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## Abstract

The evolutionary interests of father and son are eternally intertwined. Their trust in and concern for each other is natural. However, fathers across cultures complain that their sons do not listen to them, while their sons lament their lack of understanding. This paper illustrates a rational mechanism that results in this contradiction. It studies the interactive dynamics of a pair of an ordinary father and son. The son aspires to achievements and expects the father's good judgment; the father enjoys the son's success and suffers with his failure. It is shown that these ordinary properties suffice to generate a steady-state where the son, on average, consults outsiders more often. Interest bond gives no advantage to parental guidance over outsiders' endorsement in influencing the son's decisions. To enhance the father-and-son partnership, a father may transmit suggestions via trustworthy outsiders so that his useful advice can go around the son's disappointment with him and get a proper listening.

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# On the Enterprise of Self-establishment

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## Abstract

The evolutionary interests of father and son are eternally intertwined. Their trust in and concern for each other is natural. However, fathers across cultures complain that their sons do not listen to them, while their sons lament their lack of understanding. This paper illustrates a rational mechanism that results in this contradiction. It studies the interactive dynamics of an ordinary pair of father and son. The son aspires to achievements and expects the father's good judgement; the father enjoys the son's success and suffers with his failure. It is shown that these traits suffice to generate a steady-state where the son, on average, consults outsiders more often than consult his father. Interest bond gives no advantage to parental guidance over outsiders' endorsement in the son's venture decisions. To enhance the partnership with his son, a father may transmit suggestions via trustworthy outsiders so that his useful advice can surpass the son's disappointment with him and get a proper listening.

**Keywords:** father; son; outsiders; venture; population average probability of success; consulting; consenting; regret.

JEL code: C70, D70

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# On the Enterprise of Self-establishment

## I. Introduction

Most human sons grow up and establish social status with resources mainly contributed by their fathers.<sup>2</sup> Parental support often continues into to the early thirties of a son (Rosenzweig and Wolpin, 1993). Meanwhile, a father's happiness is inseparable from that of his sons because they carry his genes or surname or both. This inherent bond motivates him to invest in his sons. Case et al.(2000) demonstrates the power of genetic connection in motivating a parent to nurture the children. With respect to a man's concern for the bearers of his family name, we need only to observe that granted opportunities, most people want their good names to be kept alive in human society. Even a father with pure altruism toward his sons would like to guide them to work hard (Lindbeck and Nyberg 2006).

If a son is successful, he attains good returns to his father's investment in him. The wellbeing of both him and his father increases. In case the son commits crimes or incurs debts, the father cannot exonerate himself from the legal, emotional, and financial consequence. Despite their unbreakable bond and intertwined evolutionary interests, fathers all over the world complain that their sons do not listen to them but to outsiders and sons lament their fathers' lack of understanding. Where does such mutual complaint come from? Bonded by their connected wellbeing, why aren't father and son best partners and confidants in the son's exploring journey?

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<sup>2</sup>The analysis of this paper applies to any parent-child interaction in which children aspire to accomplishment and parents exert active parenting. As will become clear in the ensuing elaboration, the father-and-son interaction is emphasized only because historically sons obtain much more parental attention than daughters in most societies.

This study uses an infinitely-repeated extensive form game with incomplete information to investigate this issue. The goal is to provide a rational mechanism for understanding this paradoxical interaction between father and son. It is shown that a son's aspiration for achievement and expectation of his father's good advice suffice to breed mutual complaints. Even though their interests are inseparable and they resemble each other more than outsiders, a son tends to confide with outsiders more than with his father. As long as human children are born to have ambition, this defect of the father-and-son partnership will persist through generations to come. The relieving remedy is to clad fathers' opinions in friends' costume.

Specifically, this paper analyzes the interactive dynamics between an ordinary pair of father and son. The son longs to prove himself via independent achievements. A vivid example is illustrated by the migration history addressed by Grange and Renard(2001). Nevertheless, he is only one of the multitude. His chance of success is subject to the population average of his cohort. Uncertain about own fortune and confident on his father's solicitude, he expects good advice from the father. As for the father, he likes the son to expand their dominion. However, since their preferences do not perfectly match his willingness to share the potential costs of the son's venture is limited.

Although usually starting out with high self-regards( See, e.g., Justin and Dunning (1999), Van den Steen (2004), and Zájbojník(2004)), a son will inevitably encounter failures in his exploring journey. Each encounter of failure generates regret in both the son and his father. The father is disappointed because the prosperity of his offspring and thus the worth of his investment is threatened. The son regrets because the advice from his most reliable consultant is not as good as expected. Their mutual appraisal is eroded. As a result, the father raises his consenting threshold when consulted; and the

son discounts the benefit of consulting the father while contemplating a new venture. The eroding effect on mutual confidence intensifies with each frustration. Eventually, the range of the population probability of success of an intended venture over which the son wants to consult the father shrinks to its limit. Thereafter, the son elicits advice from outsiders more often than consults the father. Frequency of consulting the father never diminishes to zero only because the father's solicitude helps relieve the pain of failure.

From ancient time to modern days, kings and emperors pass thrones to their sons if only they have one. Commoners bequeath most assets to sons but barely show generosity towards daughters. Even our contemporaries place much more expectation on their sons than daughters. A president with sons vigilantly guides them to become future leaders and leaves to the Creator's decision what his daughters would become. Anticipating their sons to glorify the family someday, ordinary folks with scarce resources invest all of them in their sons; meanwhile they require daughters to facilitate the prospective success of their male siblings by all means. This is still a common practice in Asia.

As basic as nutrition need, Duncan Thomas( 1994) shows that a father tends to divert resources under his control more to his sons than daughters. Moreover, as shown by Aughinbaugh (2000) and Mazumder (2005), the earnings of a father and his son have a correlation ranging from 0.4 to 0.6. However, such a significant correlation has no counterpart in the earnings of parents and daughters. All these indicate that the bonds between fathers and sons are far more important economically and socially than other parental relations with children. Hence this analysis develops along the interaction of father and son. However, the results of this analysis apply to any case where a child

aspire to accomplishment and its parents actively participates the enterprise.

Repetti (1996) shows that boys in fourth to sixth grades are likely to vent their daily academic or social frustration in the form of aversive behavior at home. And more often than their mothers, their fathers would be irritated by these behaviors and execute punishments. This evidence shows the importance of incorporating possible mutual resentment in the analysis of father-and-son interaction. Moreover, despite their inherent comrades spirits, the preferences of father and son may not perfectly match. Interest conflict cannot be ignored. In ancient dynasties, the succession disputes sometimes lead to filicide and patricide. Similar family tragedies are repeated in modern societies across cultures and continents. Although no concern is given to those fatal extremes, this model illustrates a way in which mutual resentment between father and son accumulates over time.

Furthermore, as documented by Whitbeck et al.(1997) a father's response to a son's proposal may be influenced by his own experiences . This model accounts for this factor by allowing the father's consenting threshold regarding the son's intended venture to depend on his subjective assessment. Wirl(1999) models the relation between father and son in a static principal-agent framework to study the effect of transferring costs on the fulfillment of their common objective. Besides focusing on a different issue with a different approach, this dynamic model lets both the father and son make decisions with subjective bias, while Wirl's model allows only the agent (not the principal) to have subjective bias. Moreover, father and son have identical objective in his model, whereas their preferences are allowed to diverge in this model.

As synthesized in Collins et al. (2000), the competition between parenting and peer

influence along the process of growing up has long been recognized by psychologists. One consensus in the literature is that “...*peer influence often operates with respect to everyday behaviors and transient attitudes, not enduring personality traits or values.*” However, the realization of one’s value is composed by his daily conducts. If a young person adheres to his parents’ value, why does’nt he follow his parents’ instruction rather than rely on his friends’ confirmation? Or indeed he does and his parents’ complaint that he does not listen to them is just a matter of perspective? This paper provides the first formal mechanism based on rational choice to explain the seemingly paradox in the interaction of parents and children.

Following this introduction, Section II delineates the model. The analysis of the concerned issues and the implication of main findings are presented in Section III. Brief remarks are given in Section VI.

## **II. The Model**

At some point in his development, a son (S) starts venturing beyond the guidance of his father (F). He derives satisfaction from carrying out own ideas. He may still consult his father but no longer regards him as the director; now F is only one of the possible sources from which he elicits support. However, F is distinguished from outsiders(O) because his wellbeing cannot be completely detached from that of S. Denote the period when S attempts his first venture as period 1. Afterwards, he initiates a sequence of ventures over an indefinite horizon. Let a period  $t$ ,  $t = 1, 2, \dots, \infty$ , represent the time span from the contemplation of a venture up to the disclosure of its outcome.

For each considered venture, the Nature(N) pre-determines the population probability of success,  $0 < l_t < 1$ . It is commonly known by both F and S. In other words,



the father and son have common prior over chance moves. Uncertain about own luck in a venture, S seeks confirmation either from F or outsiders (O) before proceeding to actions. When consulted, O simply gives agreeable support, while F decides whether to consent (C) or prohibit (P) the proposed venture. If the venture is not prohibited by F, S carries out the venture; and its outcome is disclosed. Unlike the uniqueness of F, outsiders(O) are many, they have no authority to prohibit S from a venture and are not bound to share his success or failure; hence the characteristics of O as well as the interaction history between S and O are assumed to be irrelevant for their new encounter.

At their respective times to move, S and F calculate the expected payoff of each feasible action based on the population probability of success of the intended venture  $l_t$  and his experiences with his comrade over previous ventures. Since there is no available new information before taking action, an agent's belief about the other agent's characteristics is fixed over a venture. In equilibrium, every agent is required to choose the action that leads to the highest expected payoff at each turn for action. That is, an agent's actions must be sequentially rational given the probability of chance move and his belief about the other agent's likely action.

If S consults O, he carries out the venture without F's involvement. It is assumed that O never objects a venture conceived by S. In fact, as remarked in the conclusion of Kruger and Dunning (1999), O always gives agreeable opinions. If the venture succeeds, S enjoys own achievement and gets his highest satisfaction  $a_{4t}$ ; if it fails, S bears the pain alone and gets the lowest payoff  $a_{1t} < 0$ . If S consults F and obtains his consent, he ventures with the support of F. If he succeeds, sharing the glory with F, he gets payoff  $a_{3t}$  with  $0 < a_{3t} < a_{4t}$ ; and if he fails, bearing the pain with F, he gets payoff

$a_{2t}$  with  $a_{1t} < a_{2t} < 0$ . However, S may miss the actual mood of F and gets prohibited from the venture after consulting F. In this case, S obtains 0 payoff.

As for F, he gets his highest payoff  $b_{4t}$  from sharing his son's success, and the second highest payoff  $b_{3t}$  with  $0 < b_{3t} < b_{4t}$  if he celebrates the success as a spectator. While deterring S from a precarious venture exempts F from potential cost of failure, it also incurs conflict with S. Hence we assume prohibiting a venture gives F a payoff  $b_{2t} < 0$ , which is the same as observing S's failure as a bystander and higher only than that of sharing the blame of S's failure,  $b_{1t}$ , that is,  $b_{1t} < b_{2t} < 0$ .

For all outcomes jointly played out by N, S, F and O, the resulting payoff profiles are  $A_t \times B_t = \{(a_{it}, b_{jt}) | i, j = 1, 2, 3, 4 \text{ and } t = 1, 2, \dots, \infty, \}$  where  $A_t = \{a_{1t}, a_{2t}, 0, a_{3t}, a_{4t}\}$ , with  $a_{1t} < a_{2t} < 0 < a_{3t} < a_{4t}$ , is the set of the possible payoffs of S; and  $B_t = \{b_{1t}, b_{2t}, b_{3t}, b_{4t}\}$ , with  $b_{1t} < b_{2t} < 0 < b_{3t} < b_{4t}$ , the possible payoffs of F.

Formally, as S contemplates a venture in period  $t$ , the nature N first determines the population probability of its success according to the function

$$X_t : \Omega \longrightarrow \{L, U\} \text{ such that}$$

$$Prob.([\omega \in \Omega : X_t(\omega) = L]) = l_t \text{ and } Prob.([\omega \in \Omega : X_t(\omega) = U]) = 1 - l_t$$

$$\text{for } t = 1, \dots, \infty.$$

In the function specified,  $\Omega$  is the set of the states of the world relevant to the considered venture. Its elements  $\omega$ 's can be interpreted as the possible situations faced by a venturer. The functional values  $L$  and  $U$  respectively represent the lucky and unlucky paths. Finally,  $Prob.$  is the probability measure of events incurred by the random variable  $X_t$ .

Uncertain about the desired success, S seeks confirmation. He may consult the

father F or outsiders O. His choice is governed by the mapping

$$\sigma_S : \mathcal{H}_t \times \{l_t\} \longrightarrow \{F, O\},$$

where  $\mathcal{H}_t \equiv \{(T_e, T_0, T_1, T_2)\}$  is the set of possible histories at the beginning of the  $t$  period. The components of its elements  $T_e, T_0, T_1, T_2$  are, respectively, the number of ventures encouraged by O but ending unsuccessfully, the number of prohibited ventures, the number of all successful ventures, and the number of ventures consent by F but ending with failure. Moreover,  $T_e + T_0 + T_1 + T_2 \equiv T = t - 1$ , for  $t = 1, \dots, \infty$ . The set of possible histories  $\mathcal{H}_t$  molds the up to date mutual appraisal of S and F, which compounded with  $l_t$  determines their consulting and consenting choices in period  $t$ .

If S chooses to consult O, he sets forth the venture on his own and takes the consequence. Should he choose to consult F, F gets to exert his influence. F may consent(C) or prohibit(P) the proposed venture. His decision follows the mapping

$$\sigma_F : \mathcal{H}_t \times \{l_t\} \longrightarrow \{C, P\},$$

where  $\mathcal{H}_t$  and  $l_t$  are the same as previously defined.

After all involved agents make their moves, the consequence of the venture emerges. It is a mapping from sequences of moves to the pairs of S and F's payoffs:

$$V : \{L, U\} \times \{F\} \times \{C, P\} \cup \{L, U\} \times \{O\} \longrightarrow A_t \times B_t,$$

where all the notations are as defined before. Moreover, to make a venture worth trying, we assume  $|a_{1t}| < a_{4t}$ . This means the maximum potential gains of the venture exceed its largest potential loss.

The extensive game for one attempted venture and the specific correspondence

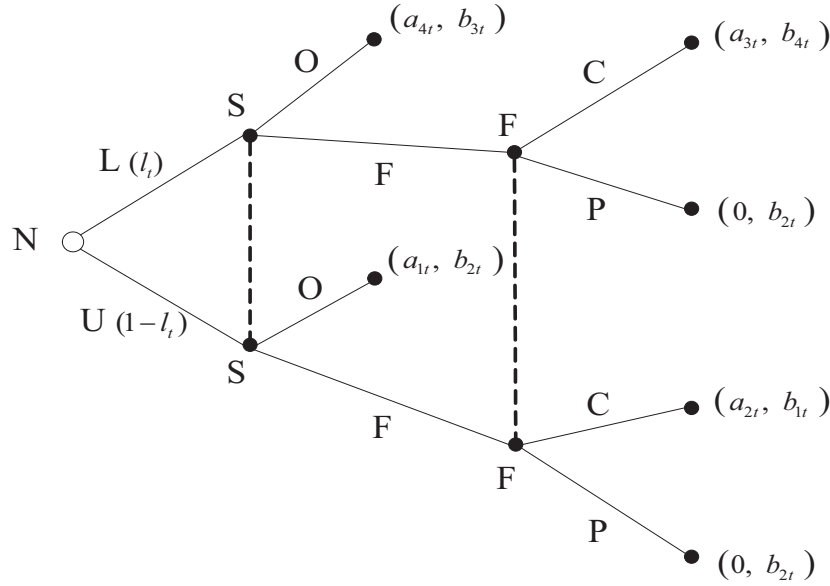


Figure 1: The interaction between the son (S) and the father (F) over a venture intended by S; where  $a_{1t} < a_{2t} < 0 < a_{3t} < a_{4t}$  are the payoffs to S and  $b_{1t} < b_{2t} < 0 < b_{3t} < b_{4t}$  are the payoffs to F for  $t = 1, 2, \dots, \infty$ . Also,  $|a_{1t}| < a_{4t}$

between terminal sequences of actions and their resulting payoffs are depicted in Figure 1.

As has been documented by psychologists and some economists (e.g., Kruger and Dunnig 1999), people tend to believe that his own capacity for success is above average. And it is particularly true for those things one has ardent interest. Reflecting this phenomenon, the analysis assumes that S believes that he can always get more successes through more ventures. This confidence induces him to venture even without F's support. However, it is also the origin of frustration when ventures fail. The pain inflicted by a failure felt by S is described in the condition  $a_{1t} < a_{2t} < 0$ ; and the pain felt by F is accounted for by the payoff order  $b_{1t} < b_{2t} < 0$ . Naturally, both S and F delight in a success. However, the delight of S is discounted slightly if the success appears to have been guided by F. The discounting effect of F's guidance is indicated

in the relation  $0 < a_{3t} < a_{4t}$  over the payoffs of S. In contrast, the joy of F over a success of S is enhanced if he has contributed advice to the venture. This property is reflected in the order  $0 < b_{3t} < b_{4t}$  of F's payoffs.

## II-1. The decisions of the father and son over the first venture

Suppose S proposes his first venture to F. Since there is no past record yet to taint their mutual appraisal, S and F make decisions according to their common prior of the probability of success of the proposed venture,  $l_1$ . Being consulted, if F consents the venture his expected payoff is  $l_1 b_{41} + (1 - l_1) b_{11}$  and if he prohibits it his expected payoff is  $l_1 b_{21} + (1 - l_1) b_{21} = b_{21}$ . Therefore, F would consent the first venture of S if and only if

$$l_1 \geq l_1^* \equiv \frac{b_{21} - b_{11}}{b_{41} - b_{11}}. \quad (1)$$

Denote  $|x|$  as the absolute value of a variable  $x$ , equation (1) renders the following fact:

### Proposition 2.1.1

*Without previous history, the father F has a lower threshold of consenting the proposed venture, if the cost of sharing its failure,  $|b_{11}|$ , is smaller or the benefit of sharing its success,  $b_{41}$ , is greater. However, F raises the threshold if the quarreling cost of prohibiting the venture,  $|b_{21}|$  is small.*

**Proof:** Straightforward from the partial derivatives of  $l_1^*$  with respect to  $b_{11}$ ,  $b_{41}$  and  $b_{21}$ . □

Rational fathers are instinctively pleased with their sons' desire for achievements. Only when their sons try out their potential would their family fortune be expanded.

A father is more permissive towards the son's venture if he is more affluent and can better afford the loss incurred by his son's failure, i.e., he has a smaller  $|b_{11}|$ ; or if he

has a larger potential gain from the success of the endeavor, i.e.,  $b_{41}$  is larger. Since a rich father can withstand big losses, permissiveness is his affordable choice. A poor father could be devastated by his son's failure. However, he may be permissive towards his son's venture as well, because taking the risk may overturn their misery of poverty. Sons of numerous impoverished families around the world venture far away from home for opportunities to depart from perpetuated poverty. Their poor fathers would bear huge shame or loss if they fail or die in the foreign lands. Nevertheless, leaning on gratifying hope, their fathers let them go even with human smugglers.

With no previous test of F's advice, S believes that F has good judgement. Knowing the rule by which F responds to his intended venture, if  $l_1 > l_1^*$  holds, S expects the payoff  $l_1 a_{31} + (1 - l_1) a_{21}$  from consulting F and  $l_1 a_{41} + (1 - l_1) a_{11}$  from consulting O and bypassing F. Recall that when a venture turns out successful, S gets payoff  $a_{3t}$  if F has consent it and  $a_{4t}$  if it is all managed by S alone; when the venture fails S gets payoff  $a_{2t}$  if F is to share the blame and  $a_{1t}$  if S digests the pain alone. In discretion, he chooses to consult F only if

$$l_1 \leq l_1^{**} \equiv \frac{a_{21} - a_{11}}{[(a_{41} - a_{31}) + (a_{21} - a_{11})]}. \quad (2)$$

In aggregate, S will consult F about the venture if and only if

$$l_1^* \equiv \frac{b_{21} - b_{11}}{b_{41} - b_{11}} \leq l_1 \leq l_1^{**} \equiv \frac{a_{21} - a_{11}}{[(a_{41} - a_{31}) + (a_{21} - a_{11})]}. \quad (3)$$

That is, S consults F if and only if he expects that F will consent his venture and that the expected benefit of sharing the pain of failure,  $(1 - l_1)(a_{21} - a_{11})$ , overcompensates the expected cost of sharing the glory,  $l_1(a_{41} - a_{31})$ .

**Proposition 2.1.2**

*The son is more willing to consult the father about his venture if the consequence of a*

*failure is more serious, i.e.  $|a_{11}|$  is greater; or the unshared glory of its success is less magnificent, i.e.  $a_{41}$  is smaller .*

**Proof:** Following from the thresholds specified in condition (3) for S to consult F.  $\square$

A son wants to establish himself through achievement. However, as a rational man, he would also like to pursue it with least hardship. Therefore, the son S is more likely to consult F if in case of failure the shared pain  $|a_{2t}|$  is much smaller than the pain borne alone  $|a_{1t}|$ . The solidarity of S and F is a natural consolation. The more serious is the consequence of a failure, the more valuable is the understanding of F. In pain, genuine empathy relieves the suffering and it comes from the one who feels the pain similarly.

## **II-2. Potential regret about the first venture**

The payoff structure of the model reflects the following characteristics of F and S. If the venture ends in success, both S and F are pleased. As for who gets his highest satisfaction, it depends on whether F was consulted. If unfortunately the venture fails, both S and F suffer negative utility. If the son had consulted the father, he questions why F did not give good advice; while the father regrets having permitted the venture and doubts the ability of S. On the other hand, if S had bypassed F and launched the venture with confirmation from O, the failure is looked at in different light by S and F. The instinct to defend the viability of own decision would convince S that the courage of undertaking a difficult challenge is what matters and venture is a necessary device to train oneself. He does not regret taking on the challenge. Meanwhile, the father thinks the venture failed because he had not been consulted. Watching the outcome as a bystander, he laments the brash decision of S but has no regret on his part.

Facing the failure, the respective extent of regret of S and F is the scope of the estimates that led them to the decisions that they now wish they had not made. With no previous record to generate prejudice, the interval over which S thinks it is worthwhile to consult F and expects consent from him is known to both S and F. It can be deduced from the commonly known population probability of success,  $l_1$ . Let the distribution of  $l_t$  be  $D(\cdot)$ , the extent of the regret of S is the probability that he has consulted F:

$$D(l_1^{**}) - D(l_1^*), \quad (4)$$

where  $l^{**}$  and  $l^*$  are the thresholds given in equation (3). Had  $l_1$  satisfied the condition specified in equation (3) such that S actually consulted F; the range shown in (4) is also the extent of F's regret. That is, the range of the distribution of  $l_1$  where he would have consented the proposal.

A son longs to establish himself through achievement stirs up the risk of failure. Had a venture been consented by F and ended in failure, both F and S have regret. The son regrets trusting a misjudgement of the father, while the father regrets consenting the venture. If F is permissive, he broadens the extent of potential regret both for S and himself. Since regret erupts with each admitted failure, we can expect a son ardently aspiring to achievements would damage the mutual appraisal with his father faster than a passive son. Consequently, their consensus intervals for ventures will shrink with a higher speed to the steady state.

### III. The Evolution of Mutual Appraisal and Decisions

In this section we study how accumulated performance records affect the mutual appraisal of S and F; and how the appraisal affects their decisions on the subsequent venture.



At the beginning of the period for a venture, each agent combines his past experience with the other agent and the population probability of success of the intended venture to form a subjective belief regarding the other agent's behavior in this period. Based on the belief each agent chooses the action that maximizes his expected payoff at each turn of his move. Since each one gets no additional information at the time to make his own decision, his belief formed at the beginning of the period remains fixed until the outcome of the considered venture emerges.

We assume that the probability of success of any venture for any individual is independently and identically distributed. Hence, by the law of large numbers or more generally by ergodic theorem (See e.g., p.86 and p. 312 of Billingsley 1995), the rate of success of S eventually converges to the population average probability of success.

Suppose up to period  $t$ , there have been  $T$  ventures attempted by S. Among the total  $T$  ventures, S sought confirmation from O, ventured alone and failed for  $T_e$  times, has  $T_1$  successes,  $T_2$  admitted failures and gets  $T_0$  prohibitions by F. On the path of pursuing success, acknowledged failures or prohibited ventures engender regret.

For any  $T_2 > 0$  or  $T_0 > 0$ , S regrets having trusted the bad judgements of F; and for  $T_2 > 0$  F regrets having allowed S to meet failures. Since new regret arises after an attempted venture fails, both S and F can accurately deduce the extent of each other's regret over the failed venture. However, regret ignites unpredictable impulsive factors. Hence S and F have only imperfect information regarding its impact on the other party's next decision. Consequently, starting with the second venture, S may miss the actual decision thresholds of F and gets prohibited when consulting F. Such incidents compose another source of S's regret.

Let  $r_{Ft}$  and  $r_{St}$  denote the respective time average intensity of regret of F and S at the beginning of period  $t$ . Their respective impact on the subsequent decision thresholds of F and S in period  $t$  are transmitted by the functions  $m(r_{Ft})$  and  $\delta(r_{St})$ . For the compatibility between mutual appraisal and action choice, the analysis requires that  $1 \leq m(r_{Ft}) \leq \frac{1}{l_t^*}$ , where  $0 < l_t^* \leq 1$  and that  $0 < \delta(r_{St}) \leq 1$ .

To determine the threshold of consenting a venture, F calculates the baseline threshold  $l_t^*$  according to the common prior  $l_t$  and compounds it with the appraisal factor  $m(r_{Ft})$  to form his actual consenting threshold for period  $t$ ,  $\beta_t^0 \equiv m(r_{Ft})l_t^*$ . It is assumed that S knows the distribution of  $m(\cdot)$  but not its exact value. Given the commonly known  $r_{Ft}$  at the beginning of period  $t$ , S estimates that the consenting threshold of F is  $\beta_t^* \equiv \overline{m_{Ft}}l_t^*$ , where  $\overline{m_{Ft}}$  is the mean of  $m(r_{Ft})$ . Meanwhile, with the up to date time average intensity of regret  $r_{St}$  over the judgement of F, S discounts the benefit of consulting F by the factor  $\delta(r_{St}) < 1$ . His threshold of consulting F is jointly determined by  $l_t$  and  $\delta(r_{St})$  such that  $\alpha_t^* \equiv \alpha(l_t, \delta(r_{St}))$ . Note that it is not important for F to know the value of  $\alpha_t^*$  because whenever S decides to bypass him, F does not have the opportunity to take actions.

Following a procedure similar to the one for the first venture, we can calculate the potential regret of S and F in period  $t \geq 2$  from the range of the distribution of  $l_t$  where the willingness of consulting and consenting intersect. However, under the shadow of past performance records, the range is now modified by the up to date subjective mutual appraisal. Incorporating the impact of regret, the consensus interval for venture  $t \geq 2$  becomes  $D(\alpha_t^*) - D(\beta_t^*)$ . Since this interval is only the subjective estimate of S rather than the true consensus interval  $D(\alpha_t^0) - D(\beta_t^0)$ , new regret of S can arise either from the failure of a venture consented by F or from F's prohibition of

the venture.

Define respectively  $D(\overline{\alpha}_T^*)$  and  $D(\overline{\beta}_T^*)$  as the time averages of  $D(\alpha_t^*)$  and  $D(\beta_t^*)$  for  $T$  attempted ventures. Accordingly, the time average intensity of regret at the beginning of period  $t = T + 1$  can be formally defined as:

$$r_{St} \equiv \frac{T - T_1}{T} (D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*))$$

and

$$r_{Ft} \equiv \frac{T - T_1}{T} (D(\overline{\beta}_T^*) - D(\overline{\beta}_T^0)), \quad \text{with } \overline{\beta}_T^* \geq \overline{\beta}_T^0,$$

for all  $T \geq 1$ . Recall that  $\beta_t^0$  represents F's actual consenting threshold and  $\beta_t^*$  S's estimate of that threshold for venture  $t \geq 2$ . Consulting F about a conceived venture, S obtains F's consent only if he has overestimated F's consenting threshold. Therefore, only if  $\overline{\beta}_T^* \geq \overline{\beta}_T^0$  holds would F, on average, have regret about letting S meet failures. Otherwise, F would, on average, have blocked all ventures when consulted and thus exonerated himself from any possible regret.

Longing for self-establishment, S regards venturing without F's participation as an indispensable part of growing process. Succeed or not, they are precious experiences. Should S choose to consult F, he does so in the hope of getting good advice. When he gets prohibition instead or the venture ends in failure, S regrets having trusted F's bad judgement. His time average intensity of regret  $r_{St}$  is the multiplication of the ratio of acknowledged unsuccessful ventures to the total attempted ventures  $\frac{T_0+T_2}{T}$  and the average probability that he had allowed F to exert his bad judgement,  $(D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*))$ .

Fostered by a different reasoning, the average density of F's regret  $r_{Ft}$  is the average probability that he had allowed S to go into failures,  $(D(\overline{\beta}_T^*) - D(\overline{\beta}_T^0))$ , multiplied with the ratio of unsuccessful ventures to the total attempted ventures,  $\frac{T-T_1}{T}$ . Note that if

S had bypassed F, then F has no regret over the failed venture. The exempt regret is accounted for by the term  $(D(\overline{\beta_T^*}) - D(\overline{\beta_T^0}))$ , in the formulation of  $r_{Ft}$ , although those failures without F's involvement are included in  $T - T_1$ .

The two equations imply that the worse is the past record, the stronger is the average intensity of regret. This is because a frustration increases the numerator of the unsuccessful ratio by 1, while it shrinks the estimated consensus interval for the next venture, which is the scope of potential new regret, only by a fraction strictly less than 1. Hence with an additional frustration, the average intensities of regret of both F and S unambiguously increase.

### III-1. The development of the second venture

Since past performance starts its influence with the second venture, a detail analysis of the interaction and potential regret of S and F over this venture helps us catch the essence of the dynamics in all further ventures.

If the initial venture is a success or has been carried out without F's participation, the recognized ratio of failures to total attempted ventures is 0. There is no regret to taint the threshold decisions for the second venture. Hence S and F still have symmetric information as they did over the first venture. The ex ante decision thresholds and potential ex post regret are measured again according to the population probability of success,  $l_2$  only. Asymmetry of information arises when previous failures create prejudice between F and S. Discrepancy thus emerges between the likely consensus intervals estimated by S and F.

Suppose the first venture was a failure or was prohibited by F. The extent of regret of F is  $D(l_1^{**}) - D(l_1^*)$  if it failed and 0 if he had prohibited it. As for S, he regrets

having let the misjudgement of F guide him whether F had consented or prohibited the venture. He has regret  $D(l_1^{**}) - D(l_1^*)$ .

When consulted about the second venture, F first derives the baseline threshold  $l_2^* \equiv \frac{b_{22}-b_{12}}{b_{42}-b_{12}}$ . Incorporating possible regret, F consents the proposed venture if and only if

$$l_2 \geq \beta_2^0 \equiv m(r_{F2}) \times l_2^*. \quad (5)$$

Being able to deduce only  $\beta_2^* \equiv \overline{m(r_{F2})}l_2^*$ , S estimates that F would consent his venture only if  $l_2 \geq \beta_2^*$ . Suppose this necessary condition holds. S expects at least the payoff  $\delta(r_{S2})[l_2a_{32} + (1 - l_2)a_{22}]$  if he consults F; and at least  $l_2a_{42} + (1 - l_2)a_{12}$  if he simply bypasses F and ventures with the confirmation of outsiders. In aggregate, S subjectively thinks it is worthwhile to consult F only if the population probability of success of the venture  $l_2$  satisfies the following condition:

$$\beta_2^* \equiv \overline{m(r_{F2})}l_2^* \leq l_2 \leq \alpha_2^* \equiv \frac{\delta_2a_{22} - a_{12}}{[(a_{42} - a_{12}) - \delta_2(a_{32} - a_{22})]}, \quad (6)$$

where  $0 < \delta_2 = \delta(r_{S2}) < 1$ , is the discounting factor on the benefit of consulting F forged by the regret of S; and  $1 \leq \overline{m(r_{F2})} \leq \frac{1}{l_2^*}$  is the mean of the impact of F's regret on his current consenting threshold. The properties of the interval of  $l_2$  necessary for S to consult F are summarized in the following.

### **Proposition 3.1.1**

*Given the commonly known population probability of success of the considered venture  $l_2$ , S is less likely to consult F if he estimates that F's regret will significantly raise his consenting threshold. On the contrary, S is more likely to consult F if he harbors less regret about F's previous judgement, or the cost of sharing the glory,  $(a_{42} - \delta_2a_{32})$ , compared with the benefit of sharing the frustration,  $|\delta_2a_{22} - a_{12}|$ , is smaller.*

**Proof:** Obvious from the necessary condition (6) for S to consult F, thus omitted.  $\square$

As we have seen in the first venture, the more permissive S thinks F is, the more likely S will consult F. However, if S is tremendously disappointed with F's misjudgment, he will significantly narrow the scope where he would consult F. In the extreme situation where  $\beta_2^* > \alpha_2^*$ , S definitely bypasses F and launches his venture on the agreeable opinion from outsiders. Such extreme case would not occur if S and F derive similar payoffs from a success  $l_t$  but S suffers far more than F does with a failure. That is, as long as a failure inflicts far more pain on S than on F, there is always a range of the distribution of the population probability of success over which S will consult F about a planned venture. This feature rules the decisions of S for any future venture.

### Potential regret at the end of the second venture

If the venture was a success, regardless of from whom S had elicited encouragement, neither S nor F regrets it. Should it fail, each has regret about the other party's rough decision.

Define  $U_{2t} \equiv (a_{2t}, b_{1t})$ , and  $U_{0t} \equiv (0, b_{2t})$ , for  $t = 1, 2, \dots, \infty$ .

Recall that  $\{a_{1t}, a_{2t}, 0, a_{3t}, a_{4t}\}$  with  $a_{1t} < a_{2t} < 0 < a_{3t} < a_{4t}$  are the possible payoffs of S and  $\{b_{1t}, b_{2t}, b_{3t}, b_{4t}\}$  with  $b_{1t} < b_{2t} < 0 < b_{3t} < b_{4t}$  the possible payoffs of F from venture  $t$ . By assumption, F regrets the venture of S should  $U_{22}$  be the consequence: Facing the failure, F regrets that he has consented the venture. For  $U_{22}$  to take place, it must be true that S had consulted F and F consented the venture. Therefore,  $U_{22}$  occurs with probability  $D(\alpha_2^*) - D(\beta_2^0)$ . It is the conditional probability that F would regret about the venture. Such extent of regret occurs in the situation where  $D(\beta_2^*) \geq D(\beta_2^0)$ .

In the opposite case where  $\beta_2^0 > \beta_2^*$ , S had underestimated F's actual consent-

ing threshold and would have been prohibited from the venture if he consulted F. In this case, F exempts himself from regretting over a failure. The relation between the consulting discretion of S and the extent of F's regret is summarized in the following.

**Proposition 3.1.2**

*F would regret the venture of S only if S had overestimated his consenting threshold. Had S underestimated the consenting threshold of F, that is, if  $D(\beta_2^*) < D(\beta_2^0)$  ex ante, F would save himself from potential regret.*

Since S has only imperfect information regarding the impact of F's regret on his present consenting threshold, his estimate may miss the real point. As a result, it is possible that he consults F about the venture in the expectation of his consent and gets prohibited from it instead. In this situation, S imagines his success in the aborted venture and resents the prohibition. On the other hand, if he did acquire the consent from F and the venture ends in failure, he laments the bad judgement of F. Therefore, in the event of  $U_{02}$  or  $U_{22}$ , S regrets the bad decision of F. The aggregate probability of these two events is

$$\begin{aligned}
 & Prob.(U_{02}) + Prob.(U_{22}) \\
 &= [D(\beta_2^0) - D(\beta_2^*)] + [D(\alpha_2^*) - D(\beta_2^0)] \tag{7} \\
 &= D(\alpha_2^*) - D(\beta_2^*).
 \end{aligned}$$

Comparing the potential extent of regret of S and F, it is easy to see that if the son S had underestimated F's consenting threshold, then the extent of his potential regret would have been greater than that of F. On the contrary, if S had overestimated F's consenting threshold, then it is F who would have greater potential regret over the venture.

So far the analysis is based on the assumption that the son S has more intense

feeling about the success and failure of own venture than the father F does. As a result, for a considered venture, there exist respective decision thresholds of S and F such that  $\alpha_t^* > \beta_t^*$ . If the Nature-determined population probability of success  $l_t$  falls in the interval  $[\beta_t^*, \alpha_t^*]$ , S would like to consult F and expects his consent. It can be understood from this interval that a son more reliant upon the consolation of his father in time of frustration is more likely to consult his father.

What would happen if  $\alpha_t^* < \beta_t^*$ , that is, if the range of the distribution of  $l_t$  over which S has incentive to consult F has no intersection with the range over which S estimates F would consent the venture if consulted?

There are two possible scenarios that can engender an extremely low  $\alpha_t^*$ . The first scenario is a son with exceptionally high self-regards, he perceives little benefit of consulting F. In this situation, he bypasses F and ventures with the outsiders' agreeable opinions. The other scenario is a son with little enthusiasm for independent achievement. Perceiving no chance of getting F's support, he considers it as unworthy to venture on his own. Since this paper aims to study the interaction of majority father and son, we leave the exceptional cases to other experts. Throughout the ensuing analysis, we continue the assumption that S has desire for independent accomplishments. He relishes independent success but also benefits from *F's* consolation.

### **III-2. The Interaction between S and F over the $t$ th venture**

In this subsection, we first analyze the interaction between S and F in any  $t$ th venture, with  $t \geq 3$ . Then, letting the number of total attempted ventures go to infinity<sup>3</sup>, we

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<sup>3</sup>Actually a sufficiently large number suffices for the analytical purpose. A person's growth is piled up by numerous actions and inactions. Starting from whether to attend kindergarten up to whether to invest in Africa, there are inexhaustible ventures awaiting a person who aspires to self-establishment.



investigate the steady state consulting behavior of S. As time passes and the outcomes of attempted ventures accumulate, S and F solidify their subjective appraisal of each other's capacity for a right decision.

Had there never been any acknowledged failure up to date, S and F use objective parameters to evaluate each other's decision for a venture. Since there has been no regret to taint their perception and hence they make decisions according to the commonly known parameter  $l_t$ , their information remains symmetric.

However, as one of the multitude, S is going to encounter failure sooner or later. Given a sufficiently large number of trials  $T^*$ , the performance record of S shall converge to the average of the population probabilities of success for all ventures. Formally, define

$$\bar{l}_T \equiv \frac{1}{T} \sum_{1 \leq k \leq T} l_k, \quad (8)$$

where  $T$  is the total attempted ventures up to date,  $l_k$  is the population probability of success for venture  $k$  as formally defined at the beginning of section II. For all  $T \geq T^*$ , by the law of large numbers, we have  $\bar{l}_T = \bar{l}$ , a constant.

With the eruption of failures, regret accumulates. And decisions of F and S are tainted by prejudice. Recall that if up to the beginning period  $t$ , S has failed for  $T_e$  times without F's involvement,  $T_1$  successes,  $T_2$  acknowledged failures, and  $T_0$  prohibitions by F among the total  $T = t - 1$  attempted ventures, the up to date time average intensity of regret is

$$r_{St} = \frac{T - T_1}{T} (D(\bar{\alpha}_T^*) - D(\bar{\beta}_T^*)) \quad (9)$$

for S; and

$$r_{Ft} = \frac{T - T_1}{T} (D(\bar{\alpha}_T^*) - D(\bar{\beta}_T^0)), \quad (10)$$

for F. The terms included in the parentheses of equations (9) and (10) are, respectively, the average extent of regret over all attempted ventures of S and F. By the strong law of large numbers, it is assured that the use of average extent of regret incurs no loss for analytical results.

The content of  $r_{St}$  shows that the more frequently his intended ventures are forbidden or fail, the more intense is the regret of S. Meanwhile, the likelihood of prohibitions and admitted failures increase with S's willingness to consult F. If S underestimates the true consenting threshold of F, he gets prohibited from the venture. In such incident S imagines that he has been deprived of a chance for success. Should S have overestimated F's consenting threshold and calculated that it was worthwhile to consult F; S attains consent from F. Once the venture fails, S complains F's misjudgement.

Similarly, the substance of  $r_{Ft}$  indicates that the worse is the past performance record of S, the deeper is the regret of F. However, the more frequently S underestimates F's actual consenting threshold, the less will be F's regret. Underestimating F's consenting threshold subjects the intended venture to prohibition. It thus exempts F from regretting a failure.

Since for any venture  $t$ , the preceding history, its population probability of success  $l_t$  and the corresponding expected payoffs of all sequences of actions are commonly known by S and F, both of them can deduce ex ante  $l_t^*, \beta_t^*$ , and  $\alpha_t^*$ . However, only F knows his own real consenting threshold  $\beta_t^0$  of the venture. S can only approximate  $\beta_t^0$  by  $\beta_t^*$  before F reveals his true position. S knows its value only after consulting F. Prohibiting or consenting the venture, F tells S his decision. When an intended venture concludes its course, both S and F knows the other party's up to date intensity

of regret. This information carries over to the beginning of next contemplated venture.

Bearing regret, F combines his up to date average intensity of regret  $r_{Ft}$  and the Nature-determined population probability of success  $l_t$  of the intended venture to determine his consenting threshold. He first calculates the baseline threshold by comparing the respective expected payoffs of consenting and of prohibiting the venture based on  $l_t$ . He would consent the venture only if

$$l_t \geq l_t^* \equiv \frac{b_{2t} - b_{1t}}{b_{4t} - b_{1t}}.$$

Incorporating his regret in his appraisal of S's capacity for success and combining it with the baseline threshold, F consents the venture if and only if

$$l_t \geq \beta_t^0 \equiv m(r_{Ft})l_t^* = m(r_{Ft})\frac{b_{2t} - b_{1t}}{b_{4t} - b_{1t}}. \quad (11)$$

It is clear from  $\beta_t^0$  that if F has a larger potential gain from the venture relative to the potential loss of sharing its failure, his consenting threshold is lower. Therefore he is more permissive.

Knowing F has regret about his past performance, S estimates F's possible reaction to his venture proposal with caution. S can calculate  $r_{Ft}$ , however he cannot predict precisely its impulsive impact on F's subsequent decision. Unable to do better otherwise, S takes the mean of  $m(r_{Ft})$  for his estimate of F's likely consenting threshold. Formally, S estimates that F would consent the venture only if

$$l_t \geq \beta_t^* \equiv \overline{m(r_{Ft})}l_t^* = \overline{m(r_{Ft})}\frac{b_{2t} - b_{1t}}{b_{4t} - b_{1t}}.$$

First, let's consider the case where  $l_t$  satisfies the estimated necessary condition. In this situation, S thinks F is likely to consent the venture if consulted. Conditional on

the likelihood of obtaining F's consent, S will actually consult F if and only if doing so brings about a greater expected payoff than bypassing F. Being disappointed with F's past bad judgement, S discounts the usefulness of consulting F by scaling down its resulting expected payoff to

$$\delta(r_{st})[l_t a_{3t} + (1 - l_t)a_{2t}].$$

While the expected payoff from bypassing F and eliciting support from outsiders remains a matter of objective luck:

$$l_{1t}a_{4t} + (1 - l_t)a_{1t}.$$

Therefore, S consults F only if

$$l_t \leq \alpha_t^* \equiv \frac{\delta a_{2t} - a_{1t}}{(a_{4t} - a_{1t}) - \delta(a_{3t} - a_{2t})}.$$

Aggregating the estimate of F's possible response and own incentive, S will consult F only if

$$\beta_t^* \leq l_t \leq \alpha_t^*. \tag{12}$$

The above interval indicates the range of the distribution of  $l_t$  over which S expects F to consent his venture and regards it is worthwhile to consult him.

If  $l_t > \alpha_t^*$ , S would not bother to consult F even he expects F would consent the venture. A large population probability of success allures S into the mood that success is within his reach. Eager to prove himself, he sees no need to wait on the judgement of F. Suppose the range of  $l_t$  is such that  $\beta_t^* \leq l_t < \beta_t^0 < \alpha_t^*$ , S chooses to consult F; however since he underestimates F's consenting threshold his intended venture will meet with F's prohibition.

Next, we examine the decision of S if  $l_t$  is strictly less than his estimate of F's consenting threshold,  $\beta_t^*$ .

**Proposition 3·2·1**

*Suppose the Nature-determined population probability of success of the venture  $l_t$  is so small that  $l_t < \beta_t^*$ . In this situation S will bypass F and set forth the venture as long as he believes that his chance of success is at least  $\underline{p} \equiv \frac{-a_{1t}}{a_{4t}-a_{1t}}$ , and  $\underline{p} < \frac{1}{2}$ .*

**Proof:** When  $l_t$  is below the estimated consenting threshold of F,  $\beta_t^*$ , S thinks he is likely to be prohibited from the venture if he consults F, and the resulting expected payoff would be 0. In this situation, if he believes that his own chance of success is  $p_t$ , he expects to obtain at least  $p_t a_{4t} + (1 - p_t) a_{1t}$  by carrying out the venture without F's interference. This payoff is greater than or equal to 0, the payoff of being prohibited from the venture, if  $p_t \geq \underline{p} \equiv \frac{-a_{1t}}{a_{4t}-a_{1t}}$ . Moreover, since  $|a_{1t}| < a_{4t}$ , we have  $\underline{p} < \frac{1}{2}$ .  $\square$

When S believes that his capacity for success is at least as large as that of the average population, he attributes past failures to bad luck, and anticipates more successes in further trials. Longing to attain the prospective success, he ventures into the hope despite the low population probability of success and F's potential objection.

As have been addressed in Section II, the indestructible connection of S and F entraps F in the shame and glory of S. Once he had consented a venture and it failed, F regrets the initiative of S. Hence the probability of the aftermath regret of F over venture  $t$  is the probability of the event  $U_{2t}$  :

$$Prob.(U_{2t}) = [D(\alpha_t^*) - D(\beta_t^0)] \tag{13}$$

Displeased with the failure, F wishes that he had not consented the venture.

As explained earlier, as long as the interference of F is absent from the decision process, S regrets nothing even the venture ends in failure. He affirms the courage to explore the unknown according to own vision. However, he has an object to blame if F prohibited his venture or F had consented it and the venture failed. Therefore, the extent of S's aftermath regret is the probability of the event  $U_{0t}$  or  $U_{2t}$  :

$$\begin{aligned} Prob.(U_{0t}) + Prob.(U_{2t}) &= [D(\beta_t^0) - D(\beta_t^*)] + [D(\alpha_t^*) - D(\beta^0)] \\ &= D(\alpha_t^*) - D(\beta_t^*). \end{aligned} \tag{14}$$

In the event of  $U_{0t}$ , he wishes that he had not given F the chance to prohibit the venture; while in the event of  $U_{2t}$ , he wishes that he had not trusted the judgement of F so much.

### III-3. Convergence of the long-run behavior

So far we have analyzed the interaction between the father F and the son S in the first venture, the second venture and any further venture. The analysis characterizes the consulting thresholds of S and consenting thresholds of F. Subsequently, it derives the extent of regret planted by their respective action choices. Regret induces subjective prejudice. The analysis then investigates how subjective prejudice affects the future consulting and consenting thresholds of S and F. Harboring regret, S discounts the expected benefit of consulting F and presses down the threshold under which he would seek F's advice. Meanwhile, regret makes F lower his estimate of his son's capacity for success and raise his consenting threshold. Failures breed regret and regret compresses the range of population probability of success of a considered venture over which S has incentive to consult F and expects consent from him. Regret continues to compress the scope where S and F would agree on an intended venture.

This progressive shrinking effect of regret implies that for any  $t$ th venture with

$t \geq 2$ , the thresholds tainted by prejudice  $\alpha_t^*$ ,  $\beta_t^*$ , and  $\beta_t^0$  have the following properties:

$$\alpha_t^* \leq l_t^{**}, \text{ and } \beta_t^*, \beta_t^0 \geq l_t^*$$

for  $t = 2, \dots, \infty$ , where  $l_t^{**}$  is the consulting threshold of S if he does not have any regret over F's judgement; and  $l_t^*$  is the consenting threshold of F had S never afflicted him with failed ventures. Both thresholds are derived based on the commonly known population probability of success  $l_t$ . Therefore, for a venture  $t$  we have

$$[\beta_t^*, \alpha_t^*] < [l_t^*, l_t^{**}] \quad (15)$$

Taking average over all ventures attempted up to date, we define  $\bar{l}_t^* \equiv \frac{1}{t} \sum_{1 \leq k \leq t} l_k^*$  and  $\bar{l}_t^{**} \equiv \frac{1}{t} \sum_{1 \leq k \leq t} l_k^{**}$  be the up to date time average of the unprejudiced thresholds. Subsequently, we have

$$D(\bar{\alpha}_t^*) - D(\bar{\beta}_t^*) \leq D(\bar{l}_t^{**}) - D(\bar{l}_t^*), \quad (16)$$

by the continuity and monotonicity of the distribution  $D(\cdot)$  with respect to its argument. This inequality indicates that overtime, the average subjective interval where F and S may have consensus is smaller than the average unprejudiced interval.

A son who aspires to achievements will inevitably encounter failures along his development. Assume an independent and identical distribution for everyone's probability of success of any venture. Given adequate trials, by the law of large numbers, the success rate of S will converge to the average of the population probabilities of success of all ventures. Formally, there exists a positive integer  $T^*$  such that for any greater number of trials  $T \geq T^*$  up to period  $t$ ,

$$\frac{T_1}{T} = \bar{l}_T \equiv \frac{1}{T} \sum_{0 \leq k \leq T} l_k \leq l \equiv \lim_{T \rightarrow \infty} \bar{l}_T, \quad (17)$$

where  $T_1$  is the total number of successes up to date. Corresponding to the limit of the average of population probabilities of success across all ventures, we have

$$1 - l = \lim_{T \rightarrow \infty} \frac{T_e + T_0 + T_2}{T} \quad (18)$$

as the limit of overall average probability of unsuccessful ventures.

Let the number  $T^*$  of total attempted ventures be reached in period  $t^*$ . At the beginning of period  $t > t^*$ , S harbors the average intensity of regret

$$r_{St} = (1 - l)[D(\overline{\alpha}_t^*) - D(\overline{\beta}_t^*)].$$

With  $1 - l$  unchanged from period  $t^*$  on, the measure of  $r_{St}$  is modified only by the magnitude of  $D(\alpha_{t-1}^*) - D(\beta_{t-1}^*)$  divided by the number of all attempted ventures  $t - 1$ . Hence  $r_{St}$  stops increasing after venture  $t^*$  is attempted. However, it will remain strictly positive and thus continue to discount the expected payoff of consulting F through the factor  $0 < \delta(r_{St}) < 1$ . The reduced expected payoff subsequently further narrows the worthy consulting interval.

For a total  $T$  attempted ventures, two sources fill the intensity of the regret of S. One is the prohibition of an intended venture by F. Without any prohibition by F, S will encounter conceded failures with the ratio  $\frac{T_2}{T}$  such that

$$(1 - l) > \frac{T_2}{T} > 0 \quad \forall T < T^*, \quad (19)$$

where  $T_2$  is the number of admitted failures among the total  $T$  ventures. Before the number of trials  $t^*$  is reached, an admitted failure rate lower than the population average makes S believe that his capacity for success is above the average population; hence, further ventures are certainly worthwhile. However, as an ordinary fellow, his delusion cannot resist the law of large numbers. His rate of unsuccessful ventures



will rise to the boundary  $1 - l$  eventually. With each failure of his consent venture, F's regret intensifies and shifts up his consenting threshold for the next venture of S. Meanwhile, before the  $T^*$ th venture, there are ventures whose  $l_t$ s make it worthwhile for S to consult F. With imperfect information on the impulsive reaction of F, S risks being prohibited from the intended ventures. Denote the number of prohibitions up to date as  $T_0$ . It composes one source of  $r_{St}$ .

The other source that reinforces the intensity of the regret of S is the wrong consent of F. Should F have consented a venture and it failed, S regrets having trusted the judgment of F. This type of frustration is accounted for by the number of admitted failures  $T_2$ . With strictly positive average intensity of regret  $r_{St}$ , S discounts the benefit of consulting F for the next venture by the factor  $\delta(r_{St}) < 1$ .

Meanwhile,  $r_{Ft} > 0$  if  $\frac{T_2}{T} > 0$ . Still, for any period  $t > t^*$ ,

$$r_{Ft} = (1 - l)[D(\overline{\alpha_t^*}) - D(\overline{\beta_t^0})].$$

Compared with  $r_{Ft-1}$ , with  $l \equiv \lim_{T \rightarrow \infty} \frac{T_2}{T}$  fixed,<sup>4</sup> the average intensity of F's regret  $r_{Ft} \forall t > t^*$  at the beginning of  $t$ th venture is to be revised only by the measure  $\frac{D(\alpha_{t-1}^*) - D(\beta_{t-1}^0)}{(t-1)}$ . Hence  $r_{Ft}$  ceases increasing after venture  $t^*$ . Nevertheless, it remains strictly positive and continues to lift F's consenting threshold by the factor  $1 < m(r_{Ft}) \leq \frac{1}{l_t}$  for any future venture  $t$ .

Given the inevitable fate of an ordinary S that his rate of success of all ventures will

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<sup>4</sup>The fact that neither S nor F has regret over those unsuccessful ventures that do not involve F is reflected in the likelihood of each agent's potential regret over a specific venture. Since the extent of potential regret of each agent has excluded those unsuccessful ventures over which neither of them would regret, the average intensity of regret of S and F capture exactly the portion of all attempted ventures that would have inflicted regret on each of them.

converge to the population average, mutual regret between F and S about each other's decisions accumulates with certainty along the exploring journey of S. Gradually, the range of  $l_t$  over which S considers it is worthwhile to consult F shrinks to its lower bound. The interaction of an ordinary pair of father and son over an indefinite horizon is summarized in the following theorem.

**Theorem 3.3.1**

*Consider an ordinary pair of father and son: The son S aspires to establish his standing through achievements and expects his father to be more reliable than outsiders when he seeks advice. The father F delights in the son's success and suffers with his failure. After certain number of ventures, S will consult outsiders more often than consult F.*

**Proof:** Define  $T^*$  as the number of total attempted ventures at which the success rate of an ordinary person converges to the population average. We want to show that  $D(\overline{\alpha}_t^*) - D(\overline{\beta}_t^*) \leq \frac{1}{2}$ ,  $\forall t \geq t^*$ , where period  $t^*$  is the period in which the  $T^*$ th venture is attempted. This smaller than one-half consulting-and-consenting interval indicates that S consult outsiders more often than consult F after  $T^*$  trials.

Recall that while making their respective consulting and consenting decisions, S and F combine their mutual appraisal with the population probability of success of the intended venture to derive the thresholds. Whenever either of them has regret about past ventures, there arise emotional factors to taint their decisions. Subsequently, they shift their consulting and consenting thresholds to stricter levels. Hence, the proof is accomplished via the induction on the periods in which the first acknowledged frustration occurs. That is, the proof first demonstrates the steady-state property of the consulting and consenting dynamics in which the first prohibition or failure of

a consented venture appears at the first venture. Assuming the identified property holds in the dynamics where the first prohibition or failure of a consented venture appears at the  $K$ th,  $K \geq 2$  venture, the proof then shows that the same property holds if the indicated frustration appears one period later. The proof is presented in the appendix. □

## VI. Concluding Remarks

The evolutionary interests of father and son are eternally intertwined. While alive, they naturally trust in and concern for each other. However, across cultures and geographies, fathers complain that their sons listen more to others than to them while their sons lament their lack of understanding. Even the most powerful or intelligent fathers agonize over the behaviors of their sons. It is said that the President 41 of the United States regrets that the President 43 has not listened more to him on foreign policy. Al Gore has the charisma to arouse world wide actions against global warming; nevertheless, he appears powerless over his son's choice of life style. These examples testify the observation of Mark Twain: It is a wise child that knows its own father, and an unusual one that unreservedly approves of him.

This paper explains this paradoxical interaction between father and son with an incomplete information and infinitely-repeated extensive-form game. It is shown that a son's aspiration for achievements and his expectation of his father's good judgement suffice to engender dynamics in which after an adequate number of ventures, he on average consults outsiders more often than consults his father.

In light of this analysis, the incentives beneath these seemingly contradictory interaction between father and son are clear. Advancing beyond the dominance of own father's opinion is an inevitable trend of the son's development. Without this deviation,

human progress would be impossible.

Sons of rich fathers can afford more ventures beyond their fathers' envision. It is because the wealth and power of their fathers can shelter them from severe consequence of failure. Sons of impoverished fathers must be more adventurous than their fathers if they want to depart from their fathers' pitiful states. As for the sons of middle class fathers, they can afford less the dire consequence of failures as compared with those better endowed sons; and they have less to gain from a risky venture in contrast to those desperate sons. Therefore, this paper predicts that sons of fathers with medium wealth are more pious to their fathers.

However, for fathers of all ranks, a son's aspiring to accomplishments and higher expectation of his father than of outsiders, together with the father's conditional support for him suffice to generate a steady-state where the son seeks advice less frequently from his father than from outsiders.

Carrying ordinary human traits, a pair of father and son would complain each other when frustrated with the son's unsuccessful ventures. The son regrets his father's bad judgement while the father regrets letting the son bring in troubles. As regret intensifies, the range of the population probability of success of an intended venture over which the son would consult his father and expect his consent shrinks. After a sufficient number of attempted ventures, their solidified average intensity of regret compresses the range to a level where the son turns to outsiders' agreeable endorsement more often than to his father's precarious opinions.

The findings of this paper do not exclude the possibility that as a son advances into older age he appreciates that his father stopped him from certain ventures; as Mark

Twain once confessed: When I was a boy of fourteen, my father was so ignorant I could hardly stand to have the old man around. But when I got to be twenty-one, I was astonished at how much the old man had learned in seven years. In this paper, a son's resentment of his father during his earlier age draws on his father's misjudgement in not preventing a potential failure or hindering a potential success. After enough frustrated ventures, the son lowers his expectation of his father to its deserved status. From then on, he consults the father less frequently. Hence there will be less occasions for him to regret his father's judgement. Consequently, the son becomes less critical of his father's intelligence.

As a strategy implication for parents, the study illuminates the rationale of exchanging sons to teach practiced by ancient Chinese. Given that a son may reject his father's reasonable advice out of his prejudice against him, transferring his advice en route of a trustworthy friend is the best way to surpass the prejudice and obtain a proper listening from the son.

### Appendix: Proof of Theorem 3· 3·1

**Proof:** Following from the analysis of the main text,  $D(\alpha_t^*) - D(\beta_t^*) \leq D(l_t^{**}) - D(l_t^*)$ ,  $\forall t \geq 2$  whenever there is regret harbored by either S or F. If  $D(l_t^{**}) - D(l_t^*) \leq \frac{1}{2} \forall t$ , the proof is complete without further argument. Hence the remaining task is to show that for  $\frac{1}{2} < D(l_t^{**}) - D(l_t^*) < 1$ , there exist a dynamics that lead to  $D(\overline{\alpha_t^*}) - D(\overline{\beta_t^*}) \leq \frac{1}{2}, \forall t \geq t^*$ , where period  $t^*$  is the period in which the success rate of S's venture  $\frac{T_1^*}{T^*}$  converges to the population average,  $l$ ; and thus the unsuccessful rate converges to  $1 - l$ .

As an ordinary person, S is subject to the population average probability of success. Even regarding part of those unsuccessful ventures as experiences rather than acknowledging all of them as failures, S will unavoidably encounter increasing number of failures before the  $T^*$ th venture is attempted, at which the success rate of his performance converges to the population average.

#### **If the first frustration appears at the first venture:**

As explained in section II, if S got a prohibition from F, S regrets having consulted F; while if F had consented a venture and it failed both S and F have regret. S regrets the bad judgement of F and F regrets having consented the venture. Denote the average intensity of F's regret at the beginning of the second venture as  $r_{F2}$ . Its impact on F's consenting decision for the second venture is  $m(r_{F2})$ . Let the average intensity of the regret of S at the beginning of the second period be  $r_{S2}$ . It makes S discount the expected payoff of consulting F for the second venture by the factor  $\delta(r_{S2}) < 1$ . Therefore, under the influence of regret, from the second venture on,

the worthy consulting interval estimated by S,  $\alpha_t^* - \beta_t^*$ , is strictly smaller than the unprejudiced worthy consulting interval  $l_2^{**} - l_2^*$ .

Given that  $m(r_{F2}) > 1$ , the actual consenting threshold of F for the second venture  $\beta_2^0$  is strictly above the baseline consenting threshold  $l_2^*$ . Being able to estimate F's consenting threshold only by the mean of  $m(r_{Ft})$ , S has an estimate  $\beta_2^* = \overline{m_{Ft}}l_2^*$  of F's consenting threshold. The discrepancy between  $\beta_2^0$  and  $\beta_2^*$  creates the possibility that S gets prohibited from the venture if he consults F. Since the probability of failure of any venture  $t$ ,  $1 - l_t$ , is strictly positive, the unsuccessful rate of all attempted ventures represented by the ratio  $\frac{T_e + T_2 + T_0}{T}$  increases with strictly positive probability from the second venture on. It continues to rise until the ratio converges to  $1 - l$ , the population average probability of unsuccessful ventures. Accumulated frustrations reinforce the respective average intensity of regret,  $r_{Ft}$  and  $r_{St}$ . They subsequently diminish the worthy consulting interval for each future venture.

We first examine how  $r_{St}$  and  $r_{Ft}$  change with an additional frustration. Suppose among the total  $T$  attempted ventures at the beginning of period  $t$ , S experienced  $T_e$  unsuccessful ventures without F's involvement,  $T_0$  prohibitions by F,  $T_1$  successes, and  $T_2$  acknowledged failures. Another acknowledged failure or prohibition occurs at the  $t$ th venture in period  $t$  increases the frustration rate by  $\frac{T - (T_0 + T_2)}{T(T+1)}$ .

However, incorporating the extent of regret rendered by the frustrated  $t$ th venture, the measure of  $[D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*)]$  is only modified by the amount of

$$\begin{aligned} & - \frac{T[D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*)] + [D(\overline{\alpha}_{T+1}^*) - D(\overline{\beta}_{T+1}^*)]}{T+1} + \frac{T[D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*)]}{T} \\ & = \frac{-1}{T+1}[D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*)] + \frac{1}{T+1}[D(\alpha_{T+1}^*) - D(\beta_{T+1}^*)]. \end{aligned}$$

Let  $A \equiv D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*)$  and  $R \equiv D(\alpha_{T+1}^*) - D(\beta_{T+1}^*)$ . The change in  $D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*)$

by one more frustration can be rewritten with the new notations as  $\frac{T(R-A)}{T(T+1)}$ . Moreover,  $\frac{TR}{T(T+1)} < \frac{T}{T(T+1)}$  because  $0 < R < 1$ . Also,  $\frac{TA}{T(T+1)} \geq \frac{T_0+T_2}{T(T+1)}$ , because  $TA$  is the portion of total attempted ventures for which S would have consulted F; and only among these ventures either  $T_0$  or  $T_2$  would arise. Therefore, adding up the two relations, we have

$$\frac{-1}{T+1}[D(\overline{\alpha}_T^*) - D(\overline{\beta}_T^*)] + \frac{1}{T+1}[D(\alpha_t^*) - D(\beta_t^*)] < \frac{T - (T_0 + T_2)}{T(T+1)}.$$

Likewise, the difference of  $D(\overline{\beta}_T^*) - D(\overline{\beta}_T^0)$  is revised only by the amount of

$$\begin{aligned} & - \frac{T[D(\overline{\beta}_T^*) - D(\overline{\beta}_T^0)] + [D(\beta_{T+1}^*) - D(\beta_{T+1}^0)]}{T+1} + \frac{T[D(\overline{\beta}_T^*) - D(\overline{\beta}_T^0)]}{T} \\ & = \frac{-1}{T+1}[D(\overline{\beta}_T^*) - D(\overline{\beta}_T^0)] + \frac{1}{T+1}[D(\beta_{T+1}^*) - D(\beta_{T+1}^0)] < \frac{T - (T_0 + T_2)}{T(T+1)}. \end{aligned}$$

Therefore, with an additional frustration, the average intensity of the regret of S,  $r_{St}$ , and that of F,  $r_{Ft}$  unambiguously increase. Given  $\frac{d\delta(\cdot)}{dr_{St}} < 0$  and  $\frac{dm(\cdot)}{dr_{Ft}} > 0$ , there exists a  $T^*$  such that for all further ventures, the range of  $l_t$  over which S considers it is worthwhile to consult F shrinks to a level such that  $D(\overline{\alpha}_t^*) - D(\overline{\beta}_t^*) \leq \frac{1}{2}$ . This conclusion apply whether failures emerge in consecutive ventures or with several ventures apart.

Furthermore, after the  $T^*$ th venture in period  $t^*$  at which the ratio of successful ventures to total attempted ventures  $\frac{T_t}{T}$  converges to the population average probability of success  $l$ , and its complement ratio  $\frac{T_e+T_0+T_2}{T}$  settles at  $1-l$ . Hence for any future venture  $t > t^*$ ,  $r_{St}$  and  $r_{Ft}$  stop increasing. However, they remain strictly positive throughout and their respective shrinking effects,  $0 < \delta(r_{St}) < 1$  on S's consulting decisions and  $1 < m(r_{Ft}) \leq \frac{1}{t^*}$  on F's consenting decisions, continue into future ventures. The lasting compressing process on the worthy consulting interval  $D(\alpha_t^*) - D(\beta_t^*)$  for each future venture  $t > t^*$  ensures that  $D(\overline{\alpha}_t^*) - D(\overline{\beta}_t^*) < \frac{1}{2}$ .

Suppose the theorem holds in the scenario where the first frustration appears at the  $K$ th attempted venture, with  $K \geq 2$ . We now show that the theorem holds if the



first frustration emerges at the  $(K + 1)th$  venture.

### **If the first frustration appears at the $(K + 1)th$ venture**

As an ordinary person, S is subject to the inescapable fate that his unsuccessful rate,  $\frac{T_e+T_0+T_2}{T}$ , will eventually increase to the population average  $1 - l$ . One period delay of the appearance of the first frustration only means that the failures to come are spread more densely over the shorter interval between the  $(K + 1)th$  and  $T^*th$  attempted venture. The intensifying of  $r_{St}$  and  $r_{Ft}$  and the shrinking of the consulting interval  $[\alpha_t^*, \beta_t^*]$  until the converging period follow a process resembling the one when the first failure occurs at the  $Kth$  venture but with a higher speed.

Although with different timing, both paths are driven by the force that the ratio  $\frac{T_e+T_0+T_2}{T}$  will increase to the population average  $1 - l$  eventually, and  $r_{St}$  and  $r_{Ft}$  unambiguously increase with the ratio until then. Although after the unsuccessful rate converges at  $1 - l$ ,  $r_{St}$  and  $r_{Ft}$  ceases to increase, they remain strictly positive. And their shrinking effects on the worthy consulting interval continue. Since the decision thresholds are continuous in them, a mean-preserving spread of successful rates does not change the mean  $D(\overline{\alpha_t^*}) - D(\overline{\beta_t^*})$ , which settles beneath  $\frac{1}{2}$  after the  $T^*th$  venture.  $\square$

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