_j(a_{jk}) = \theta a_{jk}^{-\theta} a^{\theta - 1}, \end{aligned}$$

where $H_{jk}(a_{jk})$ is country j's expected net firm value in market k relative to the fixed overhead cost $P_j^K \kappa_{jk}$, $h_{jk}(a_{jk})$ is the conditional version of $H_{jk}(a_{jk})$, $\overline{a}_{jk}(a_{jk})$ is the aggregate unit final good requirement of surviving firms, and $\mu_{jk}(a|a_{jk})$ is the probability density function conditional on survival, with $\int_0^{a_{jk}} \mu_{jk}(a|a_{jk}) da = 1$. Since an increase in a_{jk} increases $H_{jk}(a_{jk})$ by increasing the probability of survival $G_j(a_{jk})$, Eq. (16) implies that a_{jj} and $a_{jk}, k \neq j$, always move in the opposite directions. In other words, more domestic selection (i.e., a decrease in a_{jj}) implies more exports (i.e., an increase in a_{jk}), and vice versa.

Unlike the homogeneous firm model, not all entrants survive in all markets: $n_{jk} \equiv n_j^e G_j(a_{jk})$ is the number of entrants in country j surviving in market k, or the number of varieties sold from country j to country k.

The rest of the heterogeneous firm model is the same as the homogeneous firm model, except two points. First, wherever a is, we have to take expectations over it. For example, country j's revenue of selling the intermediate goods to country k is now given by $E_{jk} \equiv n_{jk} \int_0^{a_{jk}} e_{jk}(a) \mu_{jk}(a|a_{jk}) da$. Second, we have to take account of the fixed overhead costs. Specifically, country j's market-clearing condition for the knowledge good is replaced by $Q_j^K = \overline{\kappa}_j (\dot{n}_j^e + \delta n_j^e); \overline{\kappa}_j \equiv \sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e, j = 1, 2$, where $\overline{\kappa}_j$ is an entrant's expected total fixed costs in terms of the knowledge good.

In the next subsection, we characterize a BGP in the heterogeneous firm model.

5.2 Balanced growth path

Suppose that, just like the homogeneous firm model, $w_1^*, \chi^* \equiv (n_1^e/n_2^e)^*$, and $\gamma_1^* = \gamma_2^* \equiv \gamma^*; \gamma_j^* \equiv (\dot{n}_j^e/n_j^e)^*$ are constant on a BGP for $t \geq 0$. To determine the cutoffs, we use Eqs. (15) and (16). Specifically, dividing Eq. (15) by itself with $j = k$ gives $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*) = P_{j0}^K \kappa_{jk}/(P_{k0}^K \kappa_{kk}), j \neq k$, which is rewritten as (see Appendix B for derivations):

$$a_{12}^*/a_{22}^* = v^{*-1}\tau_{12}^{-\sigma/(\sigma-1)}(\kappa_{12}/\kappa_{22})^{-1/(\sigma-1)}, \tag{17}$$
$$a_{21}^*/a_{11}^* = v^*\tau_{21}^{-\sigma/(\sigma-1)}(\kappa_{21}/\kappa_{11})^{-1/(\sigma-1)}; v^* \equiv (p_1^Y/p_2^Y)^{*\sigma/(\sigma-1)}. \tag{18}$$

We call Eqs. (17) and (18) the relative competitiveness conditions in markets 2 and 1, respectively: an increase in a_{jk}^*/a_{kk}^* means that country $j(\neq k)$ becomes relatively more competitive in market k because relatively more firms from the former enter the latter. This is true if country k liberalizes its imports (i.e., τ_{jk} decreases) and/or country j's final good becomes relatively cheaper (i.e., $(p_j^Y/p_k^Y)^*$ decreases). Eqs. (16), (17), and (18) determine the four cutoffs, with $(p_1^Y/p_2^Y)^*$ given.

The rest of the system characterizing a BGP is as follows (see Appendix B for derivations):

$$P_j = \{\textstyle\sum_k n_{kj}[\tau_{kj}p_k^Y\overline{a}_{kj}(a_{kj})/(1-1/\sigma)]^{1-\sigma}\}^{1/(1-\sigma)} = (n_j^e)^{1/(1-\sigma)}p_j^Y\overline{m}_j/(1-1/\sigma); \tag{19}$$
$$\overline{m}_j \equiv \{\textstyle\sum_k(n_k^e/n_j^e)G_k(a_{kj})[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\}^{1/(1-\sigma)},$$
$$(p_1^Y/p_2^Y)^* = (w_1^*/\chi^*)(\overline{m}_1^*/\overline{m}_2^*)^{\sigma-1}, \tag{20}$$
$$0 = \{\lambda_{12}^*/[1+(\tau_{21}-1)\lambda_{12}^*]\}w_1^*L_1 - \{\lambda_{21}^*/[1+(\tau_{12}-1)\lambda_{21}^*]\}L_2; \tag{21}$$
$$\lambda_{jk}^* = (H_{jk}(a_{jk}^*)+G_j(a_{jk}^*))\kappa_{jk}/\textstyle\sum_l(H_{jl}(a_{jl}^*)+G_j(a_{jl}^*))\kappa_{jl} = H_{jk}(a_{jk}^*)\kappa_{jk}/\kappa_j^e, \tag{22}$$
$$\gamma_1^* = \gamma_2^* \equiv \gamma^*; \tag{23}$$
$$\gamma_j^* = (1-1/\sigma)L_j/\{[a_{jj}^*/(1-1/\sigma)]^{\sigma-1}\kappa_{jj}\} - \rho - \delta. \tag{24}$$

Eqs. (19) to (24) look similar to Eqs. (1) to (7), except two points. First, it is more convenient to use counties' export revenue shares, rather than their import expenditure shares, to express country j's balance of trade (21) because λ_{jk}^* is an increasing function of a_{jk}^* only.[15] Second, γ_j^* is a decreasing function of a_{jj}^* only, instead of λ_{jj}^*.

A BGP is determined in the following way. First, Eqs. (19) and (20) determine $\overline{m}_1^*, \overline{m}_2^*$, and $(p_1^Y/p_2^Y)^*$ as functions of $w_1^*, \chi^*, \{a_{jk}^*\}$, and tariffs. Second, Eqs. (16), (17), and (18) determine $\{a_{jk}^*\}$ as functions of w_1^*, χ^*, and tariffs. Third, Eq. (21) determines w_1^* as a function of χ^* and tariffs. Finally, Eq. (23) determines χ^*.

Country j's long-run welfare (expressed in flow terms) is given by (see Appendix B for derivation):

$$\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^* = \ln L_j + \ln K_j - (\sigma-1)\ln a_{jj}^* + \ln\eta_j^* + (1/\rho)\gamma^*; \tag{25}$$
$$K_j \equiv \beta\kappa_j^e n_{j0}^e/\{[1/(1-1/\sigma)]^{\sigma-1}\kappa_{jj}\},$$

where η_j^* is defined in the same way as the homogeneous firm model. Country j's consumption in the initial period of a BGP is now expressed as $E_j^*/p_j^{Y*} = L_j(K_j/a_{jj}^{*\sigma-1})\eta_j^*$, where the part $K_j/a_{jj}^{*\sigma-1}$ indicates its real wage.

Differentiating Eqs. (24) and (25), and using Eqs. (22), (23), and $0 = \sum_k\lambda_{jk}^*\widehat{a}_{jk}^*$ from Eq. (16), we obtain country j's long-run growth and welfare formulas:

[15] Eq. (31) implies that Eq. (21) is equivalent to Eq. (5).

$$d\gamma_j^* = -(\sigma-1)(\rho+\delta+\gamma^*)\widehat{a}_{jj}^* = -[(\rho+\delta+\gamma^*)/\beta]\widehat{\lambda}_{jj}^*. \tag{26}$$

$$\begin{aligned}\rho dU_j &= (\sigma/\eta_j^*)\lambda_{jk}^*\tau_{kj}\widehat{\tau}_{kj} - (\sigma-1)[1+(\sigma/\eta_j^*)(\tau_{kj}-1)\beta(1-\lambda_{jk}^*)]\widehat{a}_{jj}^* + \Gamma_j^* d\gamma^* \\ &= (\sigma/\eta_j^*)\lambda_{jk}^*\tau_{kj}\widehat{\tau}_{kj} - (1/\beta)[1+(\sigma/\eta_j^*)(\tau_{kj}-1)\beta(1-\lambda_{jk}^*)+\Omega_j^*]\widehat{\lambda}_{jj}^*, k\neq j,\end{aligned} \tag{27}$$

where Γ_j^* and Ω_j^* are defined in the same way as the homogeneous firm model. Although the long-run growth and welfare formulas in the heterogeneous firm model (26) and (27) are quantitatively different from those in the homogeneous firm model (9) and (10) due to the presence of $\beta(=\theta/(\sigma-1)>1)$, they are qualitatively the same. This implies that Proposition 1 continues to hold.

5.3 Long-run effects of a tariff change

Throughout this subsection, we omit asterisks just for notational simplicity. Using Eqs. (16) to (24), the long-run growth effects of tariff changes are derived as (see Appendix B for derivation):

$$d\gamma = -\sigma(\rho+\delta+\gamma)[\lambda_{jk}\lambda_{kj}/(\lambda_{jk}+\lambda_{kj})](\widehat{\tau}_{kj}+\widehat{\tau}_{jk}), k\neq j. \tag{28}$$

Suppose that country 1 increases its import tariff τ_{21}. With p_1^Y/p_2^Y given, this makes country 2 relatively less competitive in market 1 (i.e., decreases a_{21} from Eq. (18)). Since country 2's expected profit from exports decreases, free entry requires that its expected profit from domestic sales increases, causing more unproductive firms to stay in their domestic market (i.e., a_{22} increases from Eq. (16)). Because of easier competition with country 2's domestic firms, more firms from country 1 start exporting (i.e., a_{12} increases from Eq. (17)). This drives more of country 1's unproductive firms out of their domestic market (i.e., decreases a_{11} from Eq. (16)). Country 1's increased import protection causes less exports and less domestic selection in country 2, whereas it causes more exports and more domestic selection in country 1.

In fact, the increase in τ_{21} affects p_1^Y/p_2^Y. With country 1 exporting more and importing less, it tends to run a trade surplus. For the surplus to be cleared, w_1 and hence p_1^Y/p_2^Y increase so that country 1 becomes relatively more costly in producing the intermediate goods (see Eqs. (20) and (21)).[16] This makes country 2 relatively more competitive in market 1, implying more exports and more domestic selection (see Eqs. (16) and (18)). Similarly, country 1 becomes relatively less competitive in market 2, causing less exports and less domestic selection (see Eqs. (16) and (17)). These indirect effects work in the opposite directions of the direct effects in the previous paragraph. It turns out that the direct effects outweigh the indirect effects for country 2, whereas the opposite is true for country 1. Since domestic selection becomes weaker in both countries, both countries grow more slowly, with χ given. Finally, even if χ adjusts to equalize countries' growth rates, the new balanced growth rate is lower than the old one.[17]

Since Eq. (28) is exactly the same as Eq. (11), Proposition 2 continues to hold. This means that the negative long-run growth effect of a tariff increase in the lab-equipment model is the same, whether the intermediate good firms are homogeneous or heterogeneous. In the literature on endogenous growth and

[16] In (21), it seems that an increase in w_1 directly increases country 1's trade surplus. However, the resulting increase in p_1^Y/p_2^Y indirectly decreases its surplus by decreasing its exports but increasing its imports. Since the sum of the indirect effects is stronger than the direct effect, country 1's trade surplus is decreasing in w_1. See Eq. (67) in Appendix B for details.

[17] Eqs. (68) and (69) in Appendix B show that γ_1 is decreasing, whereas γ_2 is increasing, in χ. This implies that the heterogeneous firm model has the same stabilization mechanism as the homogeneous firm model.

heterogeneous firms (e.g., Baldwin and Robert-Nicoud, 2008; Dinopoulos and Unel, 2011; Sampson, 2016; Ourens, 2016; Naito, 2017, 2019; Impullitti and Licandro, 2018; Akcigit et al., 2018; Fukuda, forthcoming; Perla et al., 2019), all papers but Naito (2017, 2019) and Akcigit et al. (2018) are restricted to symmetric countries, and all papers but Akcigit et al. (2018) deal only with iceberg trade costs. Unlike Akcigit et al.'s (2018) numerical study, this paper analytically shows that even a unilateral tariff reduction always raises long-run growth.

Turning to the long-run welfare effects, from Eqs. (26), (27), and (28), we obtain:

$$\rho dU_j = \sigma\lambda_{jk}\{(\tau_{kj}/\eta_j)\widehat{\tau}_{kj} - [1+(\sigma/\eta_j)(\tau_{kj}-1)\beta(1-\lambda_{jk})+\Omega_j][\lambda_{kj}/(\lambda_{jk}+\lambda_{kj})](\widehat{\tau}_{kj}+\widehat{\tau}_{jk})\}, k \neq j. \quad (29)$$

Calculating $\rho\partial U_j/\partial \ln \tau_{kj}$ from Eq. (29), and evaluating it at $\tau_{kj} = 1$, we obtain exactly the same sufficient condition for the zero optimal tariff as Eq. (14). Therefore, Proposition 3 continues to hold as is.

This section has revealed that our main results including the local optimality of a zero tariff in the homogeneous firm lab-equipment model are robust to introduction of heterogeneous firms. However, we are not sure if the zero tariff is globally optimal in a relevant domain of tariffs. To see this, we make numerical experiments in the next section.

6 Global optimality of a zero tariff

The purpose of the following numerical exercises is to check the global optimality of a zero tariff for a large country in the homogeneous and heterogeneous firm lab-equipment models. To this end, we keep the analysis as simple as possible, starting from a symmetric BGP with free trade as a benchmark. Key parameters are borrowed from other work: $\rho = 0.02$ from Acemoglu (2009); and $\sigma = 4$ and $\delta = 0.025$ from Balistreri et al. (2011). We arbitrarily set $L_j = 1$ and $n^e_{20} = 1,000$ (implying that $n^e_{10} = 1,000$ at the benchmark BGP).

In the homogeneous firm model, κ^e_j is calibrated to reproduce $\gamma = 0.01684$, the average annual growth rate of GDP per capita in the world during 1994-2018 from the World Development Indicators. Under the calibrated value $\kappa^e_j = 10.233$, the model produces $p^Y_1/p^Y_2 = 1, w_1 = 1, \chi = 1, \lambda_{jk} = 0.5$, and $\gamma = 0.01684$.

In the heterogeneous firm model, we borrow $\theta = 4$ from Balistreri et al. (2011).[18] We arbitrarily set $a_{j0} = 2$. This is consistent with a calibration target $\overline{a}_{jj}(a_{jj}) = 1$, meaning that country j's aggregate unit final good requirement of domestic surviving firms is equal to one, the unit final good requirement in the homogeneous firm model. We calibrate κ^e_j, κ_{jj}, and κ_{jk} against $\gamma = 0.01684, \overline{a}_{jj}(a_{jj}) = 1$, and $G_j(a_{jk})/G_j(a_{jj}) = 0.21$ as a fraction of exporters from Bernard et al. (2003). The resulting values $\kappa^e_j = 2.554, \kappa_{jj} = 1.279$, and $\kappa_{jk} = 4.123$ produce $a_{jj} = 1.587$ (and hence $\overline{a}_{jj}(a_{jj}) = 1$), $a_{jk} = 1.075$ (and hence $G_j(a_{jk})/G_j(a_{jj}) = 0.21$), $p^Y_1/p^Y_2 = 1, w_1 = 1, \chi = 1, \lambda_{jk} = 0.404$, and $\gamma = 0.01684$.

In each of the two models, we draw country 1's iso-welfare curves (expressed in flow terms) on the (τ_{21}, τ_{12}) plane. To see how relative country size and time preference affect the shapes of country 1's iso-welfare curves, we increase or decrease L_1 by 0.1, and/or increase or decrease ρ by 0.01, from their benchmark values. Therefore, we have nine cases for each model.

Fig. 1 depicts country 1's iso-welfare curves in the homogeneous firm model. We first look at the middle center panel corresponding to the benchmark case. τ_{kj} ranges from 1 to 2, meaning that country j's ad valorem tariff rate takes from 0 to 100%. The number attached to each iso-welfare curve indicates the value

[18] Assuming that $\delta = 0.025$ from Bernard et al. (2007), and $\sigma = 3.8$ from Bernard et al. (2003), Balistreri et al. (2011) estimate that θ ranges from 3.9 to 5.2. Felbermayr et al. (2013) also assume that $\sigma = 3.8$ and $\theta = 4$.

of country 1's long-run welfare (expressed in flow terms) ρU_1. All displayed iso-welfare curves are downward-sloping, and ρU_1 increases as we move down and to the left. This implies that, with τ_{12} given, reducing τ_{21} to $\tau_{21} = 1$ maximizes ρU_1. Therefore, a zero tariff is optimal for country 1 for this relevant domain of tariffs.[19] This is in stark contrast to the existing large-country optimal tariff models, where a country's iso-welfare curves are inverted U-shaped against its tariff.

As we move up to the top center panel, where ρ increases to $\rho = 0.03$, iso-welfare curves become flatter, suggesting that the negative relationship between τ_{21} and ρU_1 becomes relatively weaker due to the decreased net growth effect on welfare Γ_1.[20] In contrast, as we move down to the bottom center panel, where $\rho = 0.01$, iso-welfare curves become steeper. This is true for all three columns.

Again starting from the middle center panel, suppose that L_1 increases to $L_1 = 1.1$. As country 1 becomes larger than country 2, the former becomes less open than the latter on a BGP with free trade: $\lambda_{12} = 0.476, \lambda_{21} = 0.524$. Country 1's iso-welfare curves become steeper because the increase in $\lambda_{21}/(\lambda_{12} + \lambda_{21})$ intensifies the negative second term in Ψ_j of Eq. (13), the losses from autarkiness. Conversely, when L_1 decreases to $L_1 = 0.9$, the smaller country 1 becomes more open than country 2 on a BGP with free trade: $\lambda_{12} = 0.526, \lambda_{21} = 0.474$, and thus the resulting decrease in $\lambda_{21}/(\lambda_{12} + \lambda_{21})$ makes country 1's iso-welfare curves flatter. This is true for all three rows.

From Fig. 1, we observe that country 1's downward-sloping iso-welfare curves become steeper, the larger L_1 is and/or the smaller ρ is. However, even in the top left panel as the most pessimistic case, all iso-welfare curves are downward-sloping, implying that a zero tariff is optimal for country 1. Therefore, regardless of the other country's tariff, a zero tariff is globally optimal for a large country in the homogeneous firm model.

In Fig. 2, we repeat the same exercises as Fig. 1 in the heterogeneous firm model. Just like Fig. 1, country 1's iso-welfare curves become steeper as L_1 increases and/or ρ decreases. An important difference is that, when country 1 is smaller than country 2, the former's iso-welfare curves become U-shaped for a sufficiently large τ_{12}. This is because an increase in τ_{12} makes country 2 less open (i.e., decreases λ_{21}), which weakens country 1's losses from autarkiness. This implies that, with τ_{12} sufficiently large, country 1's government can increase its long-run welfare by either decreasing or increasing τ_{21}. In the middle left panel, for example, starting from $(\tau_{21}, \tau_{12}) = (1.5, 1.5)$, where $5.6 < \rho U_1 < 5.7$, country 1 gains by setting either $\tau_{21} = 1$ or $\tau_{21} = 2$.[21] For a smaller country, we cannot ensure that a zero tariff is always optimal. However, the three panels in the left column also indicate that $\tau_{21} = 1$ is country 1's best response, given that the larger country 2 chooses $\tau_{12} = 1$ as its dominant strategy (as implied by the three panels in the right column).

Our numerical results are summarized as follows. First, in the homogeneous firm model, a zero tariff is optimal for both larger and smaller countries. Second, in the heterogeneous firm model, a zero tariff is optimal for a larger or equally large country, and also for a smaller country as long as its larger trading partner sets an optimal tariff of zero. Therefore, global free trade can be supported as a Nash equilibrium, whether the two countries are symmetric or asymmetric, and whether the intermediate good firms are homogeneous or heterogeneous.

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[19] The fact that all displayed iso-welfare curves are downward-sloping on the vertical axis means that Eq. (14) is satisfied in this domain.

[20] Another observation is that, for the same (τ_{21}, τ_{12}), the value of ρU_1 decreases despite that ρ increases. This implies that U_1 decreases by more than the increase in ρ.

[21] Country 1 could gain more if it could set its tariff beyond $\tau_{21} = 2$. The problem, then, is whether an optimal tariff exists.

7 Concluding remarks

Our theory has important policy implications. If national leaders take economic growth seriously, as they almost always say they do, it makes little sense for their own countries to deviate from free trade. In the face of the recent U.S.-China trade disputes, a typical argument against them by trade economists is that they could end up with a prisoner's dilemma, and committing to the reciprocity principle of the GATT/WTO would be a solution. However, the problem is that each welfare-maximizing government is tempted to increase its tariff unilaterally to improve its terms of trade. By extending the simplest and widely accepted RBR lab-equipment model of endogenous technological change to include asymmetric countries, import tariffs, and either homogeneous or heterogeneous firms, this paper provides a stronger argument that it is in each country's own interest to keep free trade even if it is large in an economic sense.

Our model is open for extensions. First, it will be natural to have more than two countries. Since it will decrease each country's market power, the optimality of free trade will be more likely. The multi-country model will also enable us to study the long-run growth and welfare effects of regional trade agreements. Second, it will be interesting to consider multiple production stages. In the present model, there is only one intermediate production stage for international trade. Introducing more than one intermediate production stage will allow us to compare the likelihood of zero optimal tariffs in different stages of global value chains.

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Appendix A. Derivations of key equations in a homogeneous firm lab-equipment model

Derivations of Eqs. (3) and (4)

Substituting $e_{kj} = \tau_{kj}^{-\sigma}[p_k^Y/(1-1/\sigma)]^{1-\sigma}P_j^\sigma X_j$ into $\zeta_{kj} = \tau_{kj}n_k^e e_{kj}/\sum_l \tau_{lj}n_l^e e_{lj}, k \neq j$, the latter is rewritten as $\zeta_{kj} = (n_k^e/n_j^e)(\tau_{kj}p_k^Y/p_j^Y)^{1-\sigma}/[1+(n_k^e/n_j^e)(\tau_{kj}p_k^Y/p_j^Y)^{1-\sigma}], k \neq j$. Combining this with $\zeta_{jj}+\zeta_{kj} = 1, k \neq j$, and noting that $\overline{m}_j^{1-\sigma} = 1+(n_k^e/n_j^e)(\tau_{kj}p_k^Y/p_j^Y)^{1-\sigma}, k \neq j$, we obtain Eq. (3).

For Eq. (4), we first rewrite $P_jX_j = \sum_k \tau_{kj}E_{kj}$ using $E_{jk} = E_{kj}, k \neq j$ and $\lambda_{jk} = E_{jk}/\sum_l E_{jl}$ as $P_jX_j = [1+(\tau_{kj}-1)\lambda_{jk}]\sum_l E_{jl}, k \neq j$, or:

$$\textstyle\sum_l E_{jl} = \{1/[1+(\tau_{kj}-1)\lambda_{jk}]\}P_jX_j, k \neq j. \tag{30}$$

Using Eq. (30), $E_{jk} = E_{kj}, k \neq j$, and $\lambda_{jk} = E_{jk}/\sum_l E_{jl}$, we obtain:

$$\zeta_{kj} = \tau_{kj}\lambda_{jk}/[1+(\tau_{kj}-1)\lambda_{jk}] \Leftrightarrow \lambda_{jk} = \zeta_{kj}/[\tau_{kj}-(\tau_{kj}-1)\zeta_{kj}], k \neq j. \tag{31}$$

Combining Eq. (31) with $\zeta_{jj}+\zeta_{kj} = 1, \lambda_{jj}+\lambda_{jk} = 1, k \neq j$, we obtain Eq. (4).

Derivations of Eqs. (6) and (7)

Using the free entry condition $\sum_k v_{jk} = P_j^K\kappa_j^e$, the asset market-clearing condition $W_j = \sum_k n_j^e v_{jk}$ is rewritten as:

$$W_j = n_j^e \textstyle\sum_k v_{jk} = n_j^e P_j^K \kappa_j^e = p_j^K\kappa_j^e; p_j^K \equiv n_j^e P_j^K. \tag{32}$$

Time differentiating Eq. (32), and using Eq. (32), $\pi_{jk} = e_{jk}/\sigma, E_{jk} = n_j^e e_{jk}$, and the no-arbitrage condition $\dot{v}_{jk} = (r_j+\delta)v_{jk} - \pi_{jk}$ (derived by time differentiating $v_{jkt} = \int_t^\infty \pi_{jks}\exp(-\int_t^s(r_{ju}+\delta)du)ds$), we obtain:

$$\dot{W}_j = W_j(\gamma_j + r_j + \delta) - (1/\sigma)\textstyle\sum_k E_{jk}; \gamma_j \equiv \dot{n}_j^e/n_j^e.$$

Applying Shephard's lemma to $P_jX_j + w_jL_j^Y = c_j^Y(P_j, w_j)Y_j$, and using $p_j^Y = c_j^Y(P_j, w_j)$, the expenditures for the intermediate goods and labor are given by, respectively:

$$P_jX_j = \alpha_j p_j^Y Y_j, \tag{33}$$

$$w_jL_j^Y = (1-\alpha_j)p_j^Y Y_j. \tag{34}$$

Using Eqs. (30) and (33), the expression for $\dot{W}_j$ is rewritten as:

$$\dot{W}_j = W_j(\gamma_j + r_j + \delta) - \{(\alpha_j/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}]\}p_j^Y Y_j, k \neq j.$$

Multiplying $Y_j = C_j + D_j + F_j$ by p_j^Y, and using Eqs. (30), (33), $\pi_{jk} = e_{jk} - p_j^Y y_{jk} = e_{jk}/\sigma, P_j^K Q_j^K = p_j^Y D_j, Q_j^K = \kappa_j^e(\dot{n}_j^e + \delta n_j^e), F_j = \sum_k n_j^e y_{jk}, E_{jk} = n_j^e e_{jk}$, and $p_j^K = n_j^e P_j^K$, $p_j^Y Y_j$ is expressed as:

$$p_j^Y Y_j = \{1/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}[E_j + p_j^K\kappa_j^e(\gamma_j+\delta)], k \neq j. \tag{35}$$

Substituting Eq. (35) into the last expression for $\dot{W}_j$, and using Eq. (32), we obtain:

$$\begin{aligned}\dot{W}_j/W_j &= r_j + \{\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}(\gamma_j+\delta) \\ &\quad - \{\{(\alpha_j/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}Z_j; \\ Z_j &\equiv E_j/W_j, k \neq j,\end{aligned}$$

where a transformed variable $Z_j = E_j/W_j$ is interpreted as country j's average propensity to consume out of asset. Substituting the above expression and the Euler equation $\dot{E}_j/E_j = r_j - \rho$ into $\dot{Z}_j/Z_j = \dot{E}_j/E_j - \dot{W}_j/W_j$ gives:

$$\begin{aligned}\dot{Z}_j/Z_j &= \{\{(\alpha_j/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}Z_j - \rho \\ &\quad - \{\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}(\gamma_j+\delta), k \neq j. \qquad (36)\end{aligned}$$

Multiplying $L_j = L_j^Y$ by w_j, and using Eqs. (32), (34), and (35), we obtain:

$$\gamma_j = \{\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}/(1-\alpha_j)\}w_j L_j/(p_j^K \kappa_j^e) - Z_j - \delta, k \neq j. \qquad (37)$$

On a BGP, both $\dot{Z}_j/Z_j$ and γ_j are constant. Since $\lambda_{jk} (\in [0,1])$ is constant on a BGP, Eq. (36) implies that Z_j is constant on a BGP. From Eqs. (36), (37), and $\dot{Z}_j/Z_j = 0$, Z_j and γ_j are solved as:

$$Z_j^* = \rho + \{\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}/(1-\alpha_j)\}w_j^* L_j/(p_j^{K*}\kappa_j^e), \qquad (38)$$

$$\gamma_j^* = [\alpha_j/(1-\alpha_j)](1/\sigma)\{1/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^* L_j/(p_j^{K*}\kappa_j^e) - \rho - \delta, k \neq j. \qquad (39)$$

Using $P_j^K = p_j^Y$ and $p_j^Y = (n_j^e)^{-1}[\overline{m}_j/(1-1/\sigma)]^{\sigma-1}w_j$, $p_j^K = n_j^e P_j^K$ is rewritten as $p_j^K = [\overline{m}_j/(1-1/\sigma)]^{\sigma-1}w_j$. From Eqs. (3), (4), $\alpha_j = (\sigma-1)/\sigma$, and $p_j^K = [\overline{m}_j/(1-1/\sigma)]^{\sigma-1}w_j$, Eq. (39) is rewritten as Eq. (7).

On a BGP, γ_j^* is constant. Then, from Eq. (7), $\lambda_{jj}^* (\in [0,1])$ is constant. Since λ_{jj}^* is a function of w_1^*, χ^*, and tariffs, whereas w_1^* is a function of χ^* and tariffs, constancy of $\lambda_{jj}^* (\in [0,1])$ requires constancy of χ^*. This implies that $\dot{\chi}^*/\chi^* = \gamma_1^* - \gamma_2^* = 0$, or Eq. (6).

Derivation of Eq. (8)

Substituting $p_{jt}^Y = p_j^{K*}/n_{jt}^e = p_j^{Y*}\exp(-\gamma^* t)$ into $U_j = \int_0^\infty (\ln E_{jt} - \ln p_{jt}^Y)\exp(-\rho t)dt$, and applying integration by parts, we obtain:

$$\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^*.$$

For E_j^*, multiplying Eq. (38) by $W_j^* = p_j^{K*}\kappa_j^e$ from Eq. (32), and noting that $\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}/(1-\alpha_j) = 1 + (\sigma-1)(\tau_{kj}-1)\lambda_{jk}^*/[1+(\tau_{kj}-1)\lambda_{jk}^*]$ from $\alpha_j = (\sigma-1)/\sigma$, we obtain:

$$
\begin{aligned}
E_j^* &= p_j^{K*}\kappa_j^e\{\rho + \{1+(\sigma-1)(\tau_{kj}-1)\lambda_{jk}^*/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j/(p_j^{K*}\kappa_j^e)\} \\
&= p_j^{K*}\kappa_j^e\rho + \{1+(\sigma-1)(\tau_{kj}-1)\lambda_{jk}^*/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j, k \neq j.
\end{aligned} \tag{40}
$$

Rewriting Eq. (39) using Eq. (6) and $\alpha_j = (\sigma-1)/\sigma$ gives:

$$
\begin{aligned}
&: \rho + \delta + \gamma^* = \{(1-1/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j/(p_j^{K*}\kappa_j^e) \\
&\Leftrightarrow p_j^{K*}\kappa_j^e = (1-1/\sigma)w_j^*L_j/\{(\rho+\delta+\gamma^*)[1+(\tau_{kj}-1)\lambda_{jk}^*]\}, k \neq j.
\end{aligned} \tag{41}
$$

Substituting Eq. (41) into Eq. (40), E_j^* is rewritten as:

$$
E_j^* = \{w_j^*L_j/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}\eta_j^*; \eta_j^* \equiv (1-1/\sigma)\rho/(\rho+\delta+\gamma^*) + 1 + \sigma(\tau_{kj}-1)\lambda_{jk}^*, k \neq j.
$$

To express w_j^*/p_j^{Y*}, we use $p_j^Y = P_j^{\alpha_j}w_j^{1-\alpha_j}$ and $\alpha_j = (\sigma-1)/\sigma$ to obtain $w_j^*/p_j^{Y*} = (p_j^{Y*}/P_j^*)^{\sigma-1}$, where $p_j^{Y*} \equiv p_{j0}^Y$ and $P_j^* \equiv P_{j0}$ are evaluated at the initial period of a BGP. Using Eqs. (1) and (3), this is further rewritten as:

$$
w_j^*/p_j^{Y*} = (p_j^{Y*}/P_j^*)^{\sigma-1} = n_{j0}^e/\{\zeta_{jj}^*[1/(1-1/\sigma)]^{\sigma-1}\}. \tag{42}
$$

Substituting Eq. (42) into the last expression for E_j^* divided by p_j^{Y*}, and using Eq. (4), we obtain:

$$
E_j^*/p_j^{Y*} = L_j\{(w_j^*/p_j^{Y*})/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}\eta_j^* = L_j(K_j/\lambda_{jj}^*)\eta_j^*; K_j \equiv n_{j0}^e/[1/(1-1/\sigma)]^{\sigma-1}, k \neq j.
$$

Substituting this into $\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^*$, we obtain Eq. (8).

Derivation of Eq. (11)

Step (i):

Logarithmically differentiating $\overline{m}_j$ in Eq. (1) gives:

$$
\widehat{\overline{m}}_j = \zeta_{kj}\{[1/(1-\sigma)]d\ln(n_k^e/n_j^e) + \widehat{\tau}_{kj} + \widehat{p}_k^Y - \widehat{p}_j^Y\}, k \neq j. \tag{43}
$$

Eq. (43) implies that:

$$
\widehat{\overline{m}}_1 - \widehat{\overline{m}}_2 = -(\zeta_{21}+\zeta_{12})[\widehat{p}_1^Y - \widehat{p}_2^Y - \widehat{\chi}/(\sigma-1)] + \zeta_{21}\widehat{\tau}_{21} - \zeta_{12}\widehat{\tau}_{12}.
$$

Substituting this into the logarithmically differentiated form of Eq. (2), $\widehat{p}_1^Y - \widehat{p}_2^Y$ is solved as:

$$
\widehat{p}_1^Y - \widehat{p}_2^Y = (1/\Delta)[\widehat{w}_1 - (1-\zeta_{21}-\zeta_{12})\widehat{\chi} + (\sigma-1)(\zeta_{21}\widehat{\tau}_{21} - \zeta_{12}\widehat{\tau}_{12})]; \Delta \equiv 1 + (\sigma-1)(\zeta_{21}+\zeta_{12}) > 1. \tag{44}
$$

Eq. (44) implies that:

$$
\widehat{p}_1^Y - \widehat{p}_2^Y - \widehat{\chi}/(\sigma-1) = (1/\Delta)\{\widehat{w}_1 - [\sigma/(\sigma-1)]\widehat{\chi} + (\sigma-1)(\zeta_{21}\widehat{\tau}_{21} - \zeta_{12}\widehat{\tau}_{12})\}.
$$

Substituting this back into Eq. (43), we obtain:

$$\widehat{\overline{m}}_1 = (\zeta_{21}/\Delta)\{-\{\widehat{w}_1 - [\sigma/(\sigma-1)]\widehat{\chi}\} + [\Delta - (\sigma-1)\zeta_{21}]\widehat{\tau}_{21} + (\sigma-1)\zeta_{12}\widehat{\tau}_{12}\}, \tag{45}$$

$$\widehat{\overline{m}}_2 = (\zeta_{12}/\Delta)\{\widehat{w}_1 - [\sigma/(\sigma-1)]\widehat{\chi} + [\Delta - (\sigma-1)\zeta_{12}]\widehat{\tau}_{12} + (\sigma-1)\zeta_{21}\widehat{\tau}_{21}\}. \tag{46}$$

Step (ii):
Logarithmically differentiating Eq. (5) gives:

$$\widehat{\zeta}_{12} - \widehat{\tau}_{12} = \widehat{\zeta}_{21} - \widehat{\tau}_{21} + \widehat{w}_1.$$

From Eq. (3) and $\zeta_{jj} + \zeta_{kj} = 1, k \neq j$, we obtain $\widehat{\zeta}_{kj} = -[(1-\zeta_{kj})/\zeta_{kj}]\widehat{\zeta}_{jj} = -[(1-\zeta_{kj})/\zeta_{kj}](\sigma-1)\widehat{\overline{m}}_j$. Then the above expression is rewritten as:

$$-(\sigma-1)[(1-\zeta_{12})/\zeta_{12}]\widehat{\overline{m}}_2 - \widehat{\tau}_{12} = -(\sigma-1)[(1-\zeta_{21})/\zeta_{21}]\widehat{\overline{m}}_1 - \widehat{\tau}_{21} + \widehat{w}_1.$$

Substituting Eqs. (45) and (46) into the above expression, we obtain:

$$\begin{aligned}
0 &= -B\widehat{w}_1 + C\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12} \Leftrightarrow \widehat{w}_1 = (1/B)(C\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}); \qquad (47)\\
B &\equiv 1 + [(\sigma-1)/\Delta](2 - \zeta_{21} - \zeta_{12}) > 1,\\
C &\equiv (\sigma/\Delta)(2 - \zeta_{21} - \zeta_{12}) > 0,\\
F_{jk} &\equiv 1 + [(\sigma-1)/\Delta]\{(1-\zeta_{jk})[\Delta - (\sigma-1)\zeta_{jk}] - (1-\zeta_{kj})(\sigma-1)\zeta_{jk}\}, k \neq j.
\end{aligned}$$

Step (iii):
Eq. (47) implies that:

$$\widehat{w}_1 - [\sigma/(\sigma-1)]\widehat{\chi} = (1/B)\{-[\sigma/(\sigma-1)]\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}\}.$$

Substituting this back into Eqs. (45) and (46), they are rewritten as:

$$\widehat{\overline{m}}_1 = (\zeta_{21}/\Delta)(1/B)\{[\sigma/(\sigma-1)]\widehat{\chi} + (\sigma-1)\widehat{\tau}_{21} + \sigma\widehat{\tau}_{12}\}, \tag{48}$$

$$\widehat{\overline{m}}_2 = (\zeta_{12}/\Delta)(1/B)\{-[\sigma/(\sigma-1)]\widehat{\chi} + (\sigma-1)\widehat{\tau}_{12} + \sigma\widehat{\tau}_{21}\}. \tag{49}$$

Logarithmically differentiating Eq. (4), and using Eq. (3): $\widehat{\zeta}_{jj} = (\sigma-1)\widehat{\overline{m}}_j$ and $\zeta_{jj} + \zeta_{kj} = 1, k \neq j$, give:

$$\widehat{\lambda}_{jj} = \{1/[\tau_{kj} - (\tau_{kj} - 1)\zeta_{kj}]\}[\zeta_{kj}\widehat{\tau}_{kj} + (\sigma-1)\widehat{\overline{m}}_j].$$

Substituting Eqs. (48) and (49) into the above expression, noting Eq. (31), and substituting the results into Eq. (9), we obtain:

$$d\gamma_1 = -(\rho + \delta + \gamma)[(\sigma/\Delta)/B]\lambda_{12}[\widehat{\chi} + (\sigma-1)\widehat{\tau}_{12} + \sigma\widehat{\tau}_{21}], \tag{50}$$

$$d\gamma_2 = -(\rho + \delta + \gamma)[(\sigma/\Delta)/B]\lambda_{21}[-\widehat{\chi} + (\sigma-1)\widehat{\tau}_{21} + \sigma\widehat{\tau}_{12}]. \tag{51}$$

Step (iv):

Substituting Eqs. (50) and (51) into the differentiated form of Eq. (6), $\widehat{\chi}$ is solved as:

$$\widehat{\chi} = [1/(\lambda_{12}+\lambda_{21})]\{[\lambda_{21}\sigma - \lambda_{12}(\sigma-1)]\widehat{\tau}_{12} - [\lambda_{12}\sigma - \lambda_{21}(\sigma-1)]\widehat{\tau}_{21}\}. \tag{52}$$

Step (v):

Substituting Eq. (52) back into Eq. (51), we obtain Eq. (11).

Appendix B. Derivations of key equations in a heterogeneous firm lab-equipment model

Derivations of Eqs. (17) and (18)

The right-hand side of $v_{jk0}(a^*_{jk})/v_{kk0}(a^*_{kk}) = P^K_{j0}\kappa_{jk}/(P^K_{k0}\kappa_{kk}), j \neq k$, is simply rewritten as $(p^Y_j/p^Y_k)^*\kappa_{jk}/\kappa_{kk}$. In the left-hand side, $v_{jk0}(a)$ is given by:

$$v_{jk0}(a) = \pi_{jk0}(a)\Delta_{jk0}(a); \Delta_{jk0}(a) \equiv \int_0^\infty \exp(-\int_0^t (r_{js} + \delta - \dot{\pi}_{jks}(a)/\pi_{jks}(a))ds)dt.$$

This implies that we have to calculate r_{js} and $\dot{\pi}_{jks}(a)/\pi_{jks}(a)$ on a BGP to calculate $\Delta_{jk0}(a)$.

Using the free entry condition (16), the asset market-clearing condition $W_j = \sum_k n_{jk}\int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk})da$ is rewritten as:

$$W_j = n^e_j \textstyle\sum_k G_j(a_{jk})\displaystyle\int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk})da = n^e_j P^K_j(\textstyle\sum_k \kappa_{jk}G_j(a_{jk}) + \kappa^e_j) = p^K_j\overline{\kappa}_j. \tag{53}$$

In the same way as the homogeneous firm model, Z^*_j and γ^*_j are derived as:

$$Z^*_j = \rho + \{\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda^*_{jk}]\}/(1-\alpha_j)\}w^*_jL_j/(p^{K*}_j\overline{\kappa}^*_j), \tag{54}$$

$$\gamma^*_j = [\alpha_j/(1-\alpha_j)](1/\sigma)\{1/[1+(\tau_{kj}-1)\lambda^*_{jk}]\}w^*_jL_j/(p^{K*}_j\overline{\kappa}^*_j) - \rho - \delta, k \neq j. \tag{55}$$

Multiplying Eq. (54) by $W^*_j = p^{K*}_j\overline{\kappa}^*_j$ from Eq. (53), and using $\alpha_j = 1 - 1/\sigma$, we obtain:

$$\begin{aligned} E^*_j &= p^{K*}_j\overline{\kappa}^*_j\{\rho + \{1+(\sigma-1)(\tau_{kj}-1)\lambda^*_{jk}/[1+(\tau_{kj}-1)\lambda^*_{jk}]\}w^*_jL_j/(p^{K*}_j\overline{\kappa}^*_j)\} \\ &= p^{K*}_j\overline{\kappa}^*_j\rho + \{1+(\sigma-1)(\tau_{kj}-1)\lambda^*_{jk}/[1+(\tau_{kj}-1)\lambda^*_{jk}]\}w^*_jL_j, k \neq j. \end{aligned} \tag{56}$$

Since $\lambda^*_{jk}(\in[0,1])$ is constant, and $\overline{\kappa}^*_j$ is constant because the cutoffs are constant from Eq. (16), E^*_j is constant from Eq. (56), as long as w^*_j and hence $p^{K*}_j = [\overline{m}^*_j/(1-1/\sigma)]^{\sigma-1}w^*_j$ are constant. This and the Euler equation imply that $r^*_j = \rho$.

For $\dot{\pi}_{jks}(a)/\pi_{jks}(a)$, noting that $P_jX_j = (\sigma-1)w_jL_j$, $\pi_{jk}(a) = \tau^{-\sigma}_{jk}[p^Y_j a/(1-1/\sigma)]^{1-\sigma}P^\sigma_k X_k/\sigma$ is rewritten as:

$$\pi_{jkt}(a) = \tau^{-\sigma}_{jk}[a/(1-1/\sigma)]^{1-\sigma}(P_{kt}/p^Y_{jt})^{\sigma-1}(1-1/\sigma)w^*_kL_k. \tag{57}$$

Dividing Eq. (19) for $j = k$ by p_{jt}^Y gives $P_{kt}/p_{jt}^Y = (n_{kt}^e)^{1/(1-\sigma)}(p_k^Y/p_j^Y)^*\overline{m}_k^*/(1-1/\sigma)$. Substituting this into Eq. (57), and noting that n_{kt}^e grows at the rate γ^*, $\pi_{jkt}(a)$ grows at the rate $-\gamma^*$: $\dot{\pi}_{jkt}(a)/\pi_{jkt}(a) = -\gamma^*$.

Substituting the results into the definition of $\Delta_{jk0}(a)$, we obtain $\Delta_{jk0}(a) = 1/(\rho+\delta+\gamma^*)$, and hence:

$$v_{jk0}(a) = \pi_{jk0}(a)/(\rho+\delta+\gamma^*). \tag{58}$$

Dividing Eq. (58) by itself with $j = k$, and using Eq. (57), we obtain $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*) = \pi_{jk0}(a_{jk}^*)/\pi_{kk0}(a_{kk}^*) = \tau_{jk}^{-\sigma}[(p_j^Y/p_k^Y)^* a_{jk}^*/a_{kk}^*]^{1-\sigma}$. Therefore, $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*) = P_{j0}^K\kappa_{jk}/(P_{k0}^K\kappa_{kk}), j \neq k$, is rewritten as:

$$\tau_{jk}^{-\sigma}[(p_j^Y/p_k^Y)^* a_{jk}^*/a_{kk}^*]^{1-\sigma} = (p_j^Y/p_k^Y)^*\kappa_{jk}/\kappa_{kk}, j \neq k.$$

Solving this for a_{12}^*/a_{22}^* and a_{21}^*/a_{11}^*, we obtain Eqs. (17) and (18), respectively.

7.1 Derivations of Eqs. (19) to (24)

$P_j^{1-\sigma} = \int_{\Theta_j} p_j(i)^{1-\sigma}di$ is now rewritten as $P_j^{1-\sigma} = \sum_k n_{kj}\int_0^{a_{kj}}[\tau_{kj}p_k^Y a/(1-1/\sigma)]^{1-\sigma}\mu_{kj}(a|a_{kj})da$. Noting that $\overline{a}_{kj}(a_{kj})^{1-\sigma} = \int_0^{a_{kj}} a^{1-\sigma}\mu_{kj}(a|a_{kj})da$, straightforward calculation implies Eq. (19).

Derivation of Eq. (20) is the same as Eq. (2), except the definition of $\overline{m}_j$.

Using $\lambda_{jk} = E_{jk}/\sum_l E_{jl}$; $E_{jk} \equiv n_{jk}\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da$, country j's balance of trade $E_{jk} = E_{kj}, k \neq j$, is rewritten as $\lambda_{jk}\sum_l E_{jl} = \lambda_{kj}\sum_l E_{kl}, k \neq j$. Also, it can be easily verified that Eq. (30) continues to apply to the heterogeneous firm model. Using this and $P_jX_j = (\sigma-1)w_jL_j$, country j's balance of trade is further rewritten as Eq. (21).

Using Eqs. (15), (58), $\pi_{jk}(a) = e_{jk}(a)/\sigma, h_{jk}(a_{jk}) = (\overline{a}_{jk}(a_{jk})/a_{jk})^{1-\sigma}-1, \overline{a}_{jk}(a_{jk})^{1-\sigma} = \int_0^{a_{jk}} a^{1-\sigma}\mu_{jk}(a|a_{jk})da$, and $n_{jk} \equiv n_j^e G_j(a_{jk})$, E_{jk}^* is rewritten as $E_{jk}^* = n_{j0}^e G_j(a_{jk}^*)(h_{jk}(a_{jk}^*)+1)\sigma(\rho+\delta+\gamma^*)P_{j0}^K\kappa_{jk}$. Substituting this into $\lambda_{jk} = E_{jk}/\sum_l E_{jl}$, we obtain $\lambda_{jk}^* = (H_{jk}(a_{jk}^*)+G_j(a_{jk}^*))\kappa_{jk}/\sum_l(H_{jl}(a_{jl}^*)+G_j(a_{jl}^*))\kappa_{jl}$. Moreover, using Eq. (16) and $H_{jk}(a_{jk}) + G_j(a_{jk}) = G_j(a_{jk})\beta/(\beta-1) = \beta H_{jk}(a_{jk})$, we obtain Eq. (22).

Using (57), (58), and $p_j^{K*} = n_{j0}^e P_{j0}^K$, the zero cutoff profit condition (15) for $k = j$ is rewritten as:

$$[a_{jj}^*/(1-1/\sigma)]^{1-\sigma}(P_j^*/p_j^{Y*})^{\sigma-1}(1-1/\sigma)w_j^*L_j/(\rho+\delta+\gamma^*) = (p_j^{K*}/n_{j0}^e)\kappa_{jj},$$

where $p_j^{Y*} \equiv p_{j0}^Y$ and $P_j^* \equiv P_{j0}$ are evaluated at the initial period of a BGP as before.

Rewriting Eq. (55) using Eq. (23) and $\alpha_j = (\sigma-1)/\sigma$ gives:

$$\begin{aligned} &: \rho+\delta+\gamma^* = \{(1-1/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j/(p_j^{K*}\overline{\kappa}_j^*) \\ &\Leftrightarrow (1-1/\sigma)w_j^*L_j/(\rho+\delta+\gamma^*) = [1+(\tau_{kj}-1)\lambda_{jk}^*]p_j^{K*}\overline{\kappa}_j^*, k \neq j. \end{aligned} \tag{59}$$

Using Eq. (16) and $H_{jk}(a_{jk}) + G_j(a_{jk}) = \beta H_{jk}(a_{jk})$, $\overline{\kappa}_j^*$ turns out to be constant:

$$\overline{\kappa}_j^* = \beta\kappa_j^e. \tag{60}$$

Substituting Eq. (60) into Eq. (59), substituting it into the above domestic zero cutoff profit condition, and solving it for $(p_j^{Y*}/P_j^*)^{\sigma-1} = w_j^*/p_j^{Y*}$, we obtain:

$$w_j^*/p_j^{Y*} = (p_j^{Y*}/P_j^*)^{\sigma-1} = [1+(\tau_{kj}-1)\lambda_{jk}^*]\beta\kappa_j^e n_{j0}^e/\{[a_{jj}^*/(1-1/\sigma)]^{\sigma-1}\kappa_{jj}\}. \tag{61}$$

Using $\alpha_j = (\sigma-1)/\sigma$, Eq. (55) is rewritten as $\gamma_j^* = \{(1-1/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}L_j/[(p_j^{K*}/w_j^*)\overline{\kappa}_j^*] - \rho - \delta, k \neq j$, where $(p_j^{K*}/w_j^*)\overline{\kappa}_j^*$ is rewritten using Eq. (60) and $p_j^K = n_j^e P_j^K = n_j^e p_j^Y$ as $(p_j^{K*}/w_j^*)\overline{\kappa}_j^* = \beta\kappa_j^e n_{j0}^e p_j^{Y*}/w_j^*$. Combining them with Eq. (61), we obtain Eq. (24).

Derivation of Eq. (25)

Substituting $p_j^{K*}\overline{\kappa}_j^*$ from Eq. (59) into Eq. (56), E_j^* is rewritten as:

$$E_j^* = \{w_j^* L_j/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}\eta_j^*; \eta_j^* \equiv (1-1/\sigma)\rho/(\rho+\delta+\gamma^*) + 1 + \sigma(\tau_{kj}-1)\lambda_{jk}^*, k \neq j.$$

Substituting Eq. (61) into the above expression divided by p_j^{Y*}, we obtain:

$$E_j^*/p_j^{Y*} = L_j\{(w_j^*/p_j^{Y*})/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}\eta_j^* = L_j(K_j/a_{jj}^{*\sigma-1})\eta_j^*;$$
$$K_j \equiv \beta\kappa_j^e n_{j0}^e/\{[1/(1-1/\sigma)]^{\sigma-1}\kappa_{jj}\}, k \neq j.$$

Substituting this into $\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^*$, we obtain Eq. (25).

7.2 Derivation of Eq. (28)

The long-run growth effects of tariff changes are derived in six steps: (i) from Eqs. (19) and (20), we solve for $\widehat{p_1^Y/p_2^Y} = \widehat{p_1^Y/p_2^Y}(\widehat{w}_1, \widehat{\chi}, \{\widehat{a}_{jk}\}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (ii) substituting the result from step (i) into the logarithmically differentiated forms of Eqs. (17) and (18), and combining them with the logarithmically differentiated form of Eq. (16), we solve for $\widehat{a}_{jk} = \widehat{a}_{jk}(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iii) substituting the result from step (ii) into the logarithmically differentiated form of Eq. (22), and substituting it into the logarithmically differentiated form of Eq. (21), we solve for $\widehat{w}_1 = \widehat{w}_1(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iv) substituting the result from step (iii) back into $\widehat{a}_{jj} = \widehat{a}_{jj}(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$, and substituting it into Eq. (26), we solve for $d\gamma_j = d\gamma_j(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (v) substituting the result from step (iv) into the differentiated form of Eq. (23), we solve for $\widehat{\chi} = \widehat{\chi}(\widehat{\tau}_{21}, \widehat{\tau}_{12})$; and (vi) substituting the result from step (v) back into $d\gamma_2 = d\gamma_2(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$, we solve for $d\gamma = d\gamma_2(\widehat{\tau}_{21}, \widehat{\tau}_{12})$.

Step (i):

Logarithmically differentiating Eq. (16), and using Eq. (22), we obtain:

$$0 = \textstyle\sum_k \lambda_{jk}\widehat{a}_{jk}. \tag{62}$$

Logarithmically differentiating $\overline{m}_j$ in Eq. (19) gives:

$$\widehat{\overline{m}}_j = [1/(1-\sigma)]\textstyle\sum_k \frac{(n_k^e/n_j^e)G_k(a_{kj})[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}}{\sum_l (n_l^e/n_j^e)G_l(a_{lj})[(\tau_{lj}p_l^Y/p_j^Y)\overline{a}_{lj}(a_{lj})]^{1-\sigma}}$$
$$\times\, d\ln\{(n_k^e/n_j^e)G_k(a_{kj})[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\}.$$

expenditure share of varieties country j buys from country k is now given by:

$$\zeta_{kj} \equiv \frac{n_{kj}\int_0^{a_{kj}} \tau_{kj}p_{kj}^f(a)x_{kj}(a)\mu_{kj}(a|a_{kj})da}{\sum_l n_{lj}\int_0^{a_{lj}} \tau_{lj}p_{lj}^f(a)x_{lj}(a)\mu_{lj}(a|a_{lj})da} = \frac{\tau_{kj}E_{kj}}{\sum_l \tau_{lj}E_{lj}}; \textstyle\sum_k \zeta_{kj} = 1.$$

Using $e_{kj}(a) = \tau_{kj}^{-\sigma}[p_k^Y a/(1-1/\sigma)]^{1-\sigma} P_j^\sigma X_j$, $\overline{a}_{kj}(a_{kj})^{1-\sigma} = \int_0^{a_{kj}} a^{1-\sigma}\mu_{kj}(a|a_{kj})da$, and $n_{kj} = n_k^e G_k(a_{kj})$, the above expression is rewritten as:

$$\zeta_{kj} = \frac{(n_k^e/n_j^e)G_k(a_{kj})[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}}{\sum_l (n_l^e/n_j^e)G_l(a_{lj})[(\tau_{lj}p_l^Y/p_j^Y)\overline{a}_{lj}(a_{lj})]^{1-\sigma}}.$$

$d\ln\{(n_k^e/n_j^e)G_k(a_{kj})[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\}$ is rewritten using $G_k(a_{kj}) = a_{k0}^{-\theta}a_{kj}^\theta$ and $\overline{a}_{kj}(a_{kj}) = [\beta/(\beta-1)]^{1/(1-\sigma)}a_{kj}$ as:

$$d\ln\{(n_k^e/n_j^e)G_k(a_{kj})[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\} = d\ln(n_k^e/n_j^e) + \theta\widehat{a}_{kj} + (1-\sigma)(\widehat{\tau}_{kj} + \widehat{p}_k^Y - \widehat{p}_j^Y + \widehat{a}_{kj}).$$

Using these expressions and Eq. (62), $\widehat{\overline{m}}_j$ is rewritten as:

$$\widehat{\overline{m}}_j = -(\beta-1)(1-\zeta_{kj})\widehat{a}_{jj} + \zeta_{kj}\{[1/(1-\sigma)]d\ln(n_k^e/n_j^e) + (\beta-1)[(1-\lambda_{kj})/\lambda_{kj}]\widehat{a}_{kk} + \widehat{\tau}_{kj} + \widehat{p}_k^Y - \widehat{p}_j^Y\}, k \neq j.$$

This implies that:

$$\begin{aligned}
&\widehat{\overline{m}}_1 - \widehat{\overline{m}}_2 = (\beta-1)(\xi_2\widehat{a}_{22} - \xi_1\widehat{a}_{11}) - (\zeta_{21}+\zeta_{12})[\widehat{p}_1^Y - \widehat{p}_2^Y - \widehat{\chi}/(\sigma-1)] + \zeta_{21}\widehat{\tau}_{21} - \zeta_{12}\widehat{\tau}_{12};\\
&\qquad \xi_j \equiv 1 - \zeta_{kj} + \zeta_{jk}(1-\lambda_{jk})/\lambda_{jk} > 0, k \neq j.
\end{aligned}$$

Substituting this into the logarithmically differentiated form of Eq. (20), $\widehat{p}_1^Y - \widehat{p}_2^Y$ is solved as:

$$\begin{aligned}
&\widehat{p}_1^Y - \widehat{p}_2^Y = (1/\Delta)[\widehat{w}_1 - (1-\zeta_{21}-\zeta_{12})\widehat{\chi}] + [(\sigma-1)/\Delta][\zeta_{21}\widehat{\tau}_{21} - \zeta_{12}\widehat{\tau}_{12} + (\beta-1)(\xi_2\widehat{a}_{22} - \xi_1\widehat{a}_{11})];\\
&\qquad \Delta \equiv 1 + (\sigma-1)(\zeta_{21}+\zeta_{12}) > 1.
\end{aligned} \tag{63}$$

We assume that country j's import expenditure share is smaller than one half due to trade costs:

$$\zeta_{kj} < 1/2 \forall j, k, k \neq j \Rightarrow \lambda_{jk} < 1/2, \tag{64}$$

where $\lambda_{jk} < 1/2, k \neq j$, follows from Eq. (31). Eq. (64) ensures that:

$$\begin{aligned}
1 - \zeta_{21} - \zeta_{12} > 0,\\
1 - \lambda_{12} - \lambda_{21} > 0.
\end{aligned}$$

This helps us to evaluate the signs of some expressions that appear in the process of derivation. However, it will turn out that our final Eq. (28) holds whether Eq. (64) is true or not.

Step (ii):
Logarithmically differentiating Eqs. (17) and (18) gives:

$$\widehat{a}_{12} - \widehat{a}_{22} = -\widehat{v} - [\sigma/(\sigma-1)]\widehat{\tau}_{12},$$
$$\widehat{a}_{21} - \widehat{a}_{11} = \widehat{v} - [\sigma/(\sigma-1)]\widehat{\tau}_{21}.$$

Substituting them into Eq. (62), we obtain:

$$(1-\lambda_{12})\widehat{a}_{11} + \lambda_{12}\widehat{a}_{22} = \lambda_{12}\{\widehat{v} + [\sigma/(\sigma-1)]\widehat{\tau}_{12}\},$$
$$\lambda_{21}\widehat{a}_{11} + (1-\lambda_{21})\widehat{a}_{22} = \lambda_{21}\{-\widehat{v} + [\sigma/(\sigma-1)]\widehat{\tau}_{21}\}.$$

Substituting Eq. (63) into $\widehat{v} = [\sigma/(\sigma-1)](\widehat{p}_1^Y - \widehat{p}_2^Y)$, and substituting it into the above expressions, they are rewritten as:

$$\widetilde{\lambda}_{11}\widehat{a}_{11} + \widetilde{\lambda}_{12}\widehat{a}_{22} = \lambda_{12}\{\widehat{V} + [\sigma/(\sigma-1) - (\sigma/\Delta)\zeta_{12}]\widehat{\tau}_{12} + (\sigma/\Delta)\zeta_{21}\widehat{\tau}_{21}\},$$
$$\widetilde{\lambda}_{21}\widehat{a}_{11} + \widetilde{\lambda}_{22}\widehat{a}_{22} = \lambda_{21}\{-\widehat{V} + [\sigma/(\sigma-1) - (\sigma/\Delta)\zeta_{21}]\widehat{\tau}_{21} + (\sigma/\Delta)\zeta_{12}\widehat{\tau}_{12}\};$$
$$\widehat{V} \equiv [\sigma/(\sigma-1)](1/\Delta)[\widehat{w}_1 - (1-\zeta_{21}-\zeta_{12})\widehat{\chi}],$$
$$\widetilde{\lambda}_{jj} \equiv 1 - \lambda_{jk} + \lambda_{jk}(\sigma/\Delta)(\beta-1)\xi_j, \widetilde{\lambda}_{jk} \equiv \lambda_{jk} - \lambda_{jk}(\sigma/\Delta)(\beta-1)\xi_k, k \neq j.$$

They are solved for $\widehat{a}_{11}$ and $\widehat{a}_{22}$ as:

$$\widehat{a}_{11} = (\lambda_{12}/|\widetilde{\lambda}|)\{\widehat{V} + \{[\sigma/(\sigma-1)]\widetilde{\lambda}_{22} - (\sigma/\Delta)\zeta_{12}\}\widehat{\tau}_{12} - \{\sigma/(\sigma-1) - (\sigma/\Delta)\zeta_{21} - [\sigma/(\sigma-1)]\widetilde{\lambda}_{22}\}\widehat{\tau}_{21}\}, \quad (65)$$
$$\widehat{a}_{22} = (\lambda_{21}/|\widetilde{\lambda}|)\{-\widehat{V} + \{[\sigma/(\sigma-1)]\widetilde{\lambda}_{11} - (\sigma/\Delta)\zeta_{21}\}\widehat{\tau}_{21} - \{\sigma/(\sigma-1) - (\sigma/\Delta)\zeta_{12} - [\sigma/(\sigma-1)]\widetilde{\lambda}_{11}\}\widehat{\tau}_{12}\}; \quad (66)$$
$$|\widetilde{\lambda}| \equiv \widetilde{\lambda}_{11}\widetilde{\lambda}_{22} - \widetilde{\lambda}_{12}\widetilde{\lambda}_{21}$$
$$= (1/\Delta)\{(1-\zeta_{21}-\zeta_{12})[1-\lambda_{12}-\lambda_{21}+\sigma(\beta-1)(\lambda_{12}+\lambda_{21})] + \sigma(\zeta_{21}+\zeta_{12})(\beta-\lambda_{12}-\lambda_{21})\} > 0.$$

Finally, $\widehat{a}_{12}$ and $\widehat{a}_{21}$ are obtained by substituting Eqs. (65) and (66) back into Eq. (62).

Step (iii):

Logarithmically differentiating Eq. (21), and using Eq. (62) and $\widehat{\lambda}_{jk} = \theta\widehat{a}_{jk}$ from Eq. (22), give:

$$-\theta\{1/[1+(\tau_{21}-1)\lambda_{12}]\}[(1-\lambda_{12})/\lambda_{12}]\widehat{a}_{11} - \zeta_{21}\widehat{\tau}_{21} + \widehat{w}_1 = -\theta\{1/[1+(\tau_{12}-1)\lambda_{21}]\}[(1-\lambda_{21})/\lambda_{21}]\widehat{a}_{22} - \zeta_{12}\widehat{\tau}_{12}.$$

Substituting Eqs. (65) and (66) into the above expression, and using Eq. (31): $\zeta_{kj} = \tau_{kj}\lambda_{jk}/[1+(\tau_{kj}-1)\lambda_{jk}] \Leftrightarrow 1-\zeta_{kj} = (1-\lambda_{jk})/[1+(\tau_{kj}-1)\lambda_{jk}], k \neq j$, we obtain:

$$\begin{aligned}
0 &= -B\widehat{w}_1 + C\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12} \Leftrightarrow \widehat{w}_1 = (1/B)(C\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}); \qquad (67)\\
B &\equiv \beta(\sigma/\Delta)(2-\zeta_{21}-\zeta_{12}) - |\widetilde{\lambda}|\\
&= (1/\Delta)\{(1-\zeta_{21}-\zeta_{12})[1+(\beta\sigma-1)(2-\lambda_{12}-\lambda_{21})] + \sigma(\lambda_{12}+\lambda_{21})\} > 0,\\
C &\equiv \beta(\sigma/\Delta)(1-\zeta_{21}-\zeta_{12})(2-\zeta_{21}-\zeta_{12}) > 0,\\
F_{jk} &\equiv \theta\{(1-\zeta_{kj})\{[\sigma/(\sigma-1)]\widetilde{\lambda}_{kk} - (\sigma/\Delta)\zeta_{jk}\}\\
&\quad + (1-\zeta_{jk})\{\sigma/(\sigma-1) - (\sigma/\Delta)\zeta_{jk} - [\sigma/(\sigma-1)]\widetilde{\lambda}_{jj}\}\} - |\widetilde{\lambda}|\zeta_{jk}, k \neq j.
\end{aligned}$$

Step (iv):

Substituting Eq. (67) into $\widehat{V} = [\sigma/(\sigma-1)](1/\Delta)[\widehat{w}_1 - (1-\zeta_{21}-\zeta_{12})\widehat{\chi}]$ gives:

$$\widehat{V} = [1/(\sigma-1)][(\sigma/\Delta)/B][|\widetilde{\lambda}|(1-\zeta_{21}-\zeta_{12})\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}].$$

Substituting this back into Eqs. (65) and (66), noting that $\widetilde{\lambda}_{11} + \widetilde{\lambda}_{22} - 1 = |\widetilde{\lambda}|$ and $\Delta - \sigma(\zeta_{21}+\zeta_{12}) = 1-\zeta_{21}-\zeta_{12}$, and substituting the results into Eq. (26), we obtain:

$$\begin{aligned}
d\gamma_1 &= -(\rho+\delta+\gamma)[(\sigma/\Delta)/B]\lambda_{12}[(1-\zeta_{21}-\zeta_{12})\widehat{\chi} + J_1\widehat{\tau}_{12} + I_1\widehat{\tau}_{21}], \qquad (68)\\
d\gamma_2 &= -(\rho+\delta+\gamma)[(\sigma/\Delta)/B]\lambda_{21}[-(1-\zeta_{21}-\zeta_{12})\widehat{\chi} + J_2\widehat{\tau}_{21} + I_2\widehat{\tau}_{12}]; \qquad (69)\\
J_j &\equiv (\beta\sigma-1)(1-\lambda_{kj})(1-\zeta_{kj}-\zeta_{jk}) + \sigma\lambda_{kj} > 0,\\
I_j &\equiv J_j + 1 - \zeta_{kj} - \zeta_{jk} > J_j, k \neq j.
\end{aligned}$$

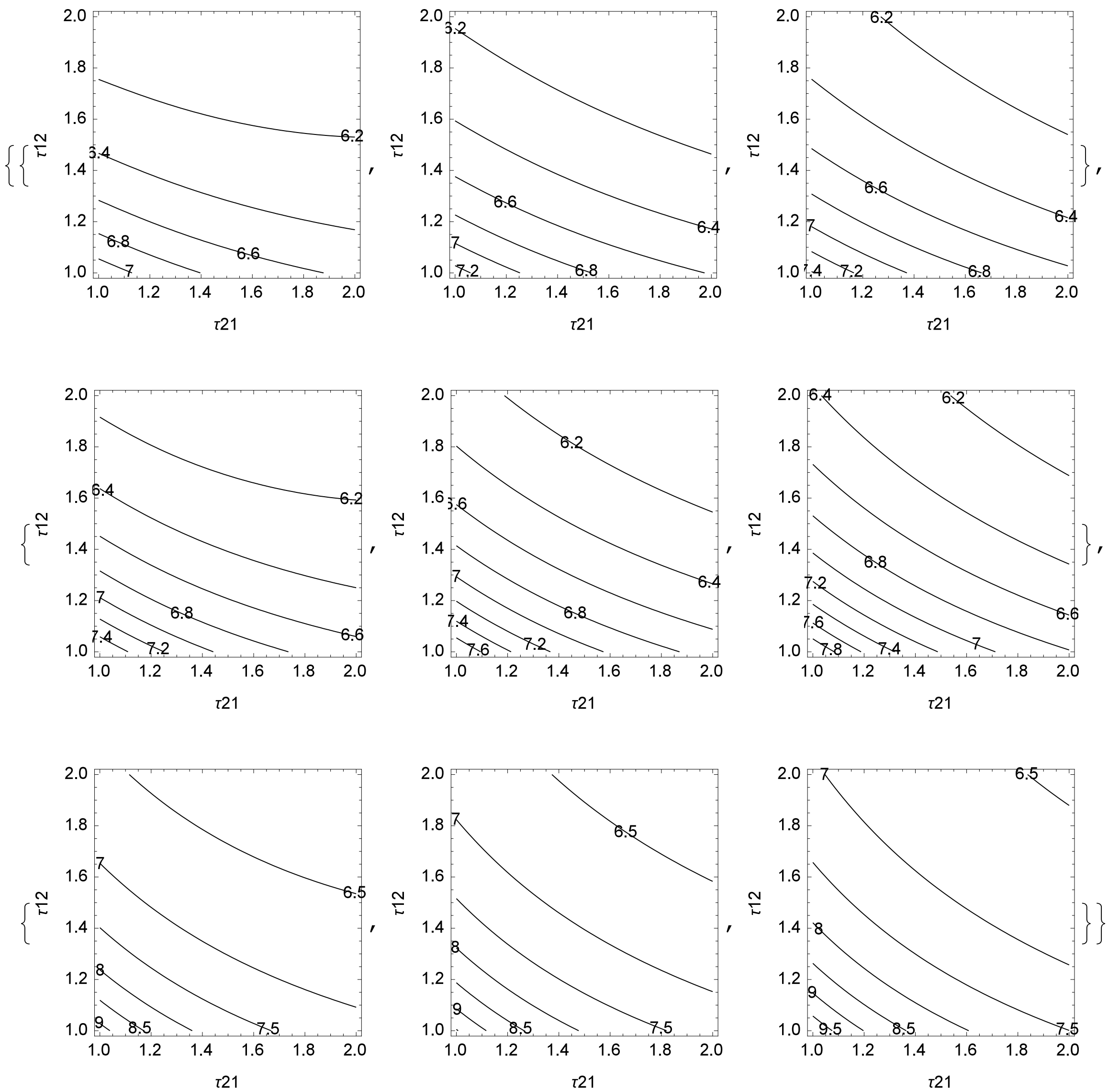
Step (v):

Substituting Eqs. (68) and (69) into the differentiated form of Eq. (23), $\widehat{\chi}$ is solved as:

$$\widehat{\chi} = [1/(1-\zeta_{21}-\zeta_{12})][1/(\lambda_{12}+\lambda_{21})][(\lambda_{21}I_2 - \lambda_{12}J_1)\widehat{\tau}_{12} - (\lambda_{12}I_1 - \lambda_{21}J_2)\widehat{\tau}_{21}]. \qquad (70)$$

Step (vi):

Substituting Eq. (70) back into Eq. (69), and noting that $I_j + J_k = \Delta B \forall j,k,k \neq j$, we obtain Eq. (28), which is independent of Eq. (64).

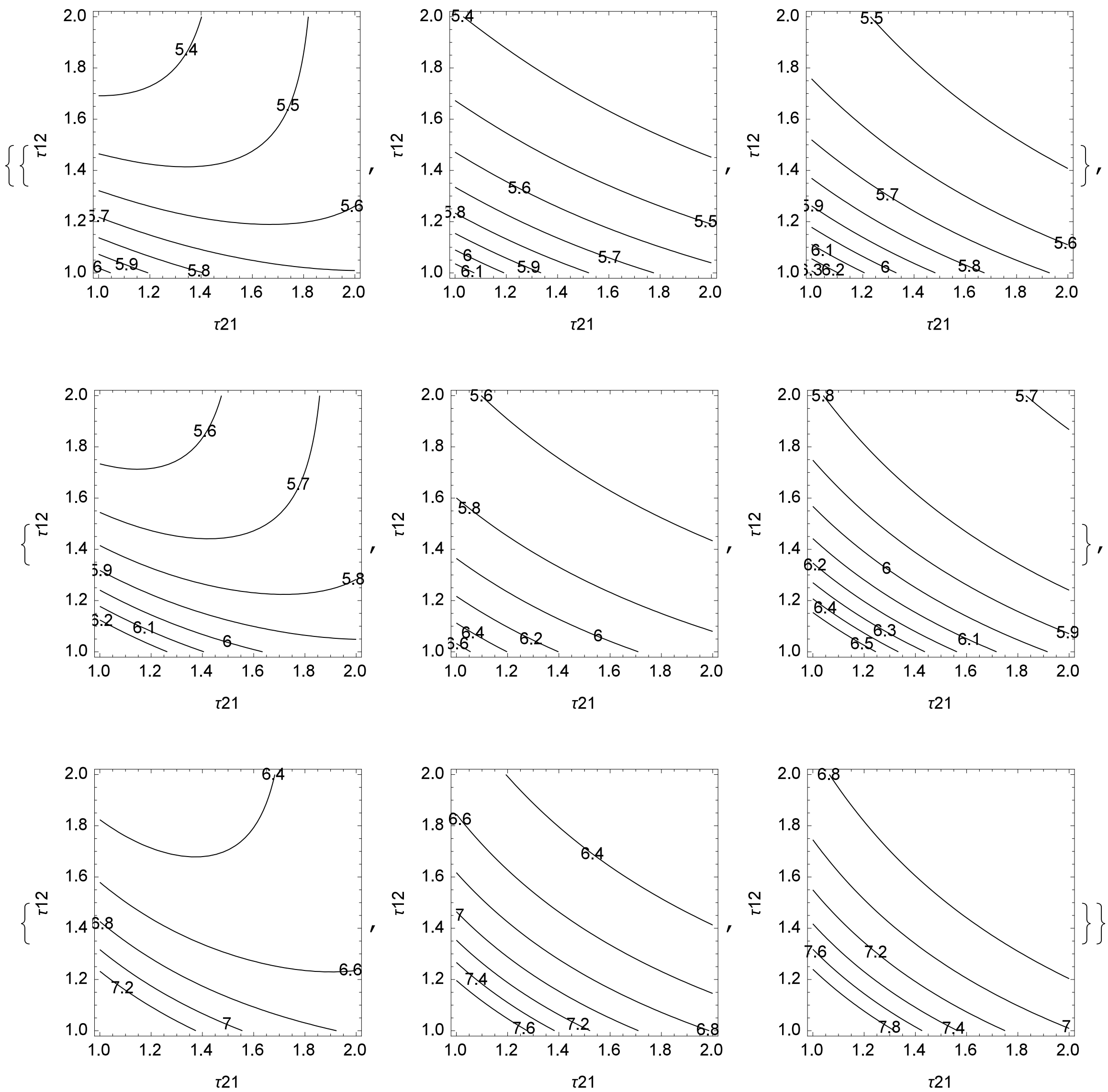


top left: $L_1 = 0.9, \rho = 0.03$ top center: $L_1 = 1.0, \rho = 0.03$ top right: $L_1 = 1.1, \rho = 0.03$

mid left: $L_1 = 0.9, \rho = 0.02$ mid center: $L_1 = 1.0, \rho = 0.02$ mid right: $L_1 = 1.1, \rho = 0.02$

bot left: $L_1 = 0.9, \rho = 0.01$ bot center: $L_1 = 1.0, \rho = 0.01$ bot right: $L_1 = 1.1, \rho = 0.01$

Fig. 1. Country 1's iso-welfare curves on the (τ_{21}, τ_{12}) plane: homogeneous firms



top left: $L_1 = 0.9, \rho = 0.03$	top center: $L_1 = 1.0, \rho = 0.03$	top right: $L_1 = 1.1, \rho = 0.03$
mid left: $L_1 = 0.9, \rho = 0.02$	mid center: $L_1 = 1.0, \rho = 0.02$	mid right: $L_1 = 1.1, \rho = 0.02$
bot left: $L_1 = 0.9, \rho = 0.01$	bot center: $L_1 = 1.0, \rho = 0.01$	bot right: $L_1 = 1.1, \rho = 0.01$

Fig. 2. Country 1's iso-welfare curves on the (τ_{21}, τ_{12}) plane: heterogeneous firms