TCER Working Paper Series

Welfare analysis of bank merger with financial instability

Akio Ino
Yusuke Matsuki

July 2020

Working Paper E-149
https://www.tcer.or.jp/wp/pdf/e149.pdf

TOKYO CENTER FOR ECONOMIC RESEARCH
1-7-10-703 Iidabashi, Chiyoda-ku, Tokyo 102-0072, Japan

Abstract

In this paper, we analyze the effect of a merger between banks by extending a structural model of banking industry with possibility of bank runs developed by Egan et al. (2017). This allows us to evaluate a merger in the banking sector, taking into account the effect on not only the merged bank itself, but also the stability of the entire financial system. We use our framework to analyse if the merger between Wells Fargo and Wachovia was beneficial to the social welfare. When the model is calibrated to the data in 2008, the merger increases the market share of the merged bank and thus allows it to set higher markup, which implies lower deposit interest rates. Through competition, this lowers the default probability of other banks in normal times. When crisis occurs to banks other than the merged bank, the default probability increases as the merged bank responds to crisis sharply. On the other hand, when the bank run occurs at the merged bank, the default probability is lower because it has higher profits. The merger increases the social welfare in normal times and when a bank run occurs at the merged bank, and decreases the social welfare when a bank run occurs at the other banks.

Akio Ino
Tokyo Foundation for Policy Research
Policy Research
Roppongi Grand Tower 34F, 3-2-1 Roppongi, Minato-ku, Tokyo, 106-6234, Japan
inoakio@gmail.com

Yusuke Matsuki
Nagasaki University
Faculty of Economics
4-2-1 Katafuchi, Nagasaki 850-8506, Japan
ymatsuki77@gmail.com

Welfare analysis of bank merger with financial instability *

Akio Ino†
Yusuke Matsuki‡

July 13, 2020

Abstract

In this paper, we analyze the effect of a merger between banks by extending a structural model of banking industry with possibility of bank runs developed by Egan et al. (2017). This allows us to evaluate a merger in the banking sector, taking into account the effect on not only the merged bank itself, but also the stability of the entire financial system. We use our framework to analyse if the merger between Wells Fargo and Wachovia was beneficial to the social welfare. When the model is calibrated to the data in 2008, the merger increases the market share of the merged bank and thus allows it to set higher markup, which implies lower deposit interest rates. Through competition, this lowers the default probability of other banks in normal times. When crisis occurs to banks other than the merged bank, the default probability increases as the merged bank responds to crisis sharply. On the other hand, when the bank run occurs at the merged bank, the default probability is lower because it has higher profits. The merger increases the social welfare in normal times and when a bank run occurs at the mergerd bank, and decreases the social welfare when a bank run occurs at the other banks.

1 Introduction

The objective of this paper is to develop a framework to study the effect of a merger in the banking sector on the social welfare when there is a possibility that the financial system can be unstable. To achieve this goal, we extend the structural model of imperfect competition in the banking sector with bank run developed by Egan et al. (2017) to include a merger between banks.

*We thank Hiroshi Uchida, Hiroshi Gunji, Iichiro Uesugi, Masaru Konishi, Arito Ono, Masanori Ohkuma, as well as seminar participants at the 13rd regional finance workshop, RIETI, and 2020 Japanese Economic Association Spring Meeting. We thank Tokyo Center for Economic Research for financial support.

†Tokyo Foundation for Policy Research, Roppongi Grand Tower 34F, 3-2-1 Roppongi, Minato-ku, Tokyo, 106-6234, Japan.

‡Faculty of Economics, Nagasaki University, 4-2-1 Katafuchi, Nagasaki 850-8506, Japan.

The objective is important for practical policy issue. Recently, there has been a conflict between the Ministry of Finance and the Fair Trade Commission in Japan about the merger of the eighteenth bank by the Fukuoka Financial Group. The merger was intended to increase the profitability of these banks so that they can survive under the situation where the population of Japanese people keep decreasing. The merger was planned to be implemented in Janurary 2017. However, the Fair Trade Commission is concerned about the higher market share and evaluate the merger proposal very carefully. If these banks are merged, the market share in Nagasaki prefecture will be higher than 70%. Although the merger was eventually implemented in April 2019 after a plan to mitigate the possibly negative effect from a higher market share, this conflict suggests that we need a framework to evaluate if the merger is beneficial to the economy.

We use the estimation result of Egan et al. (2017) which uses the data of large banks in the United States at 2008, and conduct the merger analysis of Wells Fargo and Wachovia. The merger increases the market share of the merged bank and thus allows it to set higher markup, which implies lower deposit rates. Through competition, this lowers the default probability of other banks in normal times. When crisis occurs to banks other than the merged bank, the default probability increases as the merged bank responds to crisis sharply. On the other hand, when the bank run occurs at the merged bank, the default probability is lower because it has higher profits.

1.1 Literature

This paper is related to the literature of structural models of banking. Corbae and D'erasmo (2013) builds a banking industry dynamics model where there are banks with market power. For applications, Corbae and D'Erasmo (2019) used the structural model to analyse the effect of capital requirement, and Corbae et al. (2018) conducted the stress test of banking industry based on the structural model. In addition, Egan et al. (2017) builds a structural model of banking sector with possibilities of bank runs. We contribute to this literature by extending their approach to the analysis of banking mergers.

This paper is also related to the empirical analysis of banking merger. Berger et al. (1999) summarizes the earlier literature. Recently, several studies (Sapienza (2002), Montoriol-Garriga (2008), and Erel (2011), among others) use contract level data of bank loans to study the effect of bank merger on loans. Uchino and Uesugi (2012) studies the effect of the merger between Bank of Tokyo-Mitsubishi and UFJ Bank in 2005 on the availability of funds for firms. Our paper contributes to this literature by developing a structural model of banking mergers for a counterfactual analysis, which is difficult to conduct with observational data. Akkus et al. (2016) estimated the matching function of acquirer and target banks in the merger market. Although their model is also structural, their focus is on the relationship between acquirer and target bank, rather than the merger and its implication on the financial system.

From the methodological point of view, our merger analysis is based on Nevo (2000). There is another paper which evaluate the merger analysis of this style, Bjornerstedt and Verboven (2016). In this paper we focus on the competition in the deposit market, and the deposit demand of this model is specified under BLP (Berry et al. (1995)) framework. Empirical Industrial Organization has developed tools to analyze markets with differentiated products. Here we take merger decision as exogenous. The set of brands and their characteristic are also exogenous. The set of brands does not change after a merger. Only ownership changes. The literature focus on pricing decision and hence markups. Our paper would be one of the first to study the impact of a merger on a default decision and a financial system.

The paper is organized as follows. Section 2 lays out a structural model of imperfect competition in the banking sector with bank runs. Section 3 extends the model to take into account a bank merger. Section 4 describes the calibration procedure of the model parameters. Section 5 discusses how the merger between Wells Fargo and Wachovia affect the equilibrium allocation and social welfare. Section 6 concludes.

2 The model without mergers

We first describe the model without mergers. The model in this paper is based on Egan et al. (2017). Time is discrete with infinite horizon. There are three types of agents, M^I consumers for insured deposit, M^N consumers for uninsured deposit, and K banks. Each bank supplies its own deposit brand. When we introduce mergers into the model, we assume that the merged bank supplies multiple brands. The timing of the model in each period t is as follows.

1. Each bank k sets interest rates for insured and uninsured deposits, $i^I_{k,t}$ and $i^N_{k,t}$.

2. Consumers choose where to fund.

3. Banks invest deposits and the profit shock is realized.

4. Banks choose whether to repay deposits and the coupon on the long term debt, or default.

The model is specified under the risk neutral measure.

From now on, we will specify the behaviour of each agent.

2.1 Consumers

There are M^N consumers for uninsured deposits. They have one unit of resources and need to decide which bank to deposit. Consumer j derives utilities from dealing with bank k as follow:

1. interest rate, $\alpha^N i^N_{k,t}$ where α^N is parameter,

2. $-\gamma(\gamma \geq 0)$ $(\gamma \geq 0)$ when the banks default with probability $\rho_{k,t}$,

3. bank specific fixed effect δ_k^N, and

4. i.i.d utility shock, $\epsilon_{j,k,t}^N$.

So the utility consumer j from bank k at time t is given by

$$u_{j,k,t}^N = \alpha^N i_{k,t}^N - \rho_{k,t}\gamma + \delta_k^N + \epsilon_{j,k,t}^N.$$

Insured depositors have the similar preference but they do not lose utility from default:

$$u_{j,k,t}^I = \alpha^N i_{k,t}^I + \delta_k^I + \epsilon_{j,k,t}^I.$$

where α^N is a parameter. Each consumer j chooses where to deposit by maximizing their utility: $\max_k u_{j,k,t}^N$ or $\max_k u_{j,k,t}^I$.

Assume that $\epsilon_{j,k,t}^i$ is distributed i.i.d Type 1 extreme value. Given the interest rate, the market share of bank k is given by the standard logit from:

$$s_{k,t}^I(i_{k,t}^I, \mathbf{i}_{-k,t}^I) = \frac{\exp(\alpha^I i_{k,t}^I + \delta_k^I)}{\sum_{l=1}^K \exp(\alpha^I i_{l,t}^I + \delta_l^I)}, \tag{1}$$

$$s_{k,t}^N(i_{k,t}^N, \mathbf{i}_{-k,t}^N, \rho_{k,t}, \boldsymbol{\rho}_{-k,t}) = \frac{\exp(\alpha^N i_{k,t}^N - \rho_{k,t}\gamma + \delta_k^N)}{\sum_{l=1}^K \exp(\alpha^N i_{l,t}^N - \rho_{l,t}\gamma + \delta_l^N)} \tag{2}$$

2.2 Banks

There are K banks in this model. We assume that K is exogenously given, and there is no entry and exit into the banking sector. Banks maximize the equity value by competing on deposits. Bank k receives the return on deposit net of non-interest cost, denoted by $R_{k,t} \sim N(\mu_k, \sigma_k)$. Banks need to pay additional costs c_k to serve insured deposit. Let $s_{k,t}^i$ denote the market share of bank k at time t in the market $i = I, N$. Banks have issued a Consol bond in the past, so they need to repay b_k every period. This assumption is needed to ensure that banks may choose to default with positive probabilities.

The profit of bank k at time t is then given by

$$\pi_{k,t} = M^I s_{k,t}^I(R_{k,t} - c_k - i_{k,t}^I) + M^N s_{k,t}^N(R_{k,t} - i_{k,t}^N).$$

At time t, the bank uses the net cash inflow $\pi_{k,t} - b_k$ to pay the dividend (no retained earnings).

If $\pi_{k,t} - b_k < 0$, equity holders can choose to finance the loss or not.

In the case of default, equity holders loose the claim on the future dividend.

At bankruptcy, the bank is sold and the proceed is used to repay the depositors and bondholders. Then exactly the same bank enters into the market. Although this assumption is unrealistic, this ensures that the environment is always stationary so that the computation of equilibria is very simple.

Bank default choice

Banks choose to default if amount of capital injection is larger than the future value:

$$\pi_{k,t} - b_k + \frac{1}{1+r} E_k < 0. \tag{3}$$

The optimal default policy is given by the threshold $\bar{R}_k$:

$$M^I s^I_{k,t}(\bar{R}_k - c_k - i^I_{k,t}) + M^N s^N_{k,t}(\bar{R}_k - i^N_{k,t}) - b_k + \frac{1}{1+r} E_k = 0. \tag{4}$$

After some algebra, the default threshold is give by the solution to

$$\begin{aligned}
&- M^I s^I_{k,t}(\bar{R}_k - c_k - i^I_{k,t}) - M^N s^N_{k,t}(\bar{R}_k - i^N_{k,t}) + b_k = \\
&\frac{1}{1+r}(M^I s^I_{k,t} + M^N s^N_{k,t}) \left[\mu_k - \bar{R}_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right)\right] \left[1 - F(\bar{R}_k)\right].
\end{aligned} \tag{5}$$

2.2.1 Bank interest rate choice

The Bellman equation for bank k is

$$\begin{aligned}
E_k = \max_{i^I_k, i^N_k} \int_{\bar{R}_k}^{\infty} \Big[& M^I s^I_k(i^I_k, \mathbf{i}^I_{-k})(R_k - c_k - i^I_k) \\
& + M^N s^N_k(i^N_k, \mathbf{i}^N_{-k}, \rho_k, \boldsymbol{\rho}^N_{-k})(R_k - i^N_k) \\
& - b_k + \frac{1}{1+r} E_k \Big] dF(R_k).
\end{aligned} \tag{6}$$

We can compute the (conditional) expectation analytically:

$$\begin{aligned}
E_k = \max_{i^I_k, i^N_k} \Big[& M^I s^I_k(i^I_k, \mathbf{i}^I_{-k}) \left(\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - c_k - i^I_k\right) \\
& + M^N s^N_k(i^N_k, \mathbf{i}^N_{-k}, \rho_k, \boldsymbol{\rho}^N_{-k}) \left(\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - i^N_k\right) \\
& - b_k + \frac{1}{1+r} E_k \Big] \left[1 - \Phi\left(\frac{\bar{R}_k - \mu}{\sigma_k}\right)\right].
\end{aligned} \tag{7}$$

The first order conditions with respect to the interest rates are given by

$$\begin{aligned}
i^I_k: \quad 0 =& M^I \frac{\partial s^I_k(i^I_k, \mathbf{i}^I_{-k})}{\partial i^I_k} \left(\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - c_k - i^I_k\right) \\
& - M^I s^I_k(i^I_k, \mathbf{i}^I_{-k}), \\
i^N_k: \quad 0 =& M^N \frac{\partial s^N_k(i^N_k, \mathbf{i}^N_{-k}, \rho^N_k, \boldsymbol{\rho}^N_{-k})}{\partial i^N_k} \left(\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - i^N_k\right) \\
& - M^N s^N_k(i^N_k, \mathbf{i}^N_{-k}, \rho^N_k, \boldsymbol{\rho}^N_{-k}).
\end{aligned}$$

After some algebra, the first order conditions can be written as

$$\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right) - c_k - i_k^I = \frac{1}{\alpha^I (1 - s_k^I(i_k^I, \mathbf{i}_{-k}^I))}, \tag{8}$$

$$\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right) - i_k^N = \frac{1}{\alpha^N (1 - s_k^N(i_k^N, \mathbf{i}_{-k}^N, \rho_k^N, \boldsymbol{\rho}_{-k}^N))}. \tag{9}$$

We can interpret the left hand side of these equations as the expected return on loans minus cost of loans, that is, the loan markup. These equations tells us that in this model, the loan markup is determined by the market share of the bank as well as the sensitivity of depositors to the interest rate.

2.3 Equilibrium

An equilibrium of this model consists of (i) default probabilities ρ_k (ii) default threshold, $\bar{R}_k$ (iii) interest rate on insured and unsired deposits, i_k^I and i_k^N, and (iv) market shares s_k^I and s_k^N $(k = 1, 2, \ldots K)$ such that

1. Consumers choose where to deposit optimally:

$$s_k^I(i_k^I, \mathbf{i}_{-k}^I) = \frac{\exp(\alpha^I i_k^I + \delta_k^I)}{\sum_{l=1}^K \exp(\alpha^I i_l^I + \delta_l^I)}, \tag{10}$$

$$s_k^N(i_k^N, \mathbf{i}_{-k,t}^N, \rho_k, \boldsymbol{\rho}_{-k}) = \frac{\exp(\alpha^N i_{k,t}^N - \rho_k \gamma + \delta_k^N)}{\sum_{l=1}^K \exp(\alpha^N i_{l,t}^N - \rho_{l,t} \gamma + \delta_l^N)} \tag{11}$$

2. Banks choose default threshold optimally:

$$\begin{aligned} & -M^I s_{k,t}^I(\bar{R}_k - c_k - i_{k,t}^I) - M^N s_{k,t}^N(\bar{R}_k - i_{k,t}^N) + b_k = \\ & \frac{1}{1+r}(M^I s_{k,t}^I + M^N s_{k,t}^N) \left[\mu_k - \bar{R}_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right) \right] \left[1 - F(\bar{R}_k) \right] \end{aligned} \tag{12}$$

3. Banks choose interest rates optimally:

$$\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right) - c_k - i_k^I = \frac{1}{\alpha^I (1 - s_k^I(i_k^I, \mathbf{i}_{-k}^I))} \tag{13}$$

$$\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right) - i_k^N = \frac{1}{\alpha^N (1 - s_k^N(i_k^N, \mathbf{i}_{-k}^N, \rho_k^N, \boldsymbol{\rho}_{-k}^N))} \tag{14}$$

4. Rational expectation: consumer's belief about the probability of bank default is correct in equilibrium:

$$\rho_k = P(R_k \leq \bar{R}_k) = \Phi \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right). \tag{15}$$

There are $6K$ equations for $6K$ variables. We can reduce the number of variables to $3K$ by substituting (10), (11), and (15) into (12), (13), and (14). The three reduced equations are employed for calibration and simulation as described later.

2.4 Multiple equilibria

This model can have multiple equilibria due to bank runs depending on the parameter values, especially γ. The intuition is as follow:

1. Uninsured depositors suddenly believe that a bank may default.
2. They reduce the demand of deposit from that bank.
3. Lower demand decreases the profitability of the bank.
4. Lower profitability increases the probability of default.

The bank run affects not only each individual bank, but also the entire financial system. Consider a bad equilibrium where bank k suffers from a sunspot shock. Bank k needs to offer a higher interest rate on uninsured deposit as depositors fear the possibility of default. Bank k will also offer a higher interest rate on insured deposit: it is cheaper than the uninsured deposit because of insurance. Due to competition, other banks need to offer higher interest rates, which leads to higher overall default rates. Because of this structure, this model is suitable to study the effect of a bank merger on the entire financial system.

If uninsured depositors do not care about default ($\gamma = 0$) this does not happen. The question is, how large is γ in data? Egan et al. (2017) estimated γ using the data of the United States, and found that γ is large enough that there are bank run equilibria during the financial crisis of 2008. We use their estimated parameters, so multiple equilibria arises in our simulation as well.

3 Merger analysis

Basic setting

Does bank merger lead to financial stability? In this section, We can quantitatively analyze the effect on a financial system and hence social welfare by modifying the model in the previous section by introducing mergers. Here, we focus on the case where two banks are involved in the merger. Let m denote the index for the merged bank, and m_1, m_2 denote the deposit brand the bank m owns.

To make the analysis tractable, we assume that after the merger the return on loans is equalized at $R_m = \omega R_{m_1} + (1-\omega) R_{m_2}$, $\omega \in [0,1]$, where ω is the weight on the lending technology of bank m_1. After the merger, the merged bank still pays the same insurance cost $\mathbf{c}_m \equiv \begin{bmatrix} c_{m_1}, & c_{m_2} \end{bmatrix}$. The merged bank can also pay different interest rate $\mathbf{i}_m \equiv \begin{bmatrix} i_{m_1}, & i_{m_2} \end{bmatrix}$ for each deposit brand. Then the profit of the merged bank m can be written as

$$\begin{aligned} \pi_m = & [M^I s_m^I(\mathbf{i}^I) + M^N s_m^N(\mathbf{i}^N, \rho)] R_m \\ & - M^I \mathbf{s}_m^I(\mathbf{i}^I) \mathbf{c}_m^T + (M^I \mathbf{s}_m^I(\mathbf{i}^I) + M^N \mathbf{s}_k^N(\mathbf{i}^N, \rho)) \mathbf{i}_m^T \end{aligned} \tag{16}$$

where the joint market share is defined as

$$s_m^j(\mathbf{i}^j,\rho) \equiv s_{m_1}^j(\mathbf{i}^j,\rho) + s_{m_2}^j(\mathbf{i}^j,\rho), \quad j = I, N, \tag{17}$$

and $\mathbf{s}_m$ is a vector collecting the market share of merged banks

$$\mathbf{s}_m^j(\mathbf{i}^j,\rho) = \left[s_{m_1}^j(\mathbf{i}^j,\rho), \quad s_{m_2}^j(\mathbf{i}^j,\rho)\right]. \tag{18}$$

$\mathbf{x}^T$ denote the transpose of the vector $\mathbf{x}$.

In this case, we can apply almost the same analysis in the case without merger except for the interest rate first order conditions. The default threshold is given by the solution to (market share functions omit dependency on interest rate and default probability to shorten the notation.)

$$\begin{aligned} &b_m - [M^I s_m^I + M^N s_m^N]\overline{R}_m + [M^I \mathbf{s}_m^I(\mathbf{c}_m + \mathbf{i}_m^I)^T + M^N \mathbf{s}_m^N(\mathbf{i}_m^N)^T] = \\ &\frac{1}{1+r}(M^I s_m^I + M^N s_m^N)\left[\mu_m - \bar{R}_m + \sigma_m \lambda\left(\frac{\bar{R}_k - \mu_m}{\sigma_m}\right)\right]\left[1 - F(\bar{R}_m)\right]. \end{aligned} \tag{19}$$

Since the default threshold exists, we can evaluate the RHS of the Bellman equation (The Bellman equation for the merged bank m is

$$\begin{aligned} E_m = \max_{\mathbf{i}_m^I,\mathbf{i}_m^N} \int_{\bar{R}_m}^{\infty} \Big[&[M^I s_m^I(\mathbf{i}^I,\rho) + M^N s_m^N(\mathbf{i}^I,\rho)]\bar{R}_m \\ &- M^I \mathbf{s}_m^I(\mathbf{i}^I,\rho)(\mathbf{c}_m + \mathbf{i}_m^I)^T - M^N \mathbf{s}_m^N(\mathbf{i}^I,\rho)(\mathbf{i}_m^N)^T \\ &- b_m + \frac{1}{1+r}E_m\Big] dF(R_k). \end{aligned} \tag{20}$$

We can compute the (conditional) expectation analytically:

$$\begin{aligned} E_m = \max_{\mathbf{i}_m^I,\mathbf{i}_m^N} \Big[&[M^I s_m^I(\mathbf{i}^I) + M^N s_m^N(\mathbf{i}^N,\boldsymbol{\rho})]\left[\mu_m + \sigma_m \lambda\left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right)\right] \\ &- M^I \mathbf{s}_m^I(\mathbf{i}^I,\rho)(\mathbf{c}_m + \mathbf{i}_m^I)^T - M^N \mathbf{s}_m^N(\mathbf{i}^N,\boldsymbol{\rho}^N)(\mathbf{i}_m^N)^T \\ &- b_m + \frac{1}{1+r}E_m\Big]\left[1 - \Phi\left(\frac{\bar{R}_m - \mu}{\sigma_m}\right)\right]. \end{aligned} \tag{21}$$

Note that the merged bank offers two brands, $k = m_1, m_2$. The first order condition with respect to the interest rates are given by

$$\begin{aligned} i_k^I: \quad 0 =& \frac{\partial s_m^I(\mathbf{i}^I)}{\partial i_k^I}\left[\mu_m + \sigma_m \lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right)\right] - \frac{\partial \mathbf{s}_m^I(\mathbf{i}^I)}{\partial i_k^I}(\mathbf{c}_m + \mathbf{i}_m^I)^T \\ &- s_k^I(\mathbf{i}^I), \end{aligned} \tag{22}$$

$$\begin{aligned} i_k^N: \quad 0 =& \frac{\partial s_m^N(\mathbf{i}^N,\boldsymbol{\rho}^N)}{\partial i_k^N}\left[\mu_k + \sigma_k \lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right)\right] - \frac{\partial \mathbf{s}_m^I(\mathbf{i}^N,\boldsymbol{\rho}^N)}{\partial i_k^I}(\mathbf{i}_m^N)^T \\ &- s_k^N(\mathbf{i}^N,\boldsymbol{\rho}^N) \end{aligned} \tag{23}$$

for $k = m_1, m_2$.

In the equation above, we have the additional terms $\frac{\partial s^I_{m_1}(\mathbf{i}^I)}{\partial i^I_{m_2}}$ and $\frac{\partial s^I_{m_2}(\mathbf{i}^I)}{\partial i^I_{m_1}}$ which do not appear in the case without mergers. Because the merged bank now offers two brands, when setting the interest rate it should care about the effect of brand k's interest rate on the other brand's market share.

When there are $K-1$ banks after the merger and K deposit brands are supplied, we have $3 \times (K-2)+2$ equations for $3 \times (K-2)+2$ endogenous variables, $(i^I_k, i^N_k, \rho_k)^{K-2}_{k=1}$ and $\{(i^I_k, i^N_k)_{k=m_1,m_2}, \rho_m\}$. We have only one default choice for the merged bank, although it offers two interest rates to the two deposit brands.

To simplify the notation, we omit the dependency of the market share on the interest rates. Then the FOC with respect to i_{m_1} can be written as

$$\begin{aligned} 0 =& [\alpha s_{m_1}(1-s_{m_1}) - \alpha s_{m_1} s_{m_2}] \left[\mu_m + \sigma_m \lambda \left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right)\right] \\ & - [\alpha s_{m_1}(1-s_{m_1})(c_{m_1}+i_{m_1}) - \alpha s_{m_1} s_{m_2}(c_{m_2}+i_{m_2})] \qquad (24) \\ & - s_{m_1} \\ \Rightarrow \quad \frac{1}{\alpha} =& (1-s_{m_1}-s_{m_2}) \left[\mu_m + \sigma_m \lambda \left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right)\right] \qquad (25) \\ & - [(1-s_{m_1})(c_{m_1}+i_{m_1}) - s_{m_2}(c_{m_2}+i_{m_2})]. \end{aligned}$$

In the same way, the FOC with respect to i_{m_2} is

$$\begin{aligned} \frac{1}{\alpha} =& (1-s_{m_1}-s_{m_2}) \left[\mu_m + \sigma_m \lambda \left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right)\right] \\ & - [-s_{m_1}(c_{m_1}+i_{m_1}) + (1-s_{m_2})(c_{m_2}+i_{m_2})]. \end{aligned} \qquad (26)$$

Subtracting (26) from (25), we get

$$mc_m \equiv c_{m_1} + i_{m_1} = c_{m_2} + i_{m_2}. \qquad (27)$$

Since two brands now have the same marginal cost, we can rewrite the FOC as

$$\begin{aligned} \frac{1}{\alpha} =& (1-s_{m_1}-s_{m_2}) \left[\mu_m + \sigma_m \lambda \left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right)\right] \\ & -(1-s_{m_1}-s_{m_2}) mc_m \end{aligned} \qquad (28)$$

$$\left[\mu_m + \sigma_m \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right)\right] - mc_m = \frac{1}{\alpha(1-s_m)} \qquad (29)$$

So the markup will be larger when the merged bank determine the interest rate jointly because $s_m > s_k$.

We turn to the definition of an equilibrium with a merger. We need to slightly modify the original definition of an equilibrium without merger. Let $k = 1,,2,\ldots,K-2$ be banks without merger.

An equilibrium of this model with a merger consists of (i) default probabilities ρ_k (ii) default threshold, $\bar{R}_k$ (iii) interest rate on insured and uninsured deposits, i^I_k and i^N_k, and (iv) market shares s^I_k and s^N_k $(k = 1,2,\ldots,K-2, m_1, m_2, m)$ such that

1. Consumers choose where to deposit optimally:

$$s_k^I(i_k^I, \mathbf{i}_{-k}^I) = \frac{\exp(\alpha^I i_k^I + \delta_k^I)}{\sum_l \exp(\alpha^I i_l^I + \delta_l^I)},$$
$$s_k^N(i_k^N, \mathbf{i}_{-k,t}^N, \rho_k, \boldsymbol{\rho}_{-k}) = \frac{\exp(\alpha^N i_{k,t}^N - \rho_k\gamma + \delta_k^N)}{\sum_l \exp(\alpha^N i_{l,t}^N - \rho_{l,t}\gamma + \delta_l^N)}$$

for $k = 1, 2, \ldots, K-2, m_1, m_2$, and

$$s_m^I(\mathbf{i}^I) = s_{m_1}^I(\mathbf{i}^I) + s_{m_2}^I(\mathbf{i}^I)$$
$$s_m^N(\mathbf{i}^N, \boldsymbol{\rho}) = s_{m_1}^N(\mathbf{i}^N, \boldsymbol{\rho}) + s_{m_2}^N(\mathbf{i}^N, \boldsymbol{\rho}),$$

2. Banks choose default threshold optimally:

$$-M^I s_k^I(\bar{R}_k - c_k - i_k^I) - M^N s_k^N(\bar{R}_k - i_k^N) + b_k =$$
$$\frac{1}{1+r}(M^I s_k^I + M^N s_k^N)\left[\mu_k - \bar{R}_k + \sigma_k \lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right)\right][1 - F(\bar{R}_k)]$$

for $k = 1, 2, \ldots, K-2, m,$

3. Banks choose interest rates optimally:

$$\mu_k + \sigma_k\lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - c_k - i_k^I = \frac{1}{\alpha^I(1 - s_k^I(i_k^I, \mathbf{i}_{-k}^I))}$$
$$\mu_k + \sigma_k\lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - i_k^N = \frac{1}{\alpha^N(1 - s_k^N(i_k^N, \mathbf{i}_{-k}^N, \rho_k, \boldsymbol{\rho}_{-k}))}$$

for $k = 1, 2, \ldots, K-2$ and

$$\mu_m + \sigma_m\lambda\left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right) = \frac{1}{\frac{\partial s_m^I(\mathbf{i}^I)}{\partial i_k^I}}\left(\frac{\partial \mathbf{s}_m^I(\mathbf{i}^I)}{\partial i_k^I}(\mathbf{c}_m + \mathbf{i}_m^I) + s_k^I(\mathbf{i}^I)\right)$$

$$\mu_m + \sigma_m\lambda\left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right) = \frac{1}{\frac{\partial s_m^N(\mathbf{i}^N, \boldsymbol{\rho}^N)}{\partial i_k^N}}\left(\frac{\partial \mathbf{s}_m^I(\mathbf{i}^N, \boldsymbol{\rho}^N)}{\partial i_k^I}\mathbf{i}_m^N + s_k^N(\mathbf{i}^N, \boldsymbol{\rho})\right).$$

for $k = m_1, m_2$, and

4. Rational expectation: consumer's belief about the probability of bank default is correct in equilibrium:

$$\rho_k = P(R_k \leq \bar{R}_k) = \Phi\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right)$$

for $k = 1, 2, \ldots, K-2, m.$

This system can be simplified as follow. First, since we explicitly write s_k^j as a function of interest rate and default choice, we don't need to include the market share as the definition of an equilibrium. In addition, from the rational expectation, we can write ρ_k as a function of $\overline{R}_k$. Then the resulting system of equations is

An equilibrium of this model with a merger consists of (i) default threshold, $\bar{R}_k$ for $k = 1, \ldots, K-2, m$, (ii) interest rate on insured and uninsured deposits, i_k^I and i_k^N for $k = 1, \ldots, K-2, m_1, m_2$, such that

1. Banks choose default threshold optimally:

$$
\begin{aligned}
-M^I s_{k,t}^I(\bar{R}_k - c_k - i_{k,t}^I) - M^N s_{k,t}^N(\bar{R}_k - i_{k,t}^N) + b_k = \\
\frac{1}{1+r}(M^I s_{k,t}^I + M^N s_{k,t}^N)\left[\mu_k - \bar{R}_k + \sigma_k \lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right)\right]\left[1 - F(\bar{R}_k)\right]
\end{aligned}
$$

for $k = 1, 2, \ldots, K-2, m$,

2. Banks choose interest rates optimally:

$$
\mu_k + \sigma_k \lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - c_k - i_k^I = \frac{1}{\alpha^I(1 - s_k^I(\mathbf{i}^I))}
$$

$$
\mu_k + \sigma_k \lambda\left(\frac{\bar{R}_k - \mu_k}{\sigma_k}\right) - i_k^N = \frac{1}{\alpha^N(1 - s_k^N(\mathbf{i}^N, \boldsymbol{\rho}(\overline{\mathbf{R}})))}
$$

for $k = 1, 2, \ldots, K-2$ and

$$
\mu_m + \sigma_m \lambda\left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right) = \frac{1}{\frac{\partial s_m^I(\mathbf{i}^I)}{\partial i_k^I}}\left(\frac{\partial \mathbf{s}_m^I(\mathbf{i}^I)}{\partial i_k^I}(\mathbf{c}_m + \mathbf{i}_m^I) + s_k^I(\mathbf{i}^I)\right)
$$

$$
\mu_m + \sigma_m \lambda\left(\frac{\bar{R}_m - \mu_m}{\sigma_m}\right) = \frac{1}{\frac{\partial s_m^N(\mathbf{i}^N, \boldsymbol{\rho}^N)}{\partial i_k^N}}\left(\frac{\partial \mathbf{s}_m^I(\mathbf{i}^N, \boldsymbol{\rho}(\overline{\mathbf{R}})}{\partial i_k^I}\mathbf{i}_m^N + s_k^N(\mathbf{i}^N, \boldsymbol{\rho}(\overline{\mathbf{R}}))\right).
$$

for $k = m_1, m_2$.

4 Calibration and simulation

Since we will analyze the merger which hasn't taken place, we will use the observed equilibrium without mergers to calibrate the model. As mentioned, the six equations of equilibrium conditions are reduced to three equations by solving the equilibrium FOC with respect to the supply parameters (c_k, μ_k, σ_k), given the strategic variables determined in equilibrium (i_k^I, i_k^N, ρ_k). We can obtain the closed form solution as

$$
\sigma_k = \frac{\frac{1+r}{M^I s_k^I + M^N s_k^N}(b_k - M^I s_k^I \mathcal{M}_k^I - M^I s_k^N \mathcal{M}_k^N)}{(\rho_k + r)[\tilde{R}_k - \lambda(\tilde{R}_k)]} \tag{30}
$$

$$
\mu_k = i_k^N - \sigma_k \lambda(\tilde{R}_k) + \mathcal{M}_k^N \tag{31}
$$

$$
c_k = (i_k^N + \mathcal{M}_k^N) - (i_k^I + \mathcal{M}_k^I) \tag{32}
$$

Parameter	value	description
α_I	58.79	Depositor sensitivity to interest rate (Insured)
α_N	16.64	Depositor sensitivity to interest rate (Unnsured)
γ	-12.60	Depositor sensitivity of bank default
r	0.05	Discount rate
M^I	4440000000	Insured deposit market size
M^N	4140000000	Uninsured deposit market size
ω	0.439	Weighting parameter for merged lending
b_k	[6547896, 23100000]	Consol bond
μ_k	[0.074, 0.081]	Mean return on loans
c_k	[0.046, 0.055]	Non-interest cost of loans
σ_k	[0.11, 0.29]	Standard error of loan return

Table 1: Parameter values for merger analysis

where $\mathcal{M}_k^j = 1.0/(\alpha_j * (1.0 - s_k^j))$ is the markup on loan type j and $\tilde{R}_k = \Phi^{-1}(\rho_k)$ is the normalized threshold.

If we fix (i_k^I, i_k^N, ρ_k), which are observable from data, we can recover the values of the other parameters, (μ_k, σ_k, c_k). First, in the calibration stage, we estimate the demand parameters using BLP. Once we have determined the demand parameters, the supply parameters can be obtained given the data (observed equilibrium (i_k^I, i_k^N, ρ_k) through the equations above.

For the application later, we use the estimation result from Egan et al. (2017). In their main analysis, they focus on the five largest banks (in terms of deposit shares) in the United states, Bank of America, JP Morgan, Wells Fargo, Citi bank, and Wachovia. They calibrate the model to the data of interest rates and default probability at Match 31th, 2008. The parameter value is summarized in Table 1.

Since we are calibrating to the same data as EHM, most parameter values are the same. One exception is the weighting parameter for merged bank lending ω. This parameter represents the weight of acquirer bank's lending technology to the merged bank's lending technology. We chose ω so that it corresponds to the share of Wells Fargo's lending in the total lending of Wells Fargo and Wachovia before merger, which is 0.439.

In simulation, the calibrated supply parameters (μ_k, σ_k, c_k) and the estimated demand parameters are given. Now we have the three equations with unknown (i_k^I, i_k^N, ρ_k) which possibly have multiple solutions.

To find multiple equilibria, we create a grid over the endogenous variables (i_k^I, i_k^N, ρ_k) and use them as initial guess. This does not guarantee that we can find all the solutions.

Once we calibrate the model and computed multiple equilibria, we can also compute the equilibrium with mergers by using the calibrated parameters (μ_k, σ_k, c_k) and the new first order conditions. Then we can compare the equilibrium with and without mergers, and analyze the effect of the merger on the financial system. Note that it is possible that we are comparing different equilibria because our computational procedure to find equilibria does not guaranteee that we find all the equilibria.

5 Simulation Results

In this section, we present simulation results of a case where a merger between Wells Fargo and Wachovia takes place.

5.1 Effects of mergers on interest rates and default rates

We first compute several equilibria for the case without mergers, and then use these equilibria as an initial guess to compute equilibria with mergers to reduce the possibility that we are comparing different equilibria. See Table 2. We pick up the table 4 of Egan et al. (2017), which displays observed equilibrium, best equilibrium, and bank run equilibrium at each banks.

Table 3 is our simulation results. In the observed equilibrium, as expected, lower interest rates were set, the earnings environment for banks improved, the probability of default fell, and the instability of the financial system declined.

At best equilibrium, the merged banks saw a significant drop in interest rates, but not much change for the rest of the banks. The probability of default also declined for the merged banks, but rose slightly for some banks.

In the equilibrium where the bank run occurs, the results are somewhat complicated. Even though the number of banks is decreasing, we have seen some banks setting higher interest rates.

In the equilibrium where the bank run occurs at Wells Fargo, the set interest rate at Wells Fargo is lower, while the rate at Wachovia, another branch, is considerably higher. At the equilibrium where the bank run occurs at Bank of America, the insured interest rates are higher at the merged banks and lower elsewhere. The uninsured interest rates are particularly large at the merged banks. In the equilibrium where the bank run occurs at JPMorgan, interest rates are higher at the merged banks, while at Bank of America and Citi, interest rates are lower. The default probability is smaller. Finally, in Citi's case, interest rates are lower at JPMorgan and Bank of America and higher at the combined bank; there is no change in Citi's rate setting. The default probability is lower for all but Citi.

Overall, interest rates will change in a variety of ways while the probability of default will be lower. This result roughly supports the claim that mergers reduce instability in the financial system. It should be noted, however, that if a bank run occurs at a merged bank, the probability of default for all banks will increase, and the impact will be particularly pronounced at the merged bank.

When the bank run occurs, typically the interest rates and default rates are higher under the equilibrium with the merger. When the market share of the merged bank is high, it responds to a change in the interest of the other bank more aggressively [1]:

$$\frac{\partial}{\partial i_k} s_m(\mathbf{i}) = -\alpha s_m(\mathbf{i}) s_k(\mathbf{i}) \tag{33}$$

As a result, due to higher market share, the merged bank sets a higher interest rate to keep its market share, and through competition, this increases the entire interest rates and default rates.

5.2 Effects of mergers on welfare

Once we have computed the equilibrium with and without mergers, we can compute the social welfare to evaluate if the merger is beneficial to the society. Note that we can only evaluate the social welfare for each equilibrium because we don't have any information about the likelihood of which equilibrium arises. The social welfare in this model is the sum of consumer surplus, producer surplus which is the value of banks, and the cost of deposit insurance.

Following chapter 3 of Train (2009), under the assumption that the error term follows i.i.d extreme distributions, we can write the consumer surplus as

$$CS = \frac{M^I}{\alpha^I} \ln \left[\sum_{l=1}^{K} \exp(\alpha^I i_l^I + \delta_l^I) \right] + \frac{M^N}{\alpha^N} \ln \left[\sum_{l=1}^{K} \exp(\alpha^N i_l^N + \delta_l^N + \gamma \rho_l) \right] \tag{34}$$

The annualized equity value of banks is given by

$$AEV = \sum_{l=1}^{K} r E_l. \tag{35}$$

Assuming a 40% recovery rate, the expected FDIC insurance cost is

$$EC = 0.6 \sum_{l=1}^{K} \rho_l M^I s_l^I. \tag{36}$$

Then the change in welfare can be computed as

$$\Delta W = \Delta CS + \Delta AEV - \Delta EC. \tag{37}$$

The result is shown in Table 4. Social welfare has risen in all equilibrium. While the effects of mergers vary by equilibrium, and lower interest rates may reduce consumer surplus, the positive effects of mergers on bank values and insurance costs are outweighed.

[1] The derivation of this equation is in appendix A.1.

6 Conclusion

In this paper, we extend the structural model of banking industry with possibility of bank runs to allow mergers between banks, and use it to analyse if the merger between Wells Fargo and Wachovia was beneficial to the social welfare. The merger increases the market share of the merged bank and thus allows it to set higher markup, which implies lower deposit rates. Through competition, this lowers the default probability of other banks in normal times. When crisis occurs to banks other than the merged bank, the default probability increases as the merged bank responds to crisis sharply. On the other hand, when the bank run occurs at the merged bank, the default probability is lower because it has higher profits.

So far, we have assumed that the merger is just a one time event has no future effect. However, current policy decision of whether or not the government approve the merger may affect the future merger decision as in Nocke and Whinston (2010). Taking this consideration into account is a future research.

Bank name	Obs. eqm	Best	Bank run at Wells Fargo	Bank of America	JP Morgan	Citi
Insured interest rate						
JP Morgan	1.73	0.98	2.46	2.65	10.48	3.17
Bank of America	1.98	1.53	2.13	7.34	2.44	2.46
Wells Fargo	2.13	2.05	10.05	3.06	3.57	3.68
Citi	2.23	2.11	3.01	3.21	3.72	12.26
Wachovia	2.08	2.04	2.59	2.62	2.93	2.98
Uninsured interest rate						
JP Morgan	1.73	0.94	2.41	2.56	20.35	3.02
Bank of America	1.97	1.4	1.94	11.43	2.23	2.24
Wells Fargo	2.32	2.25	17.41	3.21	3.71	3.81
Citi	2.23	2.13	2.94	3.09	3.52	24.35
Wachovia	2.23	2.19	2.67	2.71	3.00	3.04
Default probability						
JP Morgan	1.5	0.19	2.86	3.29	48.35	4.36
Bank of America	1.82	0.03	1.85	53.33	3.27	3.40
Wells Fargo	1.5	1.34	46.61	3.56	4.81	5.06
Citi	2.11	1.92	3.36	3.74	4.62	48.19
Wachovia	3.28	3.14	4.75	4.92	5.96	6.13

Table 2: Equilibria without mergers (%) from Egan et al. (2017)

Bank name	Obs. eqm	Best	Bank run at Wells Fargo	Bank of America	JP Morgan	Citi
Insured interest rate						
JP Morgan	1.66	1.0	2.62	2.22	10.48	2.47
Bank of America	1.96	1.52	2.24	7.33	1.85	1.57
Wells Fargo	1.06	1.06	7.37	3.32	3.97	4.13
Citi	2.16	2.05	3.18	2.61	2.91	12.26
Wachovia	1.11	1.1	7.41	3.37	4.02	4.17
Uninsured interest rate						
JP Morgan	1.66	0.98	2.53	2.35	20.39	2.83
Bank of America	1.96	1.41	2.08	11.43	1.81	1.65
Wells Fargo	0.9	0.92	11.66	3.28	3.95	4.06
Citi	2.17	2.07	3.06	2.85	3.28	24.41
Wachovia	0.9	0.92	11.66	3.28	3.95	4.06
Default probability						
JP Morgan	1.36	0.23	3.21	2.44	48.34	3.68
Bank of America	1.77	0.03	2.47	53.33	2.59	2.74
Wells Fargo	0.0	0.0	50.39	0.45	1.27	1.64
Citi	2.0	1.82	3.67	2.96	3.95	48.19
Wachovia	0.0	0.0	50.39	0.45	1.27	1.64

Table 3: Equilibria with mergers (%)

			Bank run at			
Bank name	Obs. eqm	Best	Wells Fargo	Bank of America	JP Morgan	Citi
Without mergers						
Insurance Cost	13.7	9.0	1080.8	979.3	1085.5	1117.3
Social Welfare	0.0	19.53	-1143.11	-1205.73	-1333.02	-1365.18
With mergers						
Insurance Cost	6.9	1.2	940.7	962.5	1074.1	1109.2
Social Welfare	7.92	23.14	-1149.34	-1125.49	-1255.22	-1295.08

Table 4: Welfare of each equilibrium (Billion dollars). The social welfare is relative to the observed equilibrium without mergers.

References

Akkus, O., J. A. Cookson, and A. Hortaçsu (2016): "The determinants of bank mergers: A revealed preference analysis," *Management Science*, 62, 2241–2258.

Berger, A. N., R. S. Demsetz, and P. E. Strahan (1999): "The consolidation of the financial services industry: Causes, consequences, and implications for the future," *Journal of Banking & Finance*, 23, 135–194.

Berry, S., J. Levinsohn, and A. Pakes (1995): "Automobile prices in market equilibrium," *Econometrica*, 63, 841–890.

Bjornerstedt, J. and F. Verboven (2016): "Does merger simulation work? evidence from the swedish analgesics market," *American Economic Journal: Applied Economics*, 8, 125–64.

Corbae, D. and P. D'Erasmo (2019): "Capital requirements in a quantitative model of banking industry dynamics," *mimeo*.

Corbae, D. and P. D'erasmo (2013): "A quantitative model of banking industry dynamics," *mimeo*.

Corbae, D., P. D'Erasmo, S. Galaasen, A. Irarrazabal, and T. Siemsen (2018): "Structural stress tests," *mimeo*.

Egan, M., A. Hortaçsu, and G. Matvos (2017): "Deposit competition and financial fragility: Evidence from the us banking sector," *American Economic Review*, 107, 169–216.

Erel, I. (2011): "The effect of bank mergers on loan prices: Evidence from the United States," *The Review of Financial Studies*, 24, 1068–1101.

Montoriol-Garriga, J. (2008): "Bank mergers and lending relationships," *ECB Working paper*.

Nevo, A. (2000): "Mergers with differentiated products: The case of the ready-to-eat cereal industry," *The RAND Journal of Economics*, 395–421.

Nocke, V. and M. D. Whinston (2010): "Dynamic merger review," *Journal of Political Economy*, 118, 1200–1251.

Sapienza, P. (2002): "The effects of banking mergers on loan contracts," *The Journal of Finance*, 57, 329–367.

Train, K. E. (2009): *Discrete choice methods with simulation*, Cambridge University Press.

Uchino, T. and I. Uesugi (2012): "The effects of a megabank merger on firm-bank relationships and borrowing costs," *mimeo*.

A Derivations

A.1 The case without mergers

In this section we derive the important equations.

First, we derive the expectation in the Bellman equation (7), which is necessary to derive the first order condition with respect to i_ks, (13) and (14).

The Bellman equation is written as

$$E_k = \max_{i_k^I, i_k^N} \int_{\bar{R}_k}^{\infty} \Big[M^I s_k^I(i_k^I, \mathbf{i}_{-k}^I)(R_k - c_k - i_k^I) + M^N s_k^N(i_k^N, \mathbf{i}_{-k}^N, \rho_k, \boldsymbol{\rho}_{-k}^N)(R_k - i_k^N) - b_k + \frac{1}{1+r} E_k \Big] dF(R_k). \tag{38}$$

The conditional expectation of R_k given $R > \overline{R}_k$ can be written as

$$E[R_k | R_k \geq \overline{R}_k] = \mu_k + \sigma_k \lambda \left(\frac{\overline{R}_k - \mu_k}{\sigma_k} \right) \tag{39}$$

where $\lambda(\cdot) \equiv \phi(\cdot)/\Phi(\cdot)$ is the Mills ratio and ϕ and Φ are the pdf and cdf of a normal random variable.

Since $\int_{\overline{R}_k}^{\infty} f(R_k) dF(R_k) = E[f(R_k) | R_k \geq \overline{R}_k] \times P(R_k \geq \overline{R}_k)$ and $P(R_k \geq \overline{R}_k) = 1 - F(\overline{R}_k) = 1 - \Phi\left(\frac{\bar{R}_k - \mu}{\sigma_k}\right)$, the whole conditional expectation can be written as

$$\begin{aligned} E_k = \max_{i_k^I, i_k^N} \Big[& M^I s_k^I(i_k^I, \mathbf{i}_{-k}^I) \left(\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right) - c_k - i_k^I \right) \\ & + M^N s_k^N(i_k^N, \mathbf{i}_{-k}^N, \rho_k, \boldsymbol{\rho}_{-k}^N) \left(\mu_k + \sigma_k \lambda \left(\frac{\bar{R}_k - \mu_k}{\sigma_k} \right) - i_k^N \right) \\ & - b_k + \frac{1}{1+r} E_k \Big] \left[1 - \Phi \left(\frac{\bar{R}_k - \mu}{\sigma_k} \right) \right]. \end{aligned} \tag{40}$$

We can solve (4) with respect to $E_k/(1+r)$ to obtain

$$\frac{1}{1+r} E_k = -M^I s_{k,t}^I (\bar{R}_k - c_k - i_{k,t}^I) - M^N s_{k,t}^N (\bar{R}_k - i_{k,t}^N) + b_k. \tag{41}$$

Substituting (41) into the Bellman equation, we obtain

$$\begin{aligned} &(1+r)[b_k - M^I s_{k,t}^I (\bar{R}_k - c_k - i_{k,t}^I) - M^N s_{k,t}^N (\bar{R}_k - i_{k,t}^N)] = \\ &\quad = \left[M^I s_k^I \left(\mu_k - \overline{R}_k + \sigma_k \lambda \left(\frac{\overline{R}_k - \mu_k}{\sigma_k} \right) \right) + M^N s_k^N \left(\mu_k - \overline{R}_k + \sigma_k \lambda \left(\frac{\overline{R}_k - \mu_k}{\sigma_k} \right) \right) \right] \\ &\qquad \times \left[1 - \Phi \left(\frac{\overline{R}_k - \mu_k}{\sigma_k} \right) \right] \end{aligned} \tag{42}$$

$$\Rightarrow \quad b_k - M^I s^I_{k,t}(\bar{R}_k - c_k - i^I_{k,t}) - M^N s^N_{k,t}(\bar{R}_k - i^N_{k,t})$$
$$= \frac{1}{1+r}[M^I s^I_k + M^N s^N_k]\left(\mu_k - \bar{R}_k + \sigma_k \lambda\left(\frac{\overline{R}_k - \mu_k}{\sigma_k}\right)\right) \times \left[1 - \Phi\left(\frac{\overline{R}_k - \mu_k}{\sigma_k}\right)\right] \tag{43}$$

We can also substitute (41) to the RHS of the Bellman equation to obtain

$$\begin{aligned} E_k = E\Big[& M^I s^I_k \left[\left(R_k - c_k - i^I_k\right) - \left(\bar{R}_k - c_k - i^I_{k,t}\right)\right] \\ & + M^N s^N_k \left[\left(R_k - i^N_k\right) - \left(\bar{R}_k - i^N_{k,t}\right)\right] \\ & - b_k + b_k | R_k \geq \overline{R}_k \Big] P(R_k \geq \overline{R}_k) \end{aligned} \tag{44}$$

$$= E\Big[M^I s^I_k \left(R_k - \bar{R}_k\right) + M^N s^N_k \left(R_k - \bar{R}_k\right) | R_k \geq \overline{R}_k\Big] P(R_k \geq \overline{R}_k) \tag{45}$$

Then we can plug in (45) to (41) to obtain the same result.

The market share of the bank k is given by

$$s_k(i_k, \mathbf{i}_{-k}) = \frac{\exp(\alpha i_k + \delta_k)}{\sum_{l=1}^{I} \exp(\alpha_l i_l + \delta_l)}. \tag{46}$$

Then the derivative of the logistic function is

$$\frac{\partial}{\partial i_k} s_k(i_k, \mathbf{i}_{-k}) = \frac{\alpha \exp(\alpha i_k + \delta_k)[\sum_{l=1}^{I} \exp(\alpha i_l + \delta_l)] - \alpha \exp(\alpha i_k + \delta_k)^2}{[\sum_{l=1}^{I} \exp(\alpha i_l + \delta_l)]^2} \tag{47}$$

$$= \frac{\alpha \exp(\alpha i_k + \delta_k)}{\sum_{l=1}^{I} \exp(\alpha i_l + \delta_l)} \frac{\sum_{l \neq k}^{I} \exp(\alpha i_l + \delta_l)}{\sum_{l=1}^{I} \exp(\alpha i_l + \delta_l)} \tag{48}$$

$$= \alpha s_k(i_k, \mathbf{i}_{-k})[1 - s_k(i_k, \mathbf{i}_{-k})]. \tag{49}$$

For $m \neq k$,

$$\frac{\partial}{\partial i_m} s_k(i_k, \mathbf{i}_{-k}) = \frac{-\alpha \exp(\alpha i_k + \delta_k) \exp(\alpha i_m + \delta_m)}{[\sum_{l=1}^{I} \exp(\alpha_l i_l + \delta_l)]^2} \tag{50}$$

$$= -\alpha \frac{\exp(\alpha i_k + \delta_k)}{\sum_{l=1}^{I} \exp(\alpha i_l + \delta_l)} \frac{\exp(\alpha i_m + \delta_m)}{\sum_{l=1}^{I} \exp(\alpha i_l + \delta_l)} \tag{51}$$

$$= -\alpha s_k(i_k, \mathbf{i}_{-k}) s_m(i_m, \mathbf{i}_{-m}). \tag{52}$$

(You can find the same calculation in page 59 of Train (2009).)

From these arguments, if we take a derivative of the market share of the merged bank s_m with respect to the interest rate of one of its brand i_{m_1}, we obtain

$$\frac{\partial s^j_m}{\partial i^j_{m_1}} = \frac{\partial}{\partial i^j_{m_1}} (s^j_{m1} + s^j_{m2}) \tag{53}$$

$$= \alpha s_{m_1}[1 - s_{m_1}] - \alpha s_{m_2} s_{m_1} \tag{54}$$

$$= \alpha s_{m_1}(1 - s_{m_1} - s_{m_2}) = \alpha s_{m_1}(1 - s_m). \tag{55}$$

We can use the same argument to conclude that $\frac{\partial s_m}{\partial s_{m_2}} = \alpha s_{m_2}(1 - s_m)$.

A.2 The case with mergers

Most analysis can be applied in the same way as in the case without mergers. The difference is that now the mean and variance of return depends on the interest rates. The Bellman equation corresponding is

$$\begin{aligned} E_m = E\Big[&[M^I s_m^I + M^N s_m^N] R_m \\ &- M^I \mathbf{s}_m^I (\mathbf{c}_m + \mathbf{i}_m^I)^T - M^N \mathbf{s}_m^N (\mathbf{i}_m^N)^T \\ &- b_m + \frac{1}{1+r} E_m | R_m \geq \overline{R}_m \Big] P(R_m \geq \overline{R}_m) \end{aligned} \tag{56}$$

The default threshold is given by

$$\frac{1}{1+r} E_m = b_m - [M^I s_m^I + M^N s_m^N] \overline{R}_m + [M^I \mathbf{s}_m^I (\mathbf{c}_m + \mathbf{i}_m^I)^T + M^N \mathbf{s}_m^N (\mathbf{i}_m^N)^T]. \tag{57}$$

We can substitute (57) into the RHS of (56) to obtain

$$E_m = E\Big[[M^I s_m^I + M^N s_m^N](R_m - \overline{R}_m) | R_m \geq \overline{R}_m\Big] P(R_m \geq \overline{R}_m). \tag{58}$$

Substituting this into (56), we get

$$\begin{aligned} &b_m - [M^I s_m^I + M^N s_m^N] \overline{R}_m + [M^I \mathbf{s}_m^I (\mathbf{c}_m + \mathbf{i}_m^I)^T + M^N \mathbf{s}_m^N (\mathbf{i}_m^N)^T] \\ =& \frac{1}{1+r} [M^I s_m^I + M^N s_m^N] \left(\mu_k - \bar{R}_m + \sigma_m \lambda \left(\frac{\bar{R}_m - \mu_m}{\sigma_m} \right) \right) \times \left[1 - \Phi \left(\frac{\overline{R}_m - \mu_m}{\sigma_m} \right) \right] \end{aligned} \tag{59}$$

FOCs with merger and single technology: For $k = m_1, m_2$,

$$\begin{aligned} i_k^I: \quad 0 =& \frac{\partial s_m^I(\mathbf{i}^I)}{\partial i_k^I} \left[\mu_m + \sigma_m \lambda \left(\frac{\bar{R}_m - \mu_m}{\sigma_m} \right) \right] - \frac{\partial \mathbf{s}_m^I(\mathbf{i}^I)}{\partial i_k^I} (\mathbf{c}_m + \mathbf{i}_m^I)^T \\ & - s_k^I(\mathbf{i}^I), \\ i_k^N: \quad 0 =& \frac{\partial s_m^N(\mathbf{i}^N, \boldsymbol{\rho}^N)}{\partial i_k^N} \left[\mu_m + \sigma_m \lambda \left(\frac{\bar{R}_m - \mu_m}{\sigma_k} \right) \right] - \frac{\partial \mathbf{s}_m^I(\mathbf{i}^N, \boldsymbol{\rho}^N)}{\partial i_k^I} (\mathbf{i}_m^N)^T \\ & - s_k^N(\mathbf{i}^N, \boldsymbol{\rho}^N). \end{aligned}$$

B Weighting parameter for merged bank : actual market share

In section 4 we calibrate the share of Wells Fargo's return in the merged bank's return, ω. We used the following equation for the calibration

$$\omega = \frac{s^I_{WF} M^I + s^N_{WF} M^N}{(s^I_{WF} + s^I_{Wachovia}) M^I + (s^N_{WF} + s^N_{Wachovia}) M^N} \tag{60}$$

where we use the data for the market share and aggregate deposits, (s^I_k, s^N_k, M^I, M^N).