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Debt intolerance: Threshold level and composition

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Abstract

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Debt Intolerance: Threshold Level and Composition*

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Abstract

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Key Words: *Generalized panel smooth transition regression, Expected public debt-to-GDP ratio, Foreign private investors, Long-term interest rate*

JEL Codes: E43,E62,H63

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"The experiences of foreign economies suggest that the relationship between debt and interest rates is complex and likely non-linear, with the influence of greater debt on interest rates rising as the debt-to-GDP ratio reaches a trajectory at which investors have concerns about its sustainability."

-Brainard (2017)

"The level of debt that is sustainable in an economy is not a constant. It can change over time and indeed has changed enormously over the last 150 years. The ratio of credit to GDP in the late Victorian British economy was under 20 percent. In the mid-twentieth century it was around 60 percent and by the early 1990s over 100 percent."

-Cunliffe (2019)

1. Introduction

As argued by [Reinhart et al. \(2003\)](#), fiscal vulnerabilities depend on both the level and composition (foreign vs. domestic) of government debt. They describe the “debt intolerance” phenomenon, in which interest rates in developing economies can spike above the “tolerance ceiling,” even if debt levels are considered manageable by advanced country standards. Long-term interest rates in advanced economies have been lower than those in emerging markets although much higher debt levels have been seen in advanced economies such as Japan, the United Kingdom and the United States (Figures 1 and 2). While research has estimated the marginal impact of public debt on long-term interest rates, the results vary widely, even when the studies control for fundamental variables such as inflation and growth expectations.¹

Meanwhile, the composition for government bond holdings varies widely across countries. The share of foreign private investors in emerging markets has been larger than in advanced economies since the global financial crisis (Figure 3).² Also, major central banks in advanced economies have been important participants in government bond markets, purchasing government bonds financed by central bank reserves through quantitative easing. This paper examines the implications of the level and composition (foreign or domestic, official or private) of government bond holdings for long-term yields.

¹Please see Section 3.4 for details.

²According to [Arslanalp and Tsuda \(2014a\)](#), the average local currency share of external public debt in 14 emerging markets increased from 24 percent in 2009Q4 to 42 percent in 2018Q2. To understand these recent dynamics, [Ottonello and Perez \(2019\)](#) develop a model of optimal choice of debt denominated in foreign and local currency and conclude that prolonged economic expansion and stabilization of inflation during this period can account for most of the observed changes in the currency composition of sovereign debt. By contrast, the temptation to reduce debt repayments in local currency through inflation is high during recessions since the marginal benefit of saving resources for consumption is high. To mitigate higher incentives to dilute debt, the government optimally chooses to tilt its currency composition towards foreign currency.

Some studies maintain that foreign investors require higher risk premia than domestic ones when a government is likely to repay domestic investors first.³ [Azzimonti and Quadrini \(2017\)](#), investigating the role of portfolio diversification on optimal default of sovereign debt, show that financial market integration increases the incentives of a given country to default. This is not only attributed to partial ownership of the defaulted debt by foreigners, but also because the macroeconomic cost of a default is smaller when the defaulting country is financially integrated. [Gros \(2013\)](#), [Ichiue and Shimizu \(2015\)](#) and [Tenreyro \(2019\)](#) also argue that if domestic financial institutions have a large share of government bonds, the losses they might incur would be large. To avoid this, governments would have a clear incentive to undertake fiscal consolidation rather than undergo default.

[Dell’Erba et al. \(2013\)](#) and [Ichiue and Shimizu \(2015\)](#) document that when an increase in debt is financed through foreign borrowing, the increase in interest rates is greater than under domestically-financed debt.⁴ Similarly, [Agca and Celasun \(2012\)](#) show that the impact of domestic public debt on syndicated loan yield spreads is not statistically significant, whereas the analogous impact for external public debt is significant. [Brzoza-Brzezina and Kotlowski \(2020\)](#), using the panel smooth transition regression (PSTR) model developed by [González et al. \(2017\)](#), show that risk premia increase slowly with deteriorating net financial asset positions, and then suddenly jump once a threshold is reached.

Meanwhile, [Reinhart and Trebesch \(2015\)](#) argue that an increase in the share of foreign investors is associated with lower long-term interest rates because these holdings supplement domestic saving in capital-scarce countries, especially in times of ample global liquidity. [Peiris \(2013\)](#) documents that while domestic investors are typically buy-and-hold investors, foreign investors are more likely to trade. Some empirical studies support

³This argument is based on the assumption that a government can default selectively on foreign investors. [Erce and Mallucci \(2018\)](#) find that selective defaults have been frequent over the period 1980-2015. Since 1980, about two-thirds of defaults have selectively involved either foreign-law bonds or domestic-law bonds. By contrast, [Sturzenegger and Zettelmeyer \(2008\)](#) conclude that with a few exceptions, domestic investors do not appear to have been treated systematically better than foreign investors between 1998 and 2005. [Broner et al. \(2014\)](#) argue that selective default can happen when the government can impose capital controls that make discrimination easier. Meanwhile, [Broner et al. \(2010\)](#) show that secondary markets both reduce the probability of default on foreigners and make it difficult for the government to discriminate among creditors.

⁴[Asonuma et al. \(2015\)](#) also show that higher home bias mitigates the upward pressure of higher public debt on bond spreads, relying on three home bias indicators: (1) Banks’ holding of domestic sovereign claims relative to banks’ total assets, (2) Banks’ holding of domestic sovereign claims relative to banks’ holding of sovereign claims and (3) Banks’ holding of domestic sovereign claims relative to public debt. Their result is line with the fiscal crisis model of [Sakuragawa and Sakuragawa \(2016\)](#), which explains why domestic investors do not require a risk premium against large outstanding debt when there is a strong home bias in the asset portfolio of domestic bondholders. The reason is that these investors turn out to have no access to any assets that hedge fiscal risk. [Ongena et al. \(2019\)](#) find that during the acute phase of the European sovereign debt crisis when the government had to roll over substantial sovereign debt, domestic banks that were smaller, less well capitalized, had a lower ratio of liquid assets, and had received government support in the past were more likely to purchase domestically-issued sovereign debt than foreign banks.

this downward effect ([Arslanalp and Poghosyan \(2016\)](#), [Carvalho and Fidora \(2015\)](#), [Ebeke and Lu \(2015\)](#) and [Warnock and Warnock \(2009\)](#)). [Azzimonti et al. \(2014\)](#) illustrate that small countries facing a larger world market relative to their own economies perceive the interest rate as less sensitive to their own debt, as in a globally integrated economy both the demand and supply of the government debt come not only from domestic investors and the government but also from foreign sources.

Although a large research program documents both positive and negative impacts of an increase in the share of foreign debt on government bond yields, only a few studies have reconciled these mixed impact estimates by investigating thresholds. Our work is closely related to [Ebeke and Lu \(2015\)](#) and [Brzoza-Brzezina and Kotlowski \(2020\)](#). [Ebeke and Lu \(2015\)](#) show that in emerging markets an increase in the share of foreign debt holdings has a negative impact on yields but if either the lagged external debt-to-GDP ratio exceeds 90 percent or the lagged short-term debt-to-GDP ratio exceeds 21.5 percent, the corresponding impacts turn positive. [Brzoza-Brzezina and Kotlowski \(2020\)](#) find that the country's risk premium gets significantly larger when the net financial assets position is below about 75 percent of GDP.

This paper contributes to two strands of the literature, extending and complementing [Ebeke and Lu \(2015\)](#) and [Brzoza-Brzezina and Kotlowski \(2020\)](#). First, it makes a methodological contribution towards the generalized panel smooth transition regression (GPSTR) model by combining two approaches: the panel smooth transition regression (PSTR) and the general logistic model (GLM). [Ebeke and Lu \(2015\)](#) and [Brzoza-Brzezina and Kotlowski \(2020\)](#) employ the interaction-term (IT) and PSTR models, respectively.⁵ The biggest difference between the GPSTR model and previous methods is the GPSTR model can allow for potential point asymmetries whereas the IT and PSTR models exhibit point symmetry. If the impact of an increase in expected public debt on the interest rate is highly non-linear, the novel model used here can improve the estimation. Here, the performance of the GPSTR model is evaluated by comparing the out-of-sample forecast error with those of other methods employed in previous studies.

Second, this paper simultaneously estimates two types of threshold values to address the following questions:

⁵The motivation of [Brzoza-Brzezina and Kotlowski \(2020\)](#) in employing the PSTR model is that the dynamic stochastic general equilibrium (DSGE) literature such as [Schmitt-Grohe and Uribe \(2003\)](#), [Adolfson et al. \(2007\)](#) and [García-Cicco et al. \(2010\)](#) that assumes a nonlinear (e.g., exponential) relationship between risk premia and foreign debt. As these DSGE studies do not assume that the government has public debt, the country risk premium used in these studies does not necessarily represent the sovereign risk premium. However, since [Brzoza-Brzezina and Kotlowski \(2020\)](#) use the PSTR model and the difference between a country's long-term and the US interest rate as a proxy for the country risk premium, their study is the most closely related to this paper.

- (i) What is the threshold of the share of foreign private investor financing that triggers an increase in the marginal impact of higher expected public debt on government bond yields?
- (ii) What is the threshold of public debt that triggers an increase in the marginal impact of a higher share of foreign private investor financing on government bond yields?

[Ebeke and Lu \(2015\)](#) and [Brzoza-Brzezina and Kotlowski \(2020\)](#) investigate only one type of threshold, whereas this paper investigates two types of threshold values simultaneously.⁶

The remainder of this paper is organized as follows. Section 2 examines non-linear models: the IT, PSTR and GPSTR to explore threshold values, based on the data for 11 advanced economies and 14 emerging markets. Section 3 presents the baseline estimation results, the model evaluation, and comparison with other studies. Section 4 concludes.

2. Empirical strategy and data

Local currency bonds can be held by both domestic and foreign investors, whereas foreign currency bonds are assumed to be held by only foreign investors. To estimate the threshold of the composition for government bond holdings (foreign vs. domestic), we employ local currency bond yields because they reflect the behavior of both domestic and foreign investors.⁷ We examine three models, namely, the (i) Interaction term model (IT), (ii) Panel Smooth Transition Regression (PSTR), and (iii) Generalized Panel Smooth Transition Regression (GPSTR) models.

The (i) IT model is a simple specification with an interaction term between the share of foreign private investors and the expected public debt level. The (ii) PSTR model, developed by [González et al. \(2017\)](#) and [Fok et al. \(2005\)](#), includes a standard logistic function that exhibits point symmetry with one slope parameter and one location parameter. [Brzoza-Brzezina and Kotlowski \(2020\)](#) is relevant to our empirical strategy because they employ the PSTR model to examine a nonlinear relationship between a country's risk premium and net

⁶These two previous studies use actual debt data, whereas this paper uses a wide range of forecast data to exclude any effects from current economic conditions on interest rates as well as to consider the forward-looking behavior of financial markets.

⁷[Ebeke and Lu \(2015\)](#) and [Brzoza-Brzezina and Kotlowski \(2020\)](#), who also use local currency bonds in emerging markets, control for exchange rate risk whereas [Moore et al. \(2013\)](#) controls for it indirectly by estimating the share of foreign investors that is affected by exchange rate risk. We include exchange rate risk in the baseline estimation but exclude it as a robustness check. [Amstad et al. \(2020\)](#), using the average credit ratings provided by Standard and Poor's, Moody's and Fitch find a slow and steady convergence of sovereign risk in local and foreign currency due to higher availability of foreign currency via FX reserves and smaller dependence on foreign currency borrowing (decline of original sin) over the past 20 years.

financial assets. To grasp the potential point asymmetry, this paper develops the (iii) GPSTR model by replacing the standard logistic function employed by [González et al. \(2017\)](#) with GLM, as suggested by [Stukel \(1988\)](#).

2.1 Interaction term (IT)

We examine how the interaction term between the share of foreign private investors and the expected public debt level affects the interest rate as follows:

$$\mathbb{E}_t L_{i,t+n} = \alpha_i + \beta_0 \mathbb{E}_t Debt_{i,t+n} + \beta_1 \mathbb{E}_t Debt_{i,t+n} \cdot f p_{i,t-1} + \delta f p_{i,t-1} + \phi \mathbf{z}_{i,t} + \varepsilon_{i,t} \quad (1)$$

where a country $i = 1, \dots, N$ at a time $t = 1, \dots, T$, $\mathbb{E}_t L_{i,t+n}$ is the n -year-ahead forward long-term interest rate, $\mathbb{E}_t Debt_{i,t+n}$ is the n -year-ahead projection for public debt (percent of GDP), $f p_{i,t-1}$ is lagged share of private foreign investors and $\mathbf{z}_{i,t}$ describes control variables.⁸ We analyze the effect of forward-looking projections of public debt on forward interest rates to exclude any effects of current economic conditions. As discussed by [Engen and Hubbard \(2005\)](#), [Ichiue and Shimizu \(2015\)](#) and [Laubach \(2009\)](#), when a deterioration of fiscal conditions and a decrease in interest rates due to expected accommodative monetary policy occur simultaneously during a recession, the upward effect of fiscal deterioration on long-term interest rates could be underestimated even if we control for other variables. In addition, the government bond yield is influenced by forward-looking variables. β_1 is expected to be larger than β_0 : domestic investors may not require a risk premium against large debt when there is strong home bias without any other assets hedging fiscal risk. δ is expected to be negative: while domestic investors are typically buy-and-hold investors, foreign investors are more likely to trade. This could be supportive of increased market liquidity.

2.2 PSTR and GPSTR

The PSTR model, developed by [González et al. \(2017\)](#) and [Fok et al. \(2005\)](#) can allow for a continuum of regimes, each one being characterized by a different value of the transition variable. Replacing $\mathbb{E}_t Debt_{i,t+n} f p_{i,t-1}$ in the IT model (1) with $\mathbb{E}_t Debt_{i,t+n} g(f p_{i,t-1}; \gamma_1, c)$ gives the PSTR model:

⁸We take a lagged value of foreign investors to avoid a simultaneity bias because the interest rate affects their behaviors. We discuss this endogeneity in Section 3.2.

$$\mathbb{E}_t L_{i,t+n} = \alpha_i + \beta_0 \mathbb{E}_t Debt_{i,t+n} + \beta_1 \mathbb{E}_t Debt_{i,t+n} \cdot g(fp_{i,t-1}; \gamma_1, c) + \delta fp_{i,t-1} + \phi \mathbf{z}_{i,t} + \varepsilon_{i,t} \quad (2)$$

where $g(fp_{i,t-1}; \gamma_1, c)$ is the standard logistic function ($\frac{1}{1+\exp(-\gamma_1(fp_{i,t-1}-c))}$, $\gamma_1 > 0$) that depends on the threshold variable $fp_{i,t-1}$, the slope parameters r_1 and location parameters c .⁹ This standard logistic function $g(fp_{i,t-1}; \gamma_1, c)$ exhibits point symmetry with one slope parameter r_1 . In other words, regardless of whether the threshold variable is larger or smaller than the location c , the impact of public debt across all regimes could be affected by one slope parameter r_1 . Figure 4 shows examples for the standard logistic function as well as the interaction term that exhibit point symmetry. To investigate potential point asymmetry, we introduce two slope parameters (γ_1 and γ_2), replacing the standard logistic function with the general logistic model suggested by [Stukel \(1988\)](#).

The generalized panel smooth transition regression (GPSTR) is as follows:

$$\mathbb{E}_t L_{i,t+n} = \alpha_i + \beta_0 \mathbb{E}_t Debt_{i,t+n} + \beta_1 \mathbb{E}_t Debt_{i,t+n} \cdot g(fp_{i,t-1}; \gamma_1, \gamma_2, c) + \delta fp_{i,t-1} + \phi \mathbf{z}_{i,t} + \varepsilon_{i,t} \quad (3)$$

Following [Stukel \(1988\)](#), the general logistic model (GLM), $g(fp_{i,t-1}; \gamma_1, \gamma_2, c)$ is defined as follows:

$$g(fp_{i,t-1}; \gamma_1, \gamma_2, c) = \frac{\exp(h(fp_{i,t-1}; \gamma_1, \gamma_2, c))}{1 + \exp(h(fp_{i,t-1}; \gamma_1, \gamma_2, c))} \quad (4)$$

For $fp_{i,t-1} - c \leq 0$,

$$h(fp_{i,t-1}; \gamma_1, c) = \begin{cases} \gamma_1^{-1}(\log(1 - \gamma_1 |fp_{i,t-1} - c|)) & \gamma_1 < 0 \\ fp_{i,t-1} - c & \gamma_1 = 0 \\ -\gamma_1^{-1}(\exp(\gamma_1 |fp_{i,t-1} - c|) - 1) & \gamma_1 > 0 \end{cases} \quad (5)$$

For $fp_{i,t-1} - c \geq 0$,

⁹When $r_1 \rightarrow \infty$, the PSTR model reduces to the two-regime panel threshold model by [Hansen \(1999\)](#).

$$h(fp_{i,t-1}; \gamma_2, c) = \begin{cases} -\gamma_2^{-1}(\log(1 - \gamma_2|fp_{i,t-1} - c|)) & \gamma_2 < 0 \\ fp_{i,t-1} - c & \gamma_2 = 0 \\ \gamma_2^{-1}(\exp(\gamma_2|fp_{i,t-1} - c|) - 1) & \gamma_2 > 0 \end{cases} \quad (6)$$

The transition functions $g(fp_{i,t-1}; \gamma_1, \gamma_2, c)$ depend on the lagged share of private foreign investors $fp_{i,t-1}$, the slope parameters r_1, r_2 and location parameters c . Figure 5 shows examples for the GLM $g(fp_{i,t-1}; \gamma_1, \gamma_2, c)$ that exhibit point asymmetry. The estimation of the parameters of the GPSTR model eliminates individual effects α_i by removing individual-specific means and then applying non-linear least squares to the transformed model. The matrix of transformed explanatory variables is

$$x^*(\gamma_1, \gamma_2, c) = \left[\mathbb{E}_t \overline{Debt}_{i,t+n} : \overline{G}_{i,t+n} : \overline{fp}_{i,t-1} : \overline{\mathbf{z}}_{i,t} \right]' \quad (7)$$

where $\overline{\mathbb{E}_t L_{i,t+n}} = \mathbb{E}_t L_{i,t+n} - \overline{L}_i$, $\mathbb{E}_t \overline{Debt}_{i,t+n} = \mathbb{E}_t Debt_{i,t+n} - \overline{Debt}_i$, $\overline{fp}_{i,t-1} = fp_{i,t-1} - \overline{fp}_i$ and $\overline{\mathbf{z}}_{i,t} = \mathbf{z}_{i,t} - \overline{\mathbf{z}}_i$. $\overline{G}_{i,t+n}(\gamma_1, \gamma_2, c) = \mathbb{E}_t \overline{Debt}_{i,t+n} \cdot g(fp_{i,t-1}; \gamma_1, \gamma_2, c) - \frac{1}{T} \sum_{t=1}^T \mathbb{E}_t Debt_{i,t+n} \cdot g(fp_{i,t-1}; \gamma_1, \gamma_2, c)$ is the transformed explanatory variables in the second regime that depends on the parameters and the transition function. Given a couple (γ_1, γ_2, c) , the parameters can be estimated by ordinary least squares, which yields:

$$\widehat{\Psi}(\gamma_1, \gamma_2, c) = \left[\sum_{i=1}^N \sum_{t=1}^T x_{i,t}^*(\gamma_1, \gamma_2, c) x_{i,t}^{*'}(\gamma_1, \gamma_2, c) \right]^{-1} \left[\sum_{i=1}^N \sum_{t=1}^T x_{i,t}^*(\gamma_1, \gamma_2, c) \overline{\mathbb{E}_t L_{i,t+n}} \right] \quad (8)$$

where $\widehat{\Psi}(\gamma_1, \gamma_2, c)$ is conditional to the values (γ_1, γ_2, c) . For the next step, by increasing the number of combinations for (γ_1, γ_2, c) , the parameters of the transition function γ_1, γ_2 and c are estimated by non-linear least squares :

$$(\hat{\gamma}_1, \hat{\gamma}_2, \hat{c}) = \underset{\{\gamma_1, \gamma_2, c\}}{\text{ArgMin}} \sum_{i=1}^N \sum_{t=1}^T \left[\overline{\mathbb{E}_t L_{i,t+n}} - \widehat{\Psi}'(\gamma_1, \gamma_2, c) x^*(\gamma_1, \gamma_2, c) \right] \quad (9)$$

Consequently, $(\hat{\beta}_0 : \hat{\beta}_1 : \hat{\delta} : \hat{\phi})' = \widehat{\Psi}(\hat{\gamma}_1, \hat{\gamma}_2, \hat{c})$.

The practical computation follows two steps.

- Step1. The initial values can be obtained by starting a grid search across the parameters γ_1, γ_2 and c where grid points are $n_{\gamma_1}=21, n_{\gamma_2}=21$ and $n_c=30$. Hence, the number of regressions is 13,230.

- Step2. We employ the Nelder–Mead simplex algorithm to find a local minimizer of the function non-linear least squares, using the initial values.¹⁰

2.3 Threshold level and composition

Using estimated parameters and solving $\frac{\partial \mathbb{E}_t L_{i,t+n}}{\partial \mathbb{E}_t Debt_{i,t+n}}=0$ and $\frac{\partial \mathbb{E}_t L_{i,t+n}}{\partial f p_{i,t-1}}=0$ can address the following two questions:

- (i) What is the threshold of foreign private investor financing share that triggers an increase in the marginal impact of higher expected public debt on government bond yields?
- (ii) What is the threshold of public debt level that triggers an increase in the marginal impact of higher share of foreign private investor financing on government bond yields?

Table 1 shows the threshold values that affect the marginal impact of expected public debt ($\mathbb{E}_t Debt_{i,t+n}$) and the share of foreign private investors ($f p_{i,t-1}$) on interest rates.

The first column shows the share of foreign private investors $f p_{i,t-1}$ that triggers a surge in government bond yields through an increase in the expected public debt. Such threshold values depend on the ratio of the first regime parameter of expected public debt (β_0) to the second regime parameter of expected public debt (β_1 , $\beta_1(f p_{i,t-1}; \gamma_1, c)$ or $\beta_1(f p_{i,t-1}; \gamma_1, \gamma_2, c)$).

The second column shows the expected public debt level $\mathbb{E}_t Debt_{i,t+n}$ that triggers a surge in government bond yields through an increase in the share of foreign investors.¹¹ The threshold of expected public debt becomes smaller if investors are more sensitive to the expected public debt. As a result, an increase in foreign private holdings of government debt would cause an increase in long-term interest rates, offsetting the downward effect on long-term interest rates from expanding market liquidity, even if foreign investors can support higher market liquidity.

¹⁰This algorithm uses a simplex of $n + 1$ points for n -dimensional vectors and discards the current worst point to reduce the difference between the current best point and other points in simplex at each step in the iteration. See Lagarias et al. (1998) and Miranda and Fackler (2002) for details. Computational codes are based on Colletaz and Hurlin (2006) and Fouquau et al. (2008).

¹¹See Appendix A.2 for the derivatives of the transition function $g(f p_{i,t-1}; \gamma_1, \gamma_2, c)$.

Table 1: Estimated threshold values of the impact of the increase in the share of foreign private investors ($fp_{i,t-1}$) and the expected public debt ($\mathbb{E}_t Debt_{i,t+n}$) on the interest rate

	Threshold composition	Threshold level
(1) IT	$fp_{i,t-1} = \frac{-\beta_0}{\beta_1}$	$\mathbb{E}_t Debt_{i,t+n} = \frac{-\delta}{\beta_1}$
(2) PSTR	$fp_{i,t-1} = \frac{-\beta_0}{\beta_1(fp_{i,t-1}; \gamma_1, c)}$	$\mathbb{E}_t Debt_{i,t+n} = \frac{-\delta}{\beta_1 \left\{ \frac{\partial g(fp_{i,t-1}; \gamma_1, c)}{\partial fp_{i,t-1}} \right\}}$
(3) GPSTR	$fp_{i,t-1} = \frac{-\beta_0}{\beta_1(fp_{i,t-1}; \gamma_1, \gamma_2, c)}$	$\mathbb{E}_t Debt_{i,t+n} = \frac{-\delta}{\beta_1 \left\{ \frac{\partial g(fp_{i,t-1}; \gamma_1, \gamma_2, c)}{\partial fp_{i,t-1}} \right\}}$

2.4 Data

2.4.1 Data coverage

Based on forecast data availability, we use bi-annual data of 25 countries: 11 advanced economies and 14 emerging markets during 2006H2-2018H2. Advanced economies include Australia, Canada, the Czech Republic, Denmark, Japan, the Republic of Korea, Norway, Sweden, Switzerland, the United Kingdom, and the United States. Emerging markets include Bulgaria, Colombia, China, Hungary, India, Indonesia, Malaysia, Mexico, Peru, the Philippines, Poland, Thailand, Turkey, and South Africa. The Euro area countries are omitted from advanced economies for the following reasons. As discussed by [Ichiue and Shimizu \(2015\)](#) and [Wright \(2011\)](#), this is because their term structures had been highly correlated with those of Germany since the introduction of the euro. Therefore, it would be difficult to identify how much long-term interest rates in the Euro area have reflected the fundamentals of these countries. In this regard, the share of foreign private investors in the euro area has been higher relative to others ([Arslanalp and Tsuda \(2014a\)](#)). In the euro area, the zero percent risk weight has been a major driver of banks' holdings of foreign debt securities denominated in the local currency due to Basel capital regulation. [Bonner \(2016\)](#) shows that capital regulation encourages banks to substitute other bonds with government bonds. Because of this regulatory impact, the impact of foreign investors on interest rates for the euro area could be different from that of other countries. Hence, euro area countries are omitted.¹²

¹²[Brzoza-Brzezina and Kotlowski \(2020\)](#) employing the PSTR model also use data from both advanced economies and emerging markets. However, they use actual data, not forecast data

2.4.2 Forward interest rates

We use forward interest rates instead of current bond yields as the dependent variable. According to [Shiller et al. \(1983\)](#), the m -to- $m+n$ -year forward nominal interest rate $\mathbb{E}_t L_{i,t+m}^n$ is implied from the year-end n - and $m+n$ -year zero-coupon interest rates as follows:

$$\mathbb{E}_t L_{i,t+n}^m = \frac{(m+n)L_{i,t}^{m+n} - nL_{i,t}^n}{m} \quad (10)$$

We employ the 5-to-10-year forward real interest as the dependent variable.¹³

2.4.3 Expected public debt

We employ projections of gross public debt to-GDP-ratio $\mathbb{E}_t Debt_{i,t+n}$ by the IMF *World Economic Outlook, Article IV Consultations*, OECD *Economic Outlook*, European Commission (EC) *European Economic Forecast*, and Economist Intelligence Unit (EIU). Following [Gruber and Kamin \(2012\)](#) and [Ichiue and Shimizu \(2015\)](#), we use two-year-ahead projections of gross public debt-to-GDP-ratio as an independent variable.¹⁴ Since the IMF *World Economic Outlook* releases its public debt projections for most countries twice a year, we use bi-annual data. However, as OECD, EC and IMF *Article IV Consultations* had made two-year-ahead projections once a year over the years, we interpolate the same projections to construct bi-annual data.

2.4.4 Foreign private holdings ratio

We use the lagged share of private foreign investors $fp_{i,t-1}$ as the threshold variable and one independent variable. The composition of government bond holdings is drawn from [Arslanalp and Tsuda \(2014a\)](#) and [Arslanalp and Tsuda \(2014b\)](#). As we use forward interest rates for government debt securities denominated in the local currency, the foreign private holdings ratio $fp_{i,t-1}$ is measured as the foreign private holdings share of government debt securities denominated in the local currency.¹⁵

¹³The zero-coupon interest rate can be obtained from Bloomberg. To deflate the nominal interest rate into the real interest rate, we use two-year-ahead projections of inflation $\mathbb{E}_t \pi_{i,t+2}$ due to data availability. The robustness check examines the nominal interest rate as well.

¹⁴Moreover, real-time vintages of data could affect long-term interest rates because fiscal data have been revised largely ([De Castro et al. \(2013\)](#)).

¹⁵While [Arslanalp and Tsuda \(2014b\)](#) provide foreign holdings share of central government debt securities denominated in local currency for emerging markets, [Arslanalp and Tsuda \(2014a\)](#) do not have the corresponding ratio in advanced economies but provide total general government debt securities, including debt denominated in both local and foreign currency. Hence, assuming that only foreign investors hold government debt securities denominated in foreign currency (data that can be obtained from the Bank

2.4.5 Other control variables

A vector of control variables $\mathbf{z}_{i,t}$ includes several variables. First, we use the lagged domestic official sector holdings ratio $do_{i,t-1}$ as a control variable. Major central banks in advanced economies have been important players in the government bond markets, purchasing government bonds financed by the creation of central bank reserves through quantitative easing — which central banks have implemented to put downward pressure on interest rates when policy rates were at or near zero. The portfolio balance channel operates when the central bank’s bond purchases, which change the relative supply of assets held by the private sector, induce equilibrating changes in the relative yields.¹⁶ In the United States, [Gagnon et al. \(2011\)](#) show a cumulative decline in the 10-year Treasury yields by about 91 basis points after announcements of quantitative easing. According to [Joyce et al. \(2011\)](#), the corresponding impacts in the United Kingdom is estimated to be 100 basis points.

Second, we add the lagged foreign official holdings ratio $fo_{i,t-1}$ using data from [Arslanalp and Tsuda \(2014a\)](#) and [Arslanalp and Tsuda \(2014b\)](#). The U.S. long-term interest rates remained low in the mid-2000s despite increases in the federal funds rate, a phenomenon Alan Greenspan labeled as a “conundrum” ([Greenspan \(2005\)](#)). [Bernanke \(2005\)](#) hypothesized that a global saving glut – driven by net savings in Asia and oil-exporting countries – lowered long-term interest rates through an accumulation of foreign exchange reserves. [Warnock and Warnock \(2009\)](#) show that a 12-month total of foreign flows of 1 percentage point of GDP is associated with a 40 basis point reduction. [Beltran et al. \(2013\)](#) find that a 10 percentage point increase in foreign official flows into and out of Treasuries lowers the five-year term premium by 135 basis points.

Third, following [Gruber and Kamin \(2012\)](#), we add two-year-ahead projections of real GDP growth rate $\mathbb{E}_t y_{i,t+2}$ which can be obtained from Consensus Economics *Consensus Forecast*, the IMF *World Economic Outlook, Article IV Consultations*, OECD *Economic Outlook*, European Commission (EC) *European Economic Forecast* and the EIU.

Finally, we control for expected exchange depreciation over a two-year horizon : the rate of change between the two-year-ahead projections of the expected exchange rate and the current year estimation of each currency’s

for International Settlements *Debt Securities Statistics*), we make an approximate estimate of the foreign holdings share of general government debt securities denominated in local currency by removing the amount of debt securities denominated in foreign currency from the total. Finally, we divide the foreign holdings ratio into a foreign private holdings ratio $fp_{i,t-1}$ and a foreign official holdings ratio $fo_{i,t-1}$ using data from [Arslanalp and Tsuda \(2014a\)](#) and [Arslanalp and Tsuda \(2014b\)](#).

¹⁶[Christensen and Rudebusch \(2012\)](#), [D’Amico et al. \(2012\)](#) and [Joyce et al. \(2017\)](#) provide further discussion.

exchange rate against the US dollar $\mathbb{E}_t \text{Exchange rate}_{i,t+2}$, using *Consensus Forecast*. As foreign investors may be measuring returns in exchange rate adjusted terms, currency depreciation would cause them to demand this risk premium. [Ebeke and Lu \(2015\)](#) control for the two-year-forward exchange rate of each currency against the US dollar.

3. Estimation results

3.1 Baseline results

Table 2 shows the baseline results. The tests for nonlinearity are significant with p-values smaller than 0.01.¹⁷ Thus, we employ the IT, PSTR and GPSTR models instead of the linear model. This result illustrates that debt composition plays an important role on the non-linear behavior of the long-term interest rate.

3.1.1 Marginal impact of the expected public debt

The main parameters of interest here are β_0 and β_1 . As expected, β_1 is larger than β_0 in the IT, PSTR and GPSTR models. These results illustrate that the more public debt is financed by foreign private investors, the greater the marginal impact of the expected public debt on interest rates. Figure 6 shows this marginal impact ($\frac{\partial \mathbb{E}_t L_{i,t+n}}{\partial \mathbb{E}_t \text{Debt}_{i,t+n}}$). One striking feature of this chart is that this impact, estimated through the GPSTR model, increases exponentially and significantly as the foreign private holdings ratio exceeds the location parameter, whereas the corresponding impact estimated through the IT model increases constantly and through the PSTR model becomes flattened. The right panel of Figure 6 shows if the share of foreign investors are 20, 30 and 40 percent, the marginal impacts of public debt on the interest rate are 1.7, 3.3 and 5.5 percentage points, respectively.¹⁸ The result of the GPSTR model is consistent with DSGE studies, such as [Schmitt-Grohe and Uribe \(2003\)](#), [Adolfson et al. \(2007\)](#), and [García-Cicco et al. \(2010\)](#) that assume an exponential relationship between risk premia and foreign debt.

By contrast, when the foreign private holding ratios are below about 20 percent, the result of the GPSTR

¹⁷In the linearity test, we follow [Luukkonen et al. \(1988\)](#) and [Terasvirta \(1994\)](#), replacing the transition function $g(\cdot)$ of the PSTR and GPSTR models with the first-order Taylor expansion (please see Appendix A.1 for details). The conversion gives the IT model. Hence, the tests of the IT, PSTR and GPSTR models for nonlinearity show the same results.

¹⁸Following [Driscoll and Kraay \(1998\)](#), standard errors are corrected for serial correlation, heteroskedasticity, and cross-sectional dependence.

model illustrates that there is no significant impact of public debt on long-term government bond yields. This implies that domestic investors are insensitive to expected public debt. In other words, domestic investors may not require a risk premium against large debt when there is strong home bias without any other assets hedging fiscal risk, which is line with Japan's debt literature ([Sakuragawa and Sakuragawa \(2016\)](#)).

Figure 8 shows the range of the time-varying impacts of expected public debt-to-GDP ratio on the interest rate during 2006H2-2018H2. Given that the share of foreign private investors in the small open economy is easily affected by the capital flows, sovereign risk can be volatile over the short-term. On the other hand, the corresponding impact would be small and stable in large economies or countries with capital control or higher home bias because the share of foreign private investors is low. According to the upper panel, a problem with the IT model is that some estimated impacts are negative in countries where the share of private foreign investors is low. This unreasonable result suggests that a highly non-linear model is appropriate. The middle and upper panels show there are no negative estimates from the PSTR and GPSTR.

3.1.2 Marginal impact of the share of foreign private investors

Another parameter of interest here is the coefficient on the lagged share of foreign private investors; δ . According to Table 2 and consistent with the literature ([Arslanalp and Poghosyan \(2016\)](#) and [Ebeke and Lu \(2015\)](#)), an increase in foreign private holdings of government debt is associated with a reduction in the interest rate as downward pressure on long-term interest rates results from expansion of market liquidity.

Figure 7 shows the marginal impact of the share of foreign investors on long-term interest rates $(\frac{\partial \mathbb{E}_t L_{i,t+n}}{\partial f_{p_{i,t-1}}})$. The left panel of Figure 7 from the result of the IT model shows that this impact varies, depending on the public debt to GDP ratio. The expected public debt-to-GDP ratio $\mathbb{E}_t Debt_{i,t+n}$ is estimated to be about 120 percent of GDP when the marginal impact of the share of foreign private investors on the interest rate is zero.¹⁹ That is, although an increase in foreign private holding of government debt is associated with a reduction in interest rates, this downward effect would be reversed if the expected public debt-to-GDP ratio exceeds this threshold.

The right panel of Figure 7 from the GPSTR model shows that the marginal impact of the share of foreign investors on long-term interest rates also depends on the composition of debt, as well as the expected public debt. Expected public debt $\mathbb{E}_t Debt_{i,t+n}$ triggers a surge in government bond yields through an increase in the

¹⁹The threshold can be calculated from $\mathbb{E}_t Debt_{i,t+n} = \frac{-\delta}{\beta_1}$ in Table 1.

share of foreign investors. Given that the marginal impact of the share of foreign private investors on the interest rate is zero, the threshold of expected public debt is high (low) when the share of foreign investors is low (high). For example, when the share of foreign investors is 15 percent, the threshold of expected public debt is 91 percent. In comparison, when the share of foreign investors is 30 percent, the threshold is 59 percent.²⁰ As the impact of public debt on interest rates based on the GPSTR model continues to increase exponentially depending on the foreign private holdings ratio, an increase in foreign private holdings of government debt would cause a rise in long-term interest rates even if the debt level is low. Hence, the result of the GPSTR model differs substantially from those of the IT and PSTR models.²¹

3.2 Robustness check

We consider a wide range of exercises to check the robustness of our headline findings, including the following specifications to control for global and country-specific factors.

3.2.1 Global factor

Robustness check 1 (controlling for U.S. monetary conditions): The baseline might already incorporate global factors because it includes the share of foreign investors. A growing literature also investigates how bond spreads especially in emerging markets, are related to global factors, especially U.S. monetary conditions. For example, [Mauro et al. \(2002\)](#) and [González-Rozada and Levy Yeyati \(2008\)](#), using sovereign bonds denominated in U.S. dollars conclude that spreads comove across emerging markets and tend to be most related to global events and global liquidity. [Foley-Fisher and Guimaraes \(2013\)](#) and [Gilchrist et al. \(2019\)](#) also examine spillovers from changes in U.S. interest rates on sovereign bond yields denominated in foreign currencies. Moreover, [Longstaff et al. \(2011\)](#) find similar evidence based on the sovereign CDS market where the reference obligation is a US dollar-denominated issue or a euro-dominated issue.

While foreign currency bonds have been widely used by previous works as discussed above, recent literature has examined local currency bonds as well. For instance, [Moore et al. \(2013\)](#) analyze the effect of US monetary

²⁰The threshold can be calculated from $\mathbb{E}_t Debt_{i,t+n} = \frac{-\delta}{\beta_1 \left\{ \frac{\partial g(J P_{i,t-1}; \gamma_1, \gamma_2, c)}{\partial J P_{i,t-1}} \right\}}$ in Table 1.

²¹As shown in the central panels of Figure 7 for the PSTR model, the marginal impacts of the share of foreign investor on the long-term interest rates over the location parameter are the same because of the assumption of the point symmetry. This result, which is difficult to interpret, is one reason why we develop the GPSTR model to investigate point asymmetry.

policy on local currency sovereign yields.

Although the baseline might already incorporate global factors through the share of foreign investors, we control for the U.S. 10-year term premia based on [Adrian and Crump \(2013\)](#), excluding the observation of the United States from the sample.²²

3.2.2 Country-specific factors

Robustness check 2 (excluding exchange rate risk): The baseline includes both the share of foreign investors and expected exchange depreciation as independent variables. Although we take a lagged value of the share of foreign investors, it may decline due to expected exchange depreciation if they exhibit forward-looking behavior. Given possible multicollinearity between these two variables, we exclude expected exchange depreciation.

Robustness check 3 (controlling for net foreign assets position): Following [Brzoza-Brzezina and Kotlowski \(2020\)](#), we control for the lagged net foreign assets to GDP ratio $NFA_{i,t-1}$. [Reinhart and Trebesch \(2015\)](#) argue that an increase in the share of foreign investors is associated with lower long-term interest rates, as these holdings supplement domestic savings in capital-scarce countries. By contrast, a chronic excess of saving over investments may cause low real interest rates.

Robustness check 4 (controlling for capital control): Capital control would lower sovereign risk because the regulation of capital flows may possibly prevent crises ([Wright \(2006\)](#)). [Fernández et al. \(2016\)](#) construct an index of capital control restrictions for bond inflow and outflow. Hence, we add this capital control dummy variable $CC_{i,t-1}$ as an additional robustness check.

Robustness check 5 (controlling for short-term interest rate): We employ forward interest rates to omit any effects of current economic conditions. However, if current monetary policy affects forward interest rates, omitted variable bias would occur due to misspecification. Hence, we control for the one-year zero coupon rate $STR_{i,t}$.

²²We directly control for U.S. monetary conditions. However, as a manifestation of the investors' risk-taking behavior amid low interest rates of advanced economies, the "search-for-yield" could cause the share of private foreign investors to increase in emerging markets. [Moore et al. \(2013\)](#) show that a 1 percentage point drop in the US 10-year Treasury bond yield leads on average to a 4.2 percentage point increase in foreign holdings of government bonds in emerging markets. Therefore, future research could control for U.S. monetary conditions indirectly by using the instrumental variables to deal with the endogeneity of the share of foreign investors without taking a lagged value. However, [Yu \(2013\)](#) finds inconsistency in the two-step-least-squares estimators in threshold models when the threshold variable is endogenous. [Caner and Hansen \(2004\)](#) use two-step least square estimators in the threshold model where they allow for endogenous regressors, but they assume the threshold variable to be exogenous. Furthermore, [Yu \(2013\)](#) points out that the estimator based on a misspecified reduced form is inconsistent even if the threshold variable is exogenous. Employing instrumental variables is an interesting future topic for research.

Robustness check 6 (dependent variable: nominal interest rates): [Ichiue and Shimizu \(2015\)](#) use the 6-to-10-year-ahead projection of the inflation rate to match the horizon reflected in the 5-to-10-year forward interest rates for 10 advanced economies. However, our baseline deflates the 5-to-10-year nominal forward rates by two-year-ahead projections of inflation due to data availability. Hence, an additional robustness check can use the nominal interest rate as the dependent variable and add the two-year-ahead projections of inflation as the independent variable.

According to Tables 3, 4, 5 and Figure 9, the results of the robustness checks are in line with the baseline results. According to the results for the GPSTR model, the impact of expected public debt on interest rates increases exponentially and significantly as the foreign private holdings ratio exceeds the location parameter. For robustness check 2, the estimated impact of a 1 percentage point increase in the U.S. 10-year term premium is about 0.5 percentage point. Instead, the marginal impact of the share of foreign investors becomes smaller than the baseline because US monetary conditions now also affect the behavior of foreign investors. Moreover, the corresponding impact of public debt is also smaller than the baseline when the share of the private investors is higher. The results of robustness checks 3 and 4 show that both higher net foreign assets position and capital controls lower the interest rates.²³

3.3 Model evaluation

The IT, PSTR, and GPSTR models are competing models, as non-linearity tests reject the null hypothesis of the linear model. As argued by [Granger \(2001\)](#), it is well known that nonlinear models are inclined to overfit the data and thus, out-of-sample forecasting evaluation is recommended. Hence, we employ cross-validation to calculate the mean squared error (MSE).

We start to estimate three models for the new in-sample by leaving out all observations of the i th cross-section. Using estimated parameters, we plug in all observations of the i th cross-section and then calculate the

²³An additional robustness check considers the expectation for fiscal consolidation so as to avoid sovereign default. This idea is derived from “fiscal limit”, i.e., a point where the government no longer has the ability to finance higher debt levels by increasing the tax rate to the revenue-maximizing tax rate ([Bi \(2012\)](#)). The sovereign risk premium is related to the fiscal limit, assuming that sovereign default occurs when government debt exceeds the fiscal limit. Empirically, [Nakamura and Yagi \(2017\)](#) use the national burden ratio, measured as the sum of tax payments and social security fees as a ratio to nominal GDP, as a variable representing expectation for fiscal consolidation. They find that a low national burden ratio keeps long-term interest rates at low levels in 23 advanced economies. However, as our sample includes emerging markets, we do not find similar results. A possible reason is that although the national burden ratio in emerging markets is lower than in advanced economies, this does not necessarily mean that the government can increase tax revenue, as many types of structural obstacles may exist (e.g., a large informal sector and dependence on a few natural resources). Please see [Besley and Persson \(2014\)](#) for further details.

MSE, comparing the estimated fitted values and the actual values as below:

$$\frac{1}{NT} \hat{e}^2 = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \left[\tilde{y}_{i,t} - \hat{\Psi}_{-i}(x_{i,t}^*) \right]^2 \quad (11)$$

where $\hat{\Psi}_{-i}(\cdot)$ is computed by leaving out all observations of the i th cross-section. $\frac{1}{NT} \hat{e}_{IT}^2$, $\frac{1}{NT} \hat{e}_{PSTR}^2$, and $\frac{1}{NT} \hat{e}_{GPSTR}^2$ are MSEs of the IT, PSTR, and GPSTR models, respectively.

Table 6 shows the comparison of MSEs across the three models and illustrates that the out-of-sample forecast errors of the GPSTR model are smaller than those of the IT and PSTR models for all cases. This means that the GPSTR model is robust to the sample selection.

3.4 Comparison with other studies

Table 7 compares our estimates with those of other studies in three dimensions: (1) the impact of an increase in expected public debt on government bond yields, (2) the impact of an increase in the share of foreign investors on government bond yield, and (3) the threshold value for the impact of an increase in the share of foreign investors.

First, significant research has been devoted to estimating the positive impact of public debt on long-term interest rates, focusing on government debt securities denominated in local currency. The estimated impacts found for advanced economies are smaller than for emerging markets (Arslanalp and Poghosyan (2016), Gruber and Kamin (2012), Ichiue and Shimizu (2015) and Jaramillo and Weber (2013)).²⁴ Our result shows the impact of an increase in expected public debt on forward interest rate rises with the share of foreign private financing in public debt. Overall, the range of our estimated impacts is consistent with that of the existing literature.

Second, our result shows that higher foreign private holdings of government debt is also associated with lower in long-term interest rates, consistent with the literature (Arslanalp and Poghosyan (2016) and Ebeke and Lu (2015)).

Third, Ebeke and Lu (2015) show that in emerging markets an increase in the share of foreign holdings has a negative impact on yield, but this impact turns positive if either the lagged external debt-to-GDP ratio

²⁴As the studies employ different sample periods, they are not necessarily comparable.

exceeds 90 percent or the lagged short-term debt-to-GDP ratio exceeds 21.5 percent. Our corresponding result from the GPSTR model indicates that the threshold of expected public debt is high (low) when the share of foreign investors is low (high). For example, when the share of foreign investors is 15 percent, the threshold of the expected public debt is 91 percent. In comparison, when the share of foreign investors is 30 percent, the threshold is 59 percent. As the marginal impact of expected public debt on interest rates under the GPSTR model increases exponentially, depending on the foreign private holdings ratio, an increase in foreign private holdings of government debt would cause a rise in long-term interest rates, even if the debt level is low. Hence, the result of the GPSTR model differs substantially from that of the IT model.

4. Conclusion

This study examines threshold values to understand the non-linear behavior of long-term interest rates using a novel approach, a panel smooth transition regression with a general logistic model. This paper examines how the interaction between the funding source and the expected public debt level affects interest rates, based on the data of 11 advanced economies and 14 emerging markets.

The main findings presented here are threefold. First, the impact of expected public debt levels on interest rates increases exponentially as the share of foreign private holdings exceeds 20 percent denominated in the local currency, even if strong home bias could mitigate this upward pressure. Second, if the expected public debt-to-GDP ratio exceeds a certain level, an increase in the share of foreign private holdings of government debt would raise long-term interest rates, offsetting the downward pressure on long-term interest rates from expanding market liquidity. Third, the out-of-sample forecasts of the generalized panel smooth transition regression model are more accurate than those of previously used methods.

The composition of government debt has a significant impact on the highly non-linear behavior of the long-term interest rate. Policy makers need to consider how government bond yields react to the level of public debt. In particular, as the share of foreign private investors in a small open economy is easily affected by capital flows, sovereign risk could fluctuate in the short-term. From a long-term perspective, [Hoshi and Ito \(2014\)](#) point out that an increase in local-currency-denominated domestic assets would slow down due to the rapid aging of the population in Japan. As the government cannot indefinitely rely on private domestic investors, foreign investors would have to step in and absorb the government debt at some point. Consequently, sovereign

risk would become more sensitive to these developments.

Future research could focus on the endogeneity of the share of foreign investors by using instrumental variables. This study used a lagged value of the foreign investor share to avoid simultaneity bias and it maintained the assumption of exogeneity. However, this assumption could be relaxed by allowing for endogenous regressors although that would lead to additional econometric challenges.

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Figure 1: 5-to-10 year forward interest rates of government debt securities denominated in local currency

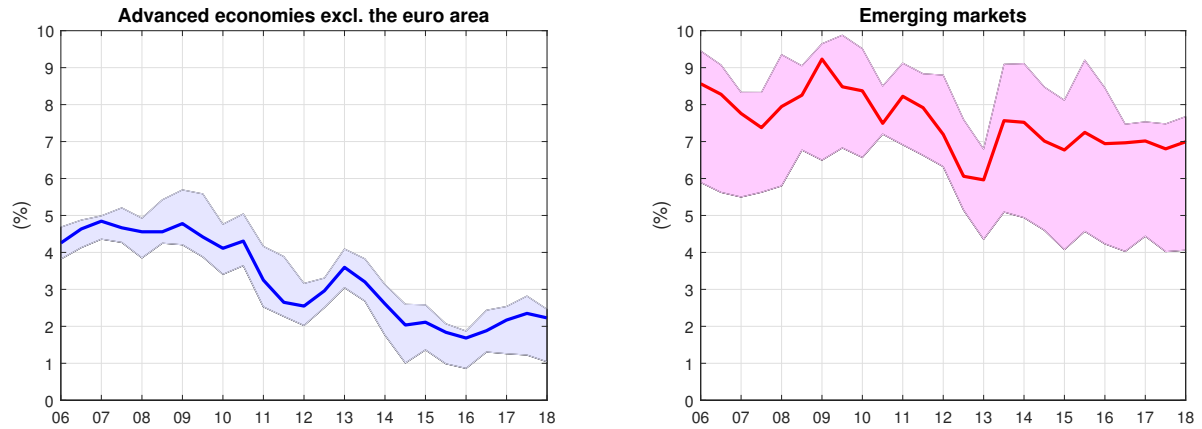


Figure 2: Two-year-ahead expected public debt to GDP ratio

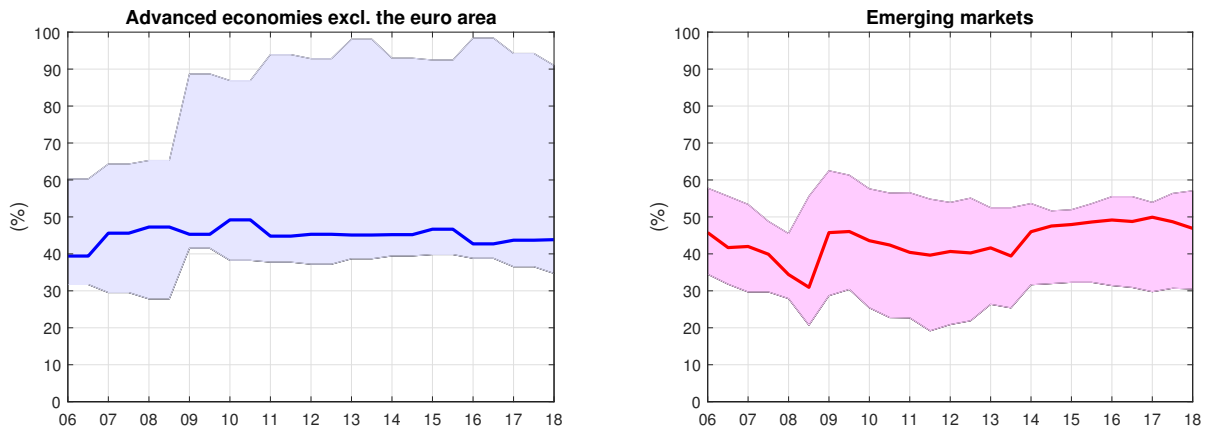
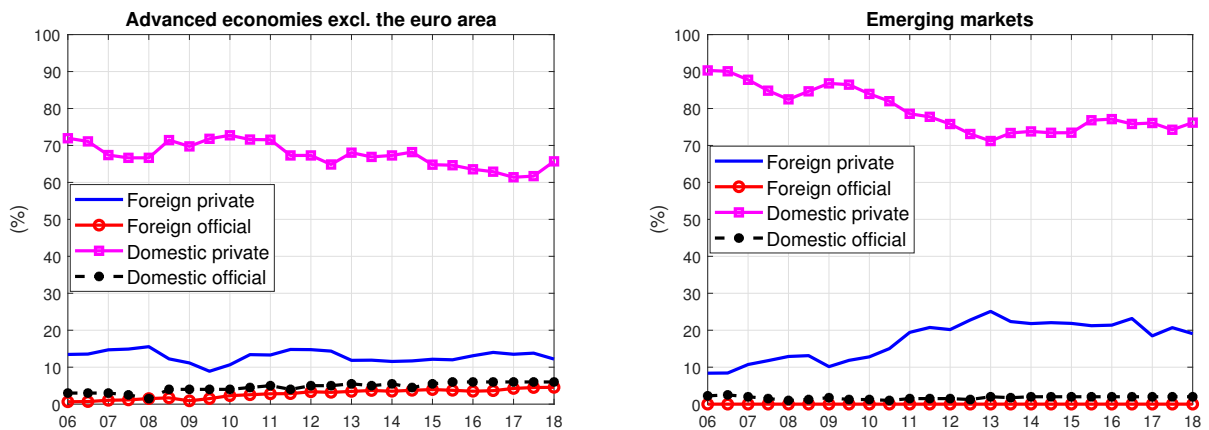


Figure 3: The share of government debt securities denominated in local currency



Notes: Advanced economies excluding the euro area(Australia, Canada, the Czech Republic, Denmark, Japan, Korea, Norway, Sweden, Switzerland , the United Kingdom and United States) and emerging markets(Bulgaria, Columbia, China, Hungary, India, Indonesia, Malaysia, Mexico, Peru, the Philippines, Poland, Thailand, Turkey and South Africa). Lines and shadow areas show the median and interquartile range within each country group, respectively. Calculations of share of government debt based on data from Arslanalp and Tsuda (2014a), Arslanalp and Tsuda (2014b) and BIS Debt Securities Statistics (See 2.4 Data for details).

Figure 4: Examples of interaction term and standard logistic model

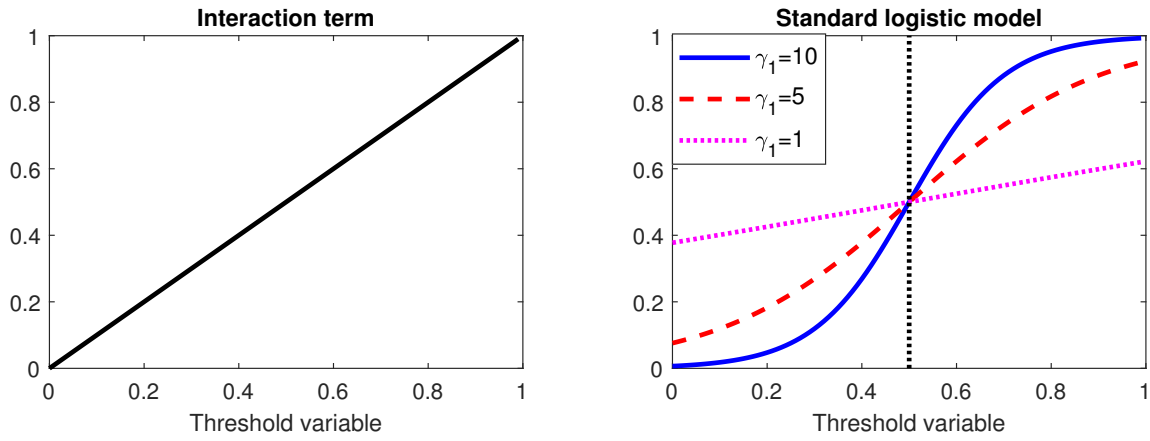
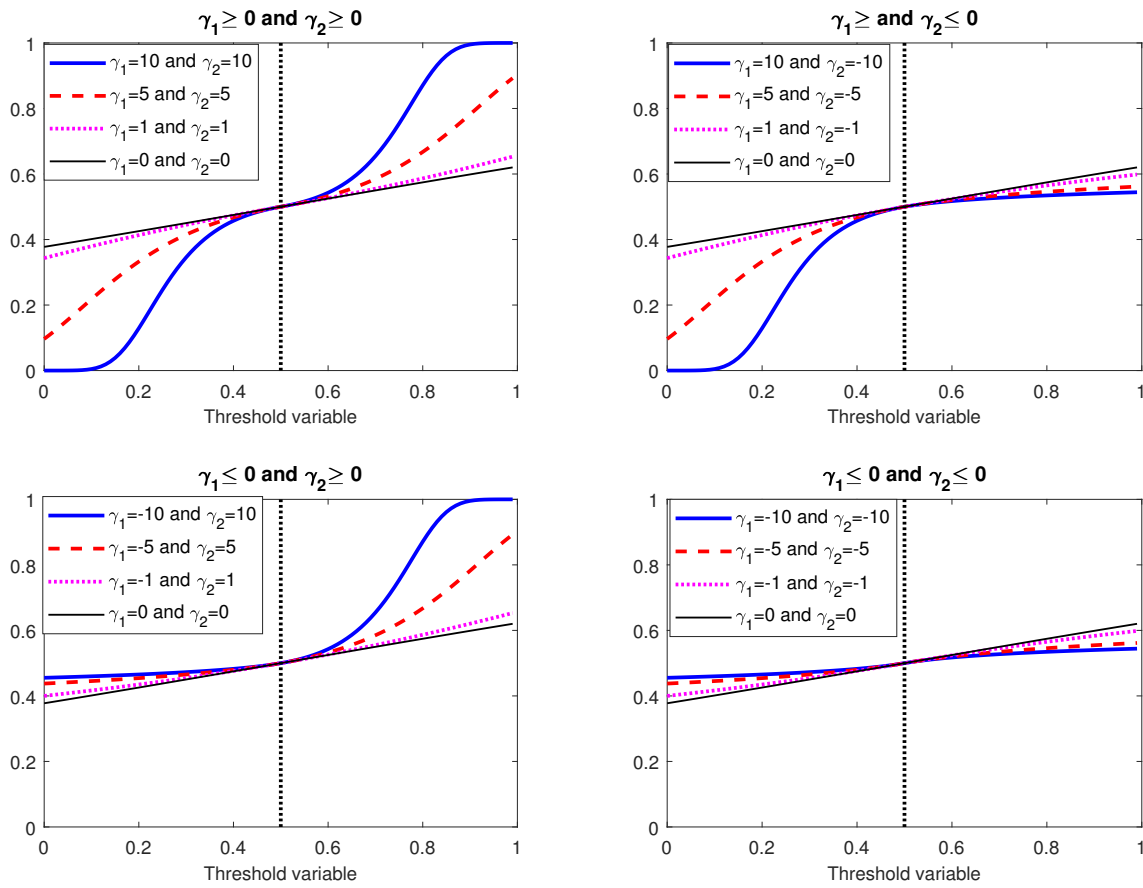


Figure 5: Examples of general logistic model (GLM)



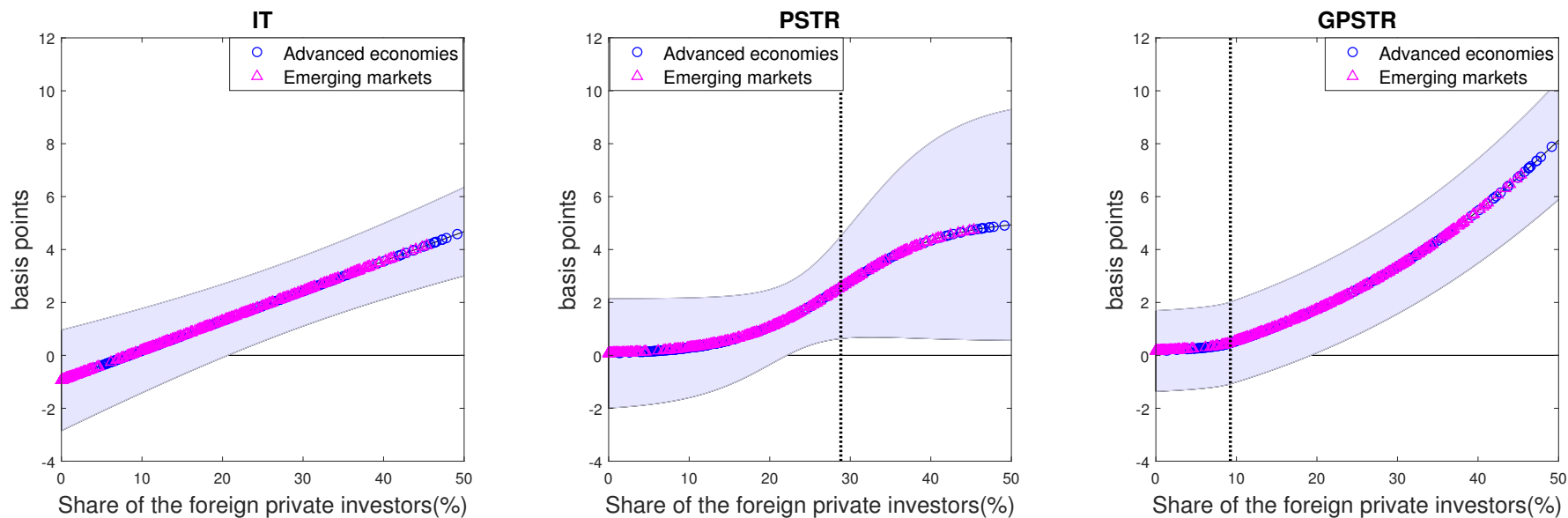
Notes: The location parameter $c=0.5$.

Table 2: Baseline results

Model	IT	PSTR	GPSTR
LM_F nonlinearity tests	10.5	10.5	10.5
p-value	0.00***	0.00***	0.00***
Location parameter(c)		0.288	0.092
Slope parameter1(γ_1)		15.3	-64.0
Slope parameter2(γ_2)			3.0
$\mathbb{E}_t Debt_{i,t+2}$	-0.008 (0.01)	0.000 (0.01)	-0.196*** (0.01)
$\mathbb{E}_t Debt_{i,t+2} \cdot fp_{t-1}$	0.120*** (0.04)		
$\mathbb{E}_t Debt_{i,t+2} \cdot g(\cdot)$		0.051* (0.03)	0.402*** (0.03)
$\mathbb{E}_t y_{i,t+2}$	-0.008 (0.13)	0.000 (0.11)	0.042 (0.13)
$do_{i,t-1}$	-0.099*** (0.01)	-0.085*** (0.01)	-0.089*** (0.01)
$fo_{i,t-1}$	-0.146*** (0.02)	-0.148*** (0.02)	-0.153*** (0.02)
$fp_{i,t-1}$	-0.104*** (0.03)	-0.105*** (0.02)	-0.104*** (0.01)
$\mathbb{E}_t Exchange\ rate_{i,t+2}$	0.042*** (0.01)	0.043*** (0.01)	0.041*** (0.01)
AIC Criterion	-8.563	-8.562	-8.573
No. of observation	625	625	625
No. of countries	25	25	25
Sample periods	06:2-18:2	06:2-18:2	06:2-18:2

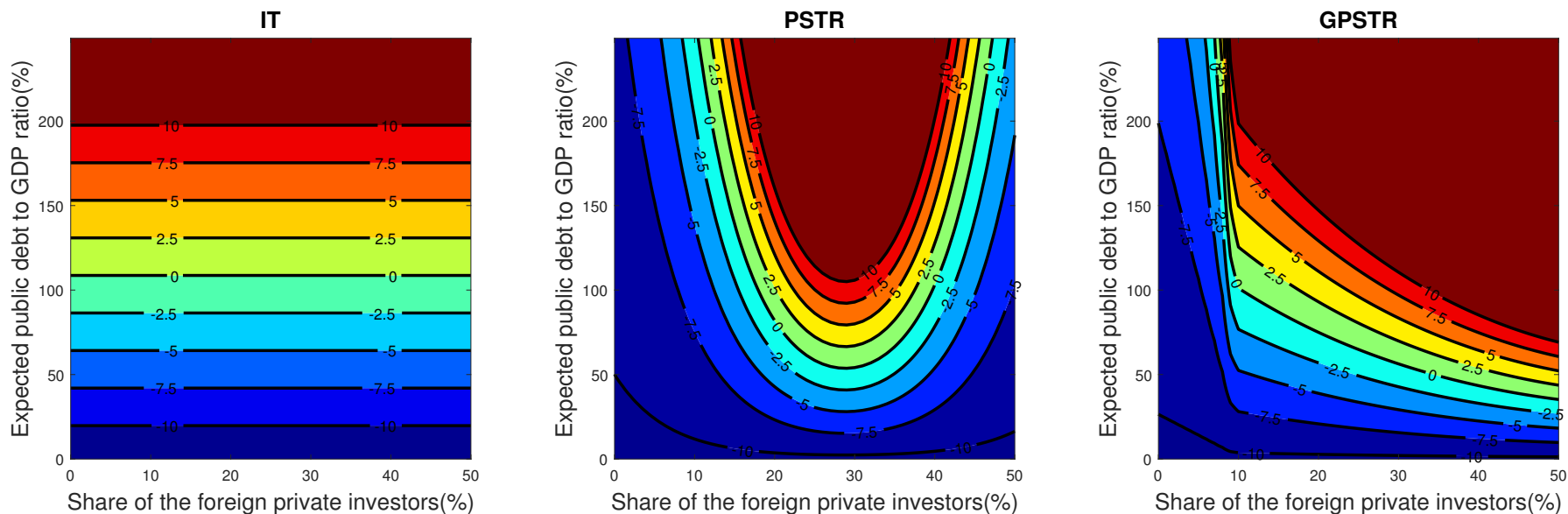
Notes: IT=Interaction term model,PSTR=Panel Smooth Transition Regression and GPSTR=General Panel Smooth Transition Regression. $\mathbb{E}_t Debt_{i,t+2}$ is two-year-ahead expected public debt to GDP ratio, $g(\cdot)$ is transition function, $\mathbb{E}_t y_{i,t+2}$ is two-year-ahead expected growth rate, $do_{i,t-1}$ is the lagged domestic official sector holdings ratio, $fo_{i,t-1}$ the lagged foreign official sector holdings ratio, $fp_{i,t-1}$ is the lagged foreign private sector holdings ratio and $\mathbb{E}_t Exchange\ rate_{i,t+2}$ is the expected exchange rate depreciation in two-year horizon. The standard errors proposed by [Driscoll and Kraay \(1998\)](#) are reported in parentheses. ***p<0.01, *p<0.05, *p<0.1.

Figure 6: Marginal impact of the expected public debt-to-GDP ratio on the interest rate (Baseline)



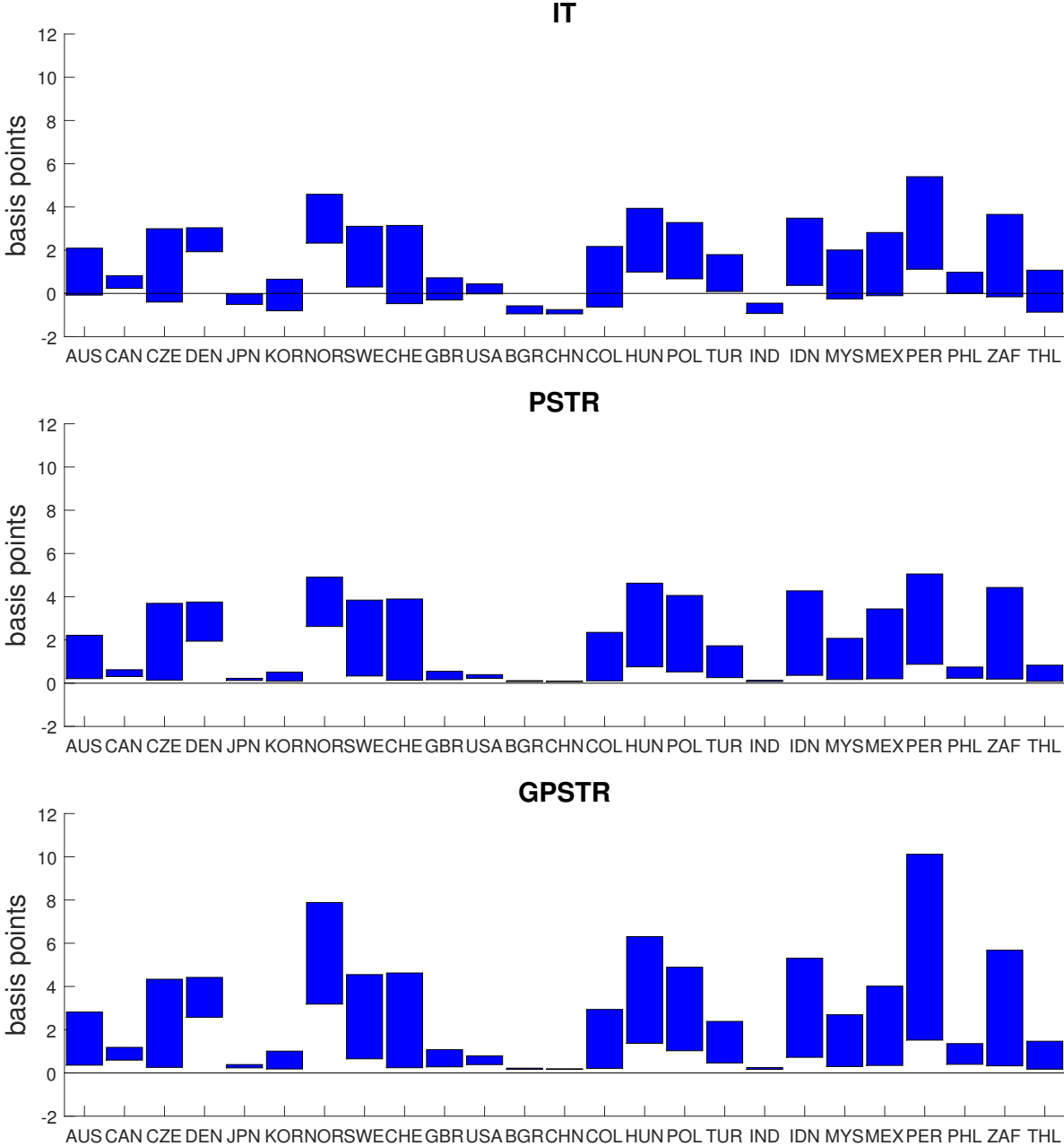
Notes: Shadow area refers to 90 percent confidence interval computed with [Driscoll and Kraay \(1998\)](#) standard errors. Circles and triangles correspond to observation for private foreign share. Broken lines represent location parameters.

Figure 7: Marginal impact of the share of foreign private investors on the interest rate (Baseline)



Notes: Basis points.

Figure 8: Range of the time-varying impact of the expected public debt-to-GDP ratio on the interest rate during 2006H2-2018H2 (Baseline)



Notes: The range is the difference between the maximum and minimum point estimate during 2006H2-2018H2. AUS:Australia, CAN:Canada, CZE:the Czech Republic, DEN:Denmark, JPN:Japan, KOR:Korea, NOR:Norway, SWE:Sweden, CHE:Switzerland, GBR:the United Kingdom, USA:the United States, BGR:Bulgaria, CHN:China, COL:Columbia, HUN:Hungary, POL:Poland, TUR:Turkey, IND:India, IDN:Indonesia, MYS: Malaysia, MEX:Mexico, PER:Peru, PHL:the Philippines, ZAF:South Africa, THL:Thailand

Table 3: Robustness Checks 1 and 2

	(R1)			(R2)		
	IT	PSTR	GPSTR	IT	PSTR	GPSTR
LM_F nonlinearity test	9.4	9.4	9.4	11.6	11.6	11.6
p-value	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***
Location parameter(c)		0.295	0.069		0.293	0.092
Slope parameter1(γ_1)		14.3	-56.6		14.2	-51.1
Slope parameter2(γ_2)			0.03			3.1
$\mathbb{E}_t Debt_{i,t+2}$	-0.008 (0.01)	-0.001 (0.01)	-0.282 (0.01)	-0.007 (0.01)	0.000 (0.01)	-0.220*** (0.01)
$\mathbb{E}_t Debt_{i,t+2} \cdot fp_{t-1}$	0.110*** (0.03)			0.127*** (0.04)		
$\mathbb{E}_t Debt_{i,t+2} \cdot g(\cdot)$		0.049** (0.02)	0.557*** (0.03)		0.056 (0.04)	0.450*** (0.03)
$\mathbb{E}_t y_{i,t+2}$	-0.117 (0.15)	-0.111 (0.15)	-0.083 (0.14)	0.011 (0.13)	0.020 (0.13)	0.063 (0.13)
$do_{i,t-1}$	-0.067*** (0.02)	-0.054*** (0.01)	-0.057*** (0.02)	-0.111*** (0.01)	-0.097*** (0.01)	-0.098*** (0.01)
$fo_{i,t-1}$	-0.098*** (0.03)	-0.100*** (0.02)	-0.093*** (0.03)	-0.153*** (0.02)	-0.154*** (0.02)	-0.159*** (0.02)
$fp_{i,t-1}$	-0.080*** (0.02)	-0.081*** (0.02)	-0.082*** (0.02)	-0.115*** (0.02)	-0.115*** (0.03)	-0.118*** (0.02)
$\mathbb{E}_t ER_{i,t+2}$	0.039*** (0.01)	0.039*** (0.01)	0.036*** (0.01)			
$US tp_{t-1}$	0.542*** (0.14)	0.543*** (0.11)	0.540*** (0.13)			
AIC Criterion	-8.636	-8.636	-8.640	-8.547	-8.545	-8.559
No. of observation	600	600	600	625	625	625
No. of countries	24	24	24	25	25	25
Sample periods	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2

Notes: IT=Interaction term model,PSTR=Panel Smooth Transition Regression and GPSTR=General Panel Smooth Transition Regression. The sample of (R1) excludes the U.S. $\mathbb{E}_t Debt_{i,t+2}$ is two-year-ahead expected public debt to GDP ratio, $g(\cdot)$ is transition function, $\mathbb{E}_t y_{i,t+2}$ is two-year-ahead expected growth rate, $do_{i,t-1}$ is the lagged domestic official sector holdings ratio, $fo_{i,t-1}$ the lagged foreign official sector holdings ratio, $fp_{i,t-1}$ is the lagged foreign private sector holdings ratio, $\mathbb{E}_t Exchange\ rate_{i,t+2}$ is the expected exchange rate depreciation in two- year horizon and $US tp_{t-1}$ is the lagged U.S. 10-year term premia. The standard errors proposed by [Driscoll and Kraay \(1998\)](#) are reported in parentheses. ***p<0.01, *p<0.05, *p<0.1

Table 4: (Continued) Robustness Checks 3 and 4

	(R3)			(R4)		
	IT	PSTR	GPSTR	IT	PSTR	GPSTR
LM_F nonlinearity test	7.0	7.0	7.0	10.0	10.0	10.0
p-value	0.01***	0.01***	0.01***	0.00***	0.00***	0.00***
Location parameter(c)		0.335	0.092		0.264	0.103
Slope parameter1(γ_1)		15.0	-35.8		16.2	-74.8
Slope parameter2(γ_2)			4.0			0.3
$\mathbb{E}_t Debt_{i,t+2}$	-0.006 (0.01)	0.002 (0.01)	-0.162*** (0.03)	-0.011 (0.01)	-0.004 (0.01)	-0.321*** (0.05)
$\mathbb{E}_t Debt_{i,t+2} \cdot fp_{t-1}$	0.099*** (0.03)			0.119*** (0.03)		
$\mathbb{E}_t Debt_{i,t+2} \cdot g(\cdot)$		0.050* (0.03)	0.331*** (0.05)		0.048* (0.03)	0.646*** (0.09)
$\mathbb{E}_t y_{i,t+2}$	-0.015 (0.14)	-0.002 (0.11)	0.030 (0.14)	-0.041 (0.12)	-0.038 (0.10)	0.037 (0.12)
$do_{i,t-1}$	-0.099*** (0.01)	-0.087*** (0.01)	-0.090*** (0.01)	-0.094*** (0.01)	-0.081*** (0.01)	-0.089*** (0.01)
$fo_{i,t-1}$	-0.142*** (0.02)	-0.144*** (0.01)	-0.149*** (0.02)	-0.170*** (0.02)	-0.170*** (0.02)	-0.179*** (0.02)
$fp_{i,t-1}$	-0.096*** (0.02)	-0.096*** (0.02)	-0.104*** (0.01)	-0.094*** (0.02)	-0.097*** (0.02)	-0.090*** (0.01)
$\mathbb{E}_t ER_{i,t+2}$	0.029*** (0.01)	0.028*** (0.01)	0.027*** (0.01)	0.038*** (0.01)	0.039*** (0.01)	0.039*** (0.01)
$NFA_{i,t-1}$	-0.014*** (0.00)	-0.015*** (0.00)	-0.014*** (0.00)			
$CC_{i,t-1}$				-3.900*** (1.10)	-3.892*** (1.00)	-4.141*** (1.09)
AIC Criterion	-8.547	-8.581	-8.640	-8.612	-8.611	-8.626
No. of observation	625	625	625	625	625	625
No. of countries	25	25	25	25	25	25
Sample periods	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2

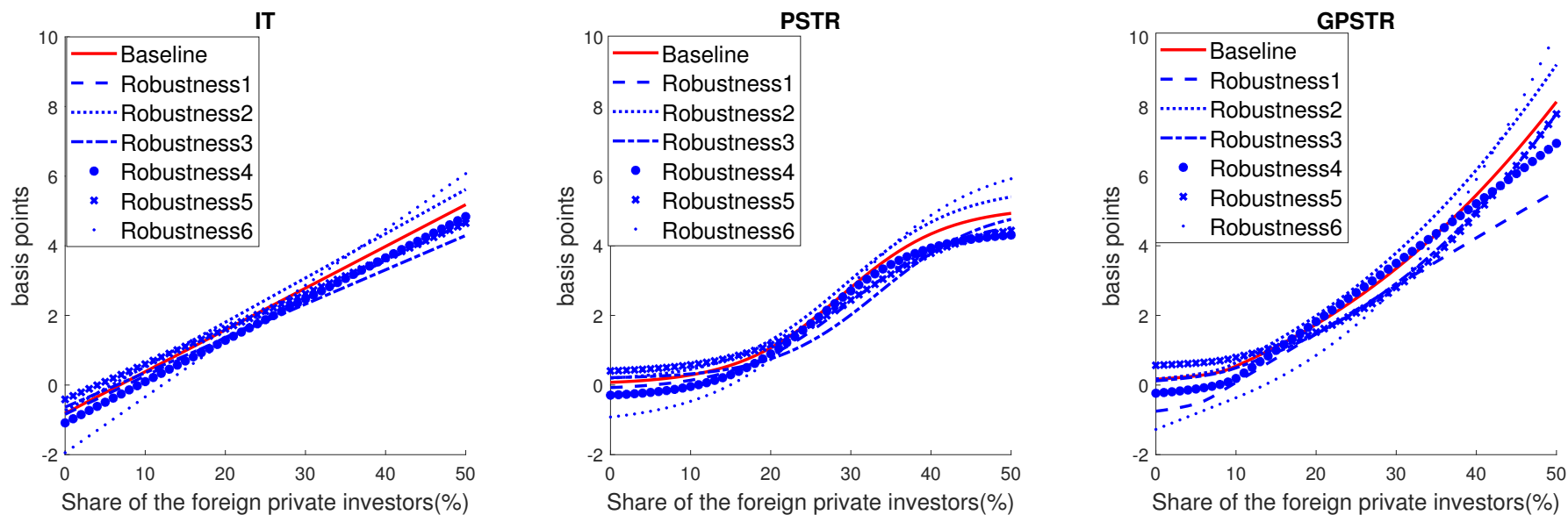
Notes: IT=Interaction term model,PSTR=Panel Smooth Transition Regression and GPSTR=General Panel Smooth Transition Regression. $\mathbb{E}_t Debt_{i,t+2}$ is two-year-ahead expected public debt to GDP ratio, $g(\cdot)$ is transition function, $\mathbb{E}_t y_{i,t+2}$ is two-year-ahead expected growth rate, $do_{i,t-1}$ is the lagged domestic official sector holdings ratio, $fo_{i,t-1}$ the lagged foreign official sector holdings ratio, $fp_{i,t-1}$ is the lagged foreign private sector holdings ratio, $\mathbb{E}_t Exchange\ rate_{i,t+2}$ is the expected exchange rate depreciation in two-year horizon, $NFA_{i,t-1}$ is the lagged net foreign assets to GDP ratio and $CC_{i,t-1}$ is the lagged capital control dummy. The standard errors proposed by [Driscoll and Kraay \(1998\)](#) are reported in parentheses. ***p<0.01, *p<0.05, *p<0.1

Table 5: (Continued) Robustness Checks 5 and 6

	(R5)			(R6)		
	IT	PSTR	GPSTR	IT	PSTR	GPSTR
LM_F nonlinearity test	9.5	9.5	9.5	20.0	20.0	20.0
p-value	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***
Location parameter(c)		0.306	0.092		0.292	0.093
Slope parameter1(γ_1)		14.0	-47.5		12.3	0.0
Slope parameter2(γ_2)			5.8			5.0
$\mathbb{E}_t Debt_{i,t+2}$	-0.004 (0.01)	0.003 (0.01)	-0.096*** (0.02)	-0.019** (0.01)	-0.011 (0.01)	-0.186*** (0.01)
$\mathbb{E}_t Debt_{i,t+2} \cdot fp_{t-1}$	0.101*** (0.04)			0.160*** (0.03)		
$\mathbb{E}_t Debt_{i,t+2} \cdot g(\cdot)$		0.044 (0.03)	0.206*** (0.03)		0.076*** (0.03)	0.363*** (0.03)
$STR_{i,t}$	0.388*** (0.05)	0.385*** (0.04)	0.381*** (0.05)			
$\mathbb{E}_t \pi_{i,t+2}$				0.358*** (0.13)	0.351*** (0.12)	0.355*** (0.12)
$\mathbb{E}_t y_{i,t+2}$	-0.060 (0.12)	-0.051 (0.11)	-0.022 (0.10)	0.087 (0.12)	0.097 (0.12)	0.127 (0.12)
$do_{i,t-1}$	-0.081*** (0.01)	-0.070*** (0.01)	-0.072*** (0.01)	-0.069*** (0.12)	-0.053*** (0.01)	-0.053*** (0.01)
$fo_{i,t-1}$	-0.099*** (0.02)	-0.101*** (0.02)	-0.105*** (0.01)	-0.139*** (0.02)	-0.140*** (0.02)	-0.145*** (0.02)
$fp_{i,t-1}$	-0.088*** (0.03)	-0.086*** (0.02)	-0.087*** (0.01)	-0.115*** (0.02)	-0.116*** (0.02)	-0.116*** (0.02)
$\mathbb{E}_t ER_{i,t+2}$	0.002 (0.01)	0.003 (0.01)	0.001 (0.01)	0.045*** (0.01)	0.046*** (0.01)	0.043*** (0.01)
AIC Criterion	-8.800	-8.796	-8.805	-8.649	-8.651	-8.657
No. of observation	625	625	625	625	625	625
No. of countries	25	25	25	25	25	25
Sample periods	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2	06:2-18:2

Notes: IT=Interaction term model,PSTR=Panel Smooth Transition Regression and GPSTR=General Panel Smooth Transition Regression. $\mathbb{E}_t Debt_{i,t+2}$ is two-year-ahead expected public debt to GDP ratio, $g(\cdot)$ is transition function, $\mathbb{E}_t \pi_{i,t+2}$ is two-year-ahead expected inflation, $\mathbb{E}_t y_{i,t+2}$ is two-year-ahead expected growth rate, $do_{i,t-1}$ is the lagged domestic official sector holdings ratio, $fo_{i,t-1}$ the lagged foreign official sector holdings ratio, $fp_{i,t-1}$ is the lagged foreign private sector holdings ratio, $\mathbb{E}_t Exchange\ rate_{i,t+2}$ is the expected exchange rate depreciation in two-year horizon, and $STR_{i,t}$ is one-year zero coupon rate. The standard errors proposed by [Driscoll and Kraay \(1998\)](#) are reported in parentheses. ***p<0.01, *p<0.05, *p<0.1

Figure 9: Marginal impact of the expected public debt-to-GDP ratio on the interest rate (Baseline and robustness check)



Notes: Basis points. The baseline and robustness1-5 employ real forward interest rates as a dependent variable. The baseline equation includes the two-year-ahead expected public debt to GDP ratio, the two-year-ahead expected growth rate, the lagged domestic official sector holdings ratio, the lagged foreign official sector holdings ratio, the lagged foreign private sector holdings ratio and the expected exchange rate depreciation in two-year horizon. Robustness1: Controlling for US monetary condition. Robustness2: Excluding the expected exchange rate depreciation. Robustness3: Controlling for the net foreign assets position. Robustness4: Controlling for the capital control. Robustness5: Controlling for the short-term interest rate. Robustness6: Dependent variable: nominal interest rates.

Table 6: Cross validations: comparison of out-of-sample forecast errors

	(1) IT vs. GPSTR	(2) PSTR vs. GPSTR
	$\frac{MSE_{GPSTR} - MSE_{IT}}{MSE_{IT}} \times 100$	$\frac{MSE_{GPSTR} - MSE_{PSTR}}{MSE_{PSTR}} \times 100$
Baseline	-7.2%	-4.0%
Robustness check1 (Controlling <i>US term premium</i>)	-5.2%	-4.0%
Robustness check2 (excl. <i>exchange rate depreciation</i>)	-6.0%	-3.0%
Robustness check3 (Controlling <i>net foreign assets</i>)	-4.4%	-2.6%
Robustness check4 (Controlling <i>capital control</i>)	-0.8%	-3.7%
Robustness check5 (Controlling <i>short – term interest rate</i>)	-8.7%	-4.8%
Robustness check6 (Dependent variable: <i>nominal interest rate</i>)	-8.2%	-0.6%

Note: MSE indicates mean squared error. $MSE = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \left[\tilde{y}_{i,t} - \hat{\Psi}_{-i}(x_{i,t}^*) \right]^2$ where $\hat{\Psi}_{-i}(\cdot)$ is computed by leaving out all observations of the i th cross-section.

Table 7: Comparison with other studies

(1) Marginal impact of expected public debt on government bond yields	basis point per 1 percent	Forecast data	Countries/Group	
This study	0.9-2.0(the share of foreign investors is 20 percent) 2.8-3.8(the share of foreign investors is 30 percent) 4.2-6.2(the share of foreign investors is 40 percent)	IMF, OECD, EC, EIU	11 AEs and 14 EMs	
Engen and Hubbard (2005)	2.8	CBO	U.S	
Laubach (2009)	3 to 4.4	CBO	U.S	
Traum and Yang (2015)	3	DEGE implied data	U.S	
Arslanalp and Poghosyan (2016)	2.7 to 2.8	IMF	22 AEs	
Gruber and Kamin (2012)	0.4 to 1.3	OECD	19 OECD countries	
Ichiue and Shimizu (2015)	0.7 to 0.9	OECD	10 OECD countries	
Jaramillo and Weber (2013)	4 to 6	EIU	26 EMs	
(2) Marginal impact of the share of foreign investors on government bond yields	basis point per 1 percent	Forecast data	Countries	Investor type
This study	-11.8 to -8.2	IMF, OECD, EC, EIU	11 AEs and 14 EMs	Foreign Private
This study	-17.9 to -14.5	IMF, OECD, EC, EIU	11 AEs and 14 EMs	Foreign Official
Arslanalp and Poghosyan (2016)	-11.5 to -6.9	IMF	22 AEs	Foreign Private
Arslanalp and Poghosyan (2016)	-6.9 to -5.5	IMF	22 AEs	Foreign Official
Ebeke and Lu (2015)	-10.6 to -6.5	Actual data	13 EMs	Foreign
(3) Estimated tipping points for the impact of the increase in foreign investors	Threshold value of debt to GDP ratio	Forecast data	Countries	Investor type
This study (Baseline)	59 percent (Expected gross public debt)	IMF, OECD, EC, EIU	11 AEs and 14 EMs	Foreign Private
	when the share of foreign investors is 30 percent			
	91 percent (Expected gross public debt)	IMF, OECD, EC, EIU	11 AEs and 14 EMs	Foreign Private
	when the share of foreign investors is 15 percent			
Ebeke and Lu (2015)	90 percent (Lagged external debt)	Actual data	13 EMs	Foreign
Ebeke and Lu (2015)	21.5 percent (Lagged short-term debt)	Actual data	13 EMs	Foreign

Notes: AEs=advanced economies and EMs=emerging markets.

Appendix

A.1 Linearity test

Testing the null hypothesis $H_0 : \beta_0 = \beta_1$ can inform us on the linearity in the GPSTR model (3). However, this test is not standard since, under H_0 , the GPSTR model contains unidentified nuisance parameters (Hansen (1996)). Following Luukkonen et al. (1988) and Terasvirta (1994), we replace the transition function $g(fp_{i,t-1}; \gamma_1, \gamma_2, c)$ with the first-order Taylor expansion around $\gamma_1 = 0$ and $\gamma_2 = 0$. The practical computation follows two steps.

- Step1. The linearized GPSTR model is given by

$$\mathbb{E}_t L_{i,t+n} = \alpha_i + \beta_0 \mathbb{E}_t Debt_{i,t+n} + \beta_1 \mathbb{E}_t Debt_{i,t+n} \cdot T_1(g(fp_{i,t-1}; \gamma_1, \gamma_2, c)) + \delta fp_{i,t-1} + \phi \mathbf{z}_{i,t} + \varepsilon_{i,t} \quad (\text{A.1})$$

where $T_1(g(fp_{i,t-1}; \gamma_1, \gamma_2, c))$ is the first-order Taylor approximation of the transition function $g(fp_{i,t-1}; \gamma_1, \gamma_2, c)$ around $\gamma_1 = 0$ and $\gamma_2 = 0$. This auxiliary regression for testing linearity can be rewritten as

$$\mathbb{E}_t L_{i,t+n} = \alpha_i + \beta_0^* \mathbb{E}_t Debt_{i,t+n} + \beta_1^* \mathbb{E}_t Debt_{i,t+n} \cdot fp_{i,t-1} + \delta fp_{i,t-1} + \phi \mathbf{z}_{i,t} + \varepsilon_{i,t}^* \quad (\text{A.2})$$

The sum of squared residuals is SSR_1 .

- Step2. To test the null hypothesis is $H_0 : \beta_1^* = 0$, the approximate likelihood ratio of H_0 is based on

$$LM_F = TN(SSR_0 - SSR_1) / SSR_0 \quad (\text{A.3})$$

where SSR_0 is the sum of squared residuals of the linear model $\mathbb{E}_t L_{i,t+n} = \alpha_i + \beta_0 \mathbb{E}_t Debt_{i,t+n} + \delta fp_{i,t-1} + \phi \mathbf{z}_{i,t} + \varepsilon_{i,t}$ and SSR_1 is that of the GPSTR model with two regimes.

If a p-value associated with LM_F leads us to reject the null hypothesis, we employ the GPSTR model. ²⁵

A.2 Derivatives of the transition function

The derivative of the transition function $g(fp_{i,t-1}; \gamma_1, \gamma_2, c)$ is needed in order to calculate the sensitivity of the long-term interest rate to the increase in the share of foreign private investors in Table 1.

²⁵González and Teräsvirta (2006) use Monte Carlo testing techniques for testing linearity against smooth transition models instead of being based on an auxiliary regression obtained by replacing the model under the alternative by approximations based on a Taylor expansion.

For $fp_{i,t-1} - c \leq 0$

$$\frac{\partial g(fp_{i,t-1}; \gamma_1, \gamma_2, c)}{\partial fp_{i,t-1}} = \begin{cases} \frac{\exp(-\gamma_1^{-1}(\log(1-\gamma_1|fp_{i,t-1}-c)))}{(1-\gamma_1|fp_{i,t-1}-c)[\exp(-\gamma_1^{-1}(\log(1-\gamma_1|fp_{i,t-1}-c)))+1]^2} & \gamma_1 < 0 \\ \frac{\exp(fp_{i,t-1}-c)}{[1+\exp(fp_{i,t-1}-c)]^2} & \gamma_1 = 0 \\ \frac{\exp(\gamma_1|fp_{i,t-1}-c)\exp(\gamma_1^{-1}(\exp(\gamma_1|fp_{i,t-1}-c)-1))}{[\exp(\gamma_1^{-1}(\exp(\gamma_1|fp_{i,t-1}-c)-1))+1]^2} & \gamma_1 > 0 \end{cases} \quad (\text{A.4})$$

For $fp_{i,t-1} - c \geq 0$

$$\frac{\partial g(fp_{i,t-1}; \gamma_1, \gamma_2, c)}{\partial fp_{i,t-1}} = \begin{cases} \frac{\exp(\gamma_2^{-1}(\log(1-\gamma_2|fp_{i,t-1}-c)))}{(1-\gamma_2|fp_{i,t-1}-c)[\exp(\gamma_2^{-1}(\log(1-\gamma_2|fp_{i,t-1}-c)))+1]^2} & \gamma_2 < 0 \\ \frac{\exp(fp_{i,t-1}-c)}{[1+\exp(fp_{i,t-1}-c)]^2} & \gamma_2 = 0 \\ \frac{\exp(\gamma_2|fp_{i,t-1}-c)\exp(-\gamma_2^{-1}(\exp(\gamma_2|fp_{i,t-1}-c)-1))}{[\exp(-\gamma_2^{-1}(\exp(\gamma_2|fp_{i,t-1}-c)-1))+1]^2} & \gamma_2 > 0 \end{cases} \quad (\text{A.5})$$

Table A.1: Sources and description of the data

Variable names	Description	Sources
Forward interest rate	Zero coupon rate on treasury securities in local currency	Bloomberg and Author's calculations based on Shiller et al. (1983) .
Expected public debt to GDP ratio	2-year-ahead projections for general government debt to nominal GDP	IMF <i>World Economic Outlook, Article IV Consultations</i> , OECD <i>Economic Outlook</i> European Commission <i>European Economic Forecast</i> Economist Intelligence Unit
Expected inflation	2-year-ahead projections for headline inflation	Consensus Economics <i>Consensus Forecast</i> IMF <i>World Economic Outlook, Article IV Consultations</i> ,
Expected growth rate	2-year-ahead projections for real GDP growth rate	OECD <i>Economic Outlook</i> European Commission <i>European Economic Forecast</i> Economist Intelligence Unit
Foreign private or official holdings ratio in emerging markets	Foreign holdings share of central government debt securities denominated in local currency	Author's calculations based on Arslanalp and Tsuda (2014a)
Foreign private or official holdings ratio in advanced economies	Foreign holdings share of general government debt securities denominated in local currency	Author's calculations based on Arslanalp and Tsuda (2014b) and BIS <i>Debt Securities Statistics</i>
Domestic central bank holdings ratio	Domestic central bank holdings share of general government debt securities	Arslanalp and Tsuda (2014a) and Arslanalp and Tsuda (2014b)
Expected exchange rate to US dollar	The rate of change between 2-year-ahead and the current estimated exchange rate	Author's calculations based on Consensus Economics <i>Consensus Forecast</i>
Treasury term premia	Treasury term premia based on Adrian and Crump (2013)	The website of Federal Reserve Bank of New York
Net foreign assets to GDP ratio	International Investment Position: Assets minus liabilities	IMF <i>International Financial Statistics</i> Haver Analytics
Capital control dummy	Capital control for bond inflow and outflow restrictions	Fernández et al. (2016)