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Mei Dong
Toshiaki Shoji
Yuki Teranishi

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1-7-10-703 Iidabashi, Chiyoda-ku, Tokyo 102-0072, Japan

Abstract

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Mei Dong
University of Melbourne
Department of Economics
Level 4, FBE Building, 111 Barry Street,
Parkville 3010, Australia
mei.dong@unimelb.edu.au

Toshiaki Shoji
Seikei University
Faculty of Economics
3-3-1 Kichijoji-Kitamachi, Musashino, Tokyo
180 - 8633, Japan
tshoji@econ.seikei.ac.jp

Yuki Teranishi
Keio University
Faculty of Business and Commerce
2-15-45 Mita, Minato, Tokyo 108-8345, Japan
yukitera@fbc.keio.ac.jp

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Mei Dong† Toshiaki Shoji‡ Yuki Teranishi§

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Abstract

This paper develops a price model with product cycles characterized by product entries and exits. Through a frictional product market with search and matching frictions, an endogenous product cycle is accompanied by a price cycle. This model nests the New Keynesian Phillips curve as a special case and generates several new phenomena in business cycle moments with product cycles. Using product-level micro data in Japan, we show that our price model well captures the observed features among product entry, number of products, demand, and price. Our model with a frictional product market replicates correlations between product matching probability and other variables. In a general equilibrium model for the Japanese economy, an endogenous product entry increases a price variation by 23 percent. This number increases to 35 percent with a price discounting after a first price. All results suggest that product cycles and search frictions play fundamental roles in describing price dynamics.

Keywords: Phillips curve; product and price cycles; search and matching

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†University of Melbourne. E-mail: `mei.dong@unimelb.edu.au`

‡Seikei University. E-mail: `tshoji@econ.seikei.ac.jp`

§Keio University. E-mail: `yukitera@fbc.keio.ac.jp`

1 Introduction

> "We have all visited several stores to check prices and/or to find the right item or the right size. Similarly, it can take time and effort for a worker to find a suitable job with suitable pay and for employers to receive and evaluate applications for job openings. Search theory explores the workings of markets once facts such as these are incorporated into the analysis. Adequate analysis of market frictions needs to consider how reactions to frictions change the overall economic environment: not only do frictions change incentives for buyers and sellers, but the responses to the changed incentives also alter the economic environment for all the participants in the market. Because of these feedback effects, seemingly small frictions can have large effects on outcomes."
>
> Peter Diamond

> "Price dynamics in imperfectly competitive markets result from the interplay of sellers' and buyers' strategies. Understanding the microeconomic determinants of price setting and their welfare or macroeconomic implication - such as the role of friction in monopolistic competition or the effects of inflation - therefore requires an analysis which incorporates the decision problems of both types agents."
>
> Roland Benabou

Product creation and destruction indicate the existence of product cycles. The behavior of prices during a product cycle forms a price cycle. We use Nikkei point of sale (POS) scanner data to document several stylized facts about product cycles and price cycles in Japan. We then develop a new model with a frictional product market that endogenously generates product cycles to understand product entries and exits, price dynamics, and their interactions.

Recent observations from micro data reveal interesting and new facts about these product cycles and price cycles. Broda and Weinstein (2010) find, when using the universe of products data, that the product turnover rate in the US is about 25 percent annually. They reveal that these product replacements have a significant effect on the aggregate price index. Nakamura and Steinsson (2012) highlight that product turnover is a key mechanism for explaining price changes using micro data on trade price indices in a discussion of the so-called a product replacement bias. A first price has a nontrivial effect on price dynamics. The authors also show that 40 percent of products are replaced without any price change after being introduced into markets. The first price and subsequent prices for a product behave differently during the product cycle. Ueda et al. (2019) confirm the same facts for Japan using matched samples of the Nikkei POS data. They reveal that the product turnover rate is 30 percent annually. Price adjustment occurs in times of product turnover and more than half of products do not experience price changes until their exits from the market.[1] On average, they show that a product's price declines after the first price, and that the price increases when replacing an existing product by a new product. This is another pattern of the price cycle during a product cycle.[2]

Using Nikkei POS scanner data for Japan, we provide further evidence about product cycle features. We find that the product cycle at the product level is about nine quarters. During the product cycle, prices generally decline, and first prices are 38 percent higher than average prices. In terms of business cycle moments, we document that product entry and price/demand have a weak positive correlation, but the number of products and price/demand exhibit a strong positive correlation.

We propose a new model that embeds features of product and price cycles in Japan to explain product entry and exit, price dynamics, and their correlations observed in the Nikkei data. We explicitly model entry and exit in a frictional product market where producers and retailers search for each other to form a match. Each new match is considered a new product. We endogenize entry decisions and leave exits as exogenous. Retailers decide to enter into the market when the benefit from selling the new product can cover the cost of entering the market. As the number of new matches determines the number of new products, the total number of products in the market varies according to business cycles.

[1] For Japanese data, Abe et al. (2017) show that first prices have a significant effect on the price index for daily necessities and foods in Japan.

[2] Abe et al. (2016) also find that prices decline after first prices using data of the most popular price comparison website in Japan. Their data include home electrical appliances and digital consumer electronics.

As for price setting, we assume that first (new) prices are set upon matching and that the subsequent prices in the match follow an exogenous path given the first price. We consider two types of price cycles after the first price in the simple model: a fixed price and a declining price at a constant discount rate that is prevalent in Japanese data. In both cases, only first prices are flexibly set. As the aggregate price index includes both new prices and existing prices, the fraction of new products has an important role in determining the aggregate price index. The model naturally links product cycles with price dynamics. In particular, the aggregate price index responds to business cycles through an extensive margin effect and an intensive margin effect. For example, in response to a positive demand shock, more retailers will enter the market which leads to more matches in the product market. This makes the aggregate price more flexible thanks to more new prices. Moreover, in each new match, the positive demand shock raises the total trading surplus, which leads to a higher new price. Overall, both entry and prices can be positively correlated with demand. The endogenous product cycles allow our model to generate rich price dynamics.

For the quantitative analysis, we use the Japanese Nikkei data to calibrate our model's key parameters. The simulation results show that our model can replicate the observed facts in the data related to product cycles and price cycles. Overall, the model with a declining price pattern performs the best because it builds in the price discounting feature that is prevalent in the Nikkei data.

We extend the simple model to a general equilibrium model where retailers sell their products to households in a monopolistic competition market. In addition to the constant and declining price patterns, we introduce endogenous price discounting by assuming depreciation of quality/preference. Calibrating the model to the Japanese economy, we find that endogenizing product entry into the frictional product market can increase the standard deviation of the inflation rate by 23 percent. This number increases to 35 percent with price discounting. This result confirms the importance of embedding product cycles and price cycles into a model to understand price dynamics.

To explain price dynamics, we review many former studies incorporating different concepts and specifications. Many papers on the New Keynesian Phillips curve assumes Calvo (1983) and Yun (1996) price adjustment, in which firms optimally change prices with a certain probability. Their price adjustment mechanism provides a useful proxy for price stickiness.[3] Golosov and Lucas (2007) set up a menu cost model in which a price is changed when a firm can pay a real menu cost under idiosyncratic productivity shocks and general inflation. Their model explains a mechanism behind the New Keynesian Phillips curve that is based on exogenous probability of a price change.[4] Mankiw and Reis (2002) develop a sticky-information model. They assume that information diffusion is slow and that information updating makes it costly to reset a good's price. In contrast to these previous studies, our model with a frictional product market highlights the role of product entries and exits in determining the aggregate price index.

Several studies have considered goods markets or product markets with search and matching frictions.[5] Michaillat and Saez (2015) assume a search and matching product market. They show that productive capacity is idle in the US and such an operating rate implies that sellers face search frictions to find buyers. They match the model to data and show that a fixed-price model describes the data better than a flexible price model does. Petrosky-Nadeau and Wasmer (2015) develop a DSGE model with search and

[3] Christiano et al. (2005) and Smets and Wouters (2007) show that this New Keynesian Phillips curve based on Calvo (1983) and Yun (1996) price adjustment fits macro data well.

[4] Gertler and Leahy (2008) develop a tractable state-dependent Phillips curve in contrast to a time-dependent Phillips curve based on the Calvo mechanism. They assume that firms that are in a position to obtain benefit over cost are able to reset an optimal new price. This Phillips curve with an Ss foundation has the same form as the New Keynesian Phillips curve. The only difference between the two Phillips curves is a larger response to demand reflecting greater flexibility of price adjustment in the Phillips curve with an Ss foundation. Furthermore, Woodford (2009) shows the similarities and differences between a state-dependent pricing model and a time-dependent pricing model under limited information.

[5] In the international trade literature, Drozd and Nosal (2012) introduce search and matching frictions into goods trading between two countries to solve puzzles regarding the correlation between prices of real exports and imports and volatilities of the real exchange rate. Eaton et al. (2016) assume a search and matching process for international buyer-seller connections to explain various empirical issues. These papers support the validity of embedding search and matching into a goods market.

matching frictions in credit, labor and goods markets. Their main goal is to understand how frictions in these markets affect labor market dynamics in response to productivity shocks. Bai et al. (2017) consider a frictional goods market and argue that demand shocks that induce more search can increase output. Their quantitative results show that demand shocks can explain a large share of business cycle fluctuations. None of these papers with a frictional goods or product market, however, focus on the role of product cycles on price dynamics.

Empirical studies, such as that by Barrot and Sauvagnat (2016), show that there exist search and matching frictions in production networks using firm-level data. They find that the occurrence of natural disasters on suppliers reduces output to their customers when these suppliers produce specific input goods. This implies that specific input goods are not traded in a centralized market where search frictions are absent. Carvalho et al. (2021) also show that individual firms cannot quickly find suitable alternatives in a decentralized goods market with search frictions when firms experience a supply-chain disruption caused by a natural disaster in Japan.

The rest of our paper is organized as follows. Section 2 describes our data and the main empirical observations. Section 3 introduces the simple partial equilibrium model with product cycles. We compare three versions of the simple model by inspecting the corresponding New Keynesian Phillips curve in Section 4. Section 5 provides quantitative analysis using the Nikkei data. The general equilibrium version of our model is discussed in Section 6. We consider a few extensions of our model in Section 7. Section 8 concludes.

2 Data and Observations

We use Nikkei POS scanner data.[6] Our data include sales prices and quantities for each product at each retail shop on each day from April 1988 to December 2017. The retail shops in our data set consist of supermarkets in Japan, where typically food products

[6]See Appendix A for details of data for an average price, the entry rate, the exit rate, and product varieties.

and daily necessities are sold. At the start of our sample period there are more than 200 supermarkets in our data set, increasing to 300 at the end of the sample. In our analysis, we basically restrict our data set to the 11 supermarkets that appear in the full sample to exclude any bias in price setting caused by shop bankruptcy. We interpret the data from the 11 supermarkets as a random sample to observe product cycles and price cycles at the product level as shown below.

A barcode including the Japanese Article Number (JAN) code is printed on all products and products are distinguished by fairly detailed classifications.[7] In addition, the barcodes provide information about the product category (such as butter, yogurt, or shampoo) and the producer of each product. In our 11-supermarkets sample, the data include about 890,000 products in total, about 100,000 products on average per year, and about 30,000 products on average per retailer per year. Our scanner data cover 170 of the 588 items in the Consumer Price Index (CPI) in Japan.[8]

An advantage of these data is that we can observe product cycles. An individual product's life cycle can be clearly identified through its entry into and exit out of the product market. We can also observe price cycles, by which we refer to how the individual product's price changes during its life cycle. The price information can provide new evidence about price setting behavior. For example, one interesting feature in our data is that the first price of a product behaves differently from the subsequent prices. With entries and exits of products, both the weights and prices of new products can affect the aggregate price index. In addition, our sample period is long enough to cover several business cycles in Japan so that we can examine how product cycles and price cycles interact with business cycles.

[7] In the JAN code, the first seven digits indicate the company code and the last six digits indicate the individual product. When JAN codes are different for the same type of products by the same company, these products are different in terms of packaging, ingredients, etc.

[8] Our data do not include fresh food, recreational durable goods, such as computers and cell phones, and services such as housing rent and utilities.

Observation 1: product cycles. The product cycle at the product level is about nine quarters. The product entry rate is more volatile than the product exit rate.

Our data set contains information about the prices and quantities of products sold in Japanese supermarkets. The JAN codes allow us to identify the numbers of new products and exiting products, which are used to calculate product entry and exit rates. We calculate the duration of a product by taking the inverse of the product exit rate. Table 1 shows basic statistics of quarterly product entry and exit rates. The product entry rate is calculated as the number of newly introduced products by producers in a given quarter divided by the total number of products in that quarter. We identify a new product when the new product appears in at least one retail shop. The product exit rate is calculated as the number of exiting products in a given quarter divided by the total number of products in that quarter. We classify an existing product as exiting from the market when none of the retail shops sell the product. Note that we define entry rates and exit rates at the product level and not at the shop level.[9] The average product entry rate of all products is 0.12 and its standard deviation is 0.023. However, the average product exit rate is 0.11 and its standard deviation is 0.012. The product exit rate implies that the product cycle is on average nine quarters.

Figure 1 shows the product entry rate and product exit rate at the product level over the sample period. We observe that both the product entry rate and product exit rate vary over the business cycle. The product entry rate is more volatile than the product exit rate, which is evident from their standard deviations.

Observation 2: price cycles. The average of new prices is 38 percent higher than the average price. Prices exhibit a declining price pattern on average.

We analyze prices at the product level and focus on regular prices that are given by modal prices from daily data and exclude temporary sales prices.[10] Based on regular

[9]See more details of the definitions of the entry rate and the exit rate in Appendix A.

[10]See details of the definitions of prices in Appendix A.

prices, we calculate average prices across products and shops for each quarter. We use weights equal to the sales amount to calculate average prices. As shown in the third row of Table 1, the standard deviation of the average price is 0.57 when we normalize the average price to 2.7, which is the steady-state value of the average price used in the simulations in the latter sections.

Regarding the first (new) price and the subsequent prices for one specific product, we find that the first prices behave differently from the subsequent prices. The fourth row of Table 1 shows that the average of new prices is 38 percent higher than the average price. This implies that first prices are set higher than the subsequent prices, and that prices decline thereafter. Figure 2 shows how price changes after entry on average.[11] Prices decline after entry.[12] This is consistent with Ueda et al. (2019), who use the same data as ours but match successor products and predecessor products to account for product turnover, creating a so-called matched sample. In their calculation, the new price of successor products is about 10 percent higher than that of the predecessor product on average and a price declines after the first price. Ueda et al. (2019) rationalize this declining price pattern by a fashion effect. This effect suggests that new products attract higher demand and therefore have higher prices. Then, prices start to decline with the reduction of demand over time.

Figure 3 shows the percentages of products experiencing price increases, price decreases, or no price change over their life cycles.[13] On average, 23 percent of products experience a declining price pattern and 16 percent of products experience an increasing price pattern after entry. In the last 10 years, the ratio of products with a declining price pattern increases to 29 percent on average, although the ratio of products with an increasing price pattern does not change. The fraction of products that experience either

[11]Note that products are restricted to those with life-spans of 20 quarters or more. As shown in Figure A2, we observe a similar declining price pattern for shorter life-span products.

[12]Similar declining price patterns have been found in Melser and Syed (2014) and Abe et al. (2016). Melser and Syed (2014) use a large US scanner data set on supermarket products and find that prices decline as items age on average. Abe et al. (2016) use online price data and show that new product prices decrease gradually after entry and the speed of price decline varies considerably across products.

[13]A similar figure appears in Ueda et al. (2019), although they use a matched sample.

no price change or a declining price after entry is 84 percent on average. On average, prices exhibit a declining price pattern. It is essential to incorporate these features in our analysis of price dynamics in Japan.

We argue that regular prices at retail shops reflect producers' prices – prices set by producers for retailers. In contrast, temporary sales prices are normally set by retail shops to attract customers and may not necessarily reflect producers' prices. This is strongly supported by the relationship between the CPI and the Corporate Goods Price Index (CGPI) in Japan. Here, CPI corresponds to prices between consumers and retail shops and CGPI corresponds to prices between producers and retail shops. Note that CPI excludes temporary sales prices. Figure 4 shows two indices that correspond to product categories in the Nikkei data.[14] The first log differences of these indices co-move closely and their correlation is as high as 0.91 at the quarterly frequency. Moreover, Nakamura and Zerom (2010) show that the majority of incomplete pass-through of a cost-push shock arise at the level of manufacturer prices rather than retail prices in the coffee industry in the US. This implies that producers' prices for retailers and retailers' prices for consumers should be highly correlated.

Observation 3: business cycle moments. Product entry and price/demand have a weak positive correlation. The number of products and price/demand have a strong positive correlation. First prices are more volatile than average prices.

Table 2 shows the correlations among product entry rates, product exit rates, number of products, prices, and demand. To show the robustness of these statistics, we show correlations at both the quarterly frequency and the annual frequency. There is a weak correlation between product entry rates and prices. The correlation between product entry rates and prices is 0.1 at the quarterly frequency and 0.44 at the annual frequency.

[14]CPI is published by the Statistics Bureau of Japan and CGPI is published by the Bank of Japan. Nikkei data cover 17 percent of the prices in the CPI and 13 percent in the CGPI in a sales weight base, respectively.

Product entry rates and demand also exhibit a weak correlation. The correlation between the entry rate and demand is 0.12 at the quarterly frequency and 0.5 at the annual frequency.[15] We calculate demand as final sales at retail shops including special sales in addition to regular sales. Note that this is not sales between retailers and producers. These correlations are not so strong because search frictions in product markets can reduce these correlations. Moreover, these results are accompanied by a positive correlation between prices and demand. The correlation between prices and demand is 0.84 at the quarterly frequency and 0.8 at the annual frequency. Furthermore, the total number of products in the market is also related to prices and demand. The data show pro-cyclicality between the number of products and price/demand. The correlation between the number of products and prices is 0.74 at the quarterly frequency and 0.79 at the annual frequency. The correlation between the number of products and demand is 0.81 at the quarterly frequency and 0.87 at the annual frequency.

The different nature of first prices and subsequent prices is also reflected by the observation that the standard deviation of first prices is much larger than that of average prices. Figure 5 shows year-to-year changes of the average of new prices and the average price. First prices are more volatile than subsequent prices and first prices tend to decide movements of the average prices. The ratio of the standard deviation of new price averages to the standard deviation of average prices is 2.34, as shown in Table 1. As shown in Figure 3, on average 61 percent of products do not experience any price change after entry in Japan. No price change after entry contributes to different standard deviations between first prices and existing prices. If price setting for a new product and an existing product were the same, the ratio of their standard deviations should be close to one. Moreover, as observed in a price cycle, a declining price after entry may induce producers to set a sufficiently high first price. This can also contribute to a higher standard deviation of first prices.

[15] Ueda et al. (2019) also find a weak correlation between the entry rate and demand using matched samples.

3 Model with Product and Price Cycles

We begin with a simple partial equilibrium model with search frictions in the product market. The search frictions naturally give rise to endogenous product cycles. We embed features of product cycles and price cycles shown in Observation 1 and Observation 2 into our model to explore whether the model can generate the key business cycle moments that are identified in Observation 3.

3.1 Setting

Time is discrete and continues forever. There are two types of firms: producers and retailers. They trade product A in a decentralized market. In particular, producers can produce product A. Retailers have demand for product A, but cannot produce product A. Therefore, producers and retailers search for each other in a decentralized product market. We can interpret retailers that repackage intermediate products produced by producers to supply final goods to households. The measure of producers is 1. Retailers can choose to enter the product market at a cost κ.

Let the measure of unmatched producers be u_t at time t and the measure of vacant retailers be v_t. The matching function exhibits a constant return to scale and is given by

$$m\left(u_t, v_t\right) = \chi u_t^{1-\alpha} v_t^{\alpha} \text{ where } \alpha \in (0,1)\,. \tag{1}$$

Define market tightness as $\theta_t = v_t/u_t$. The probability of a vacant retailer finding an unmatched producer is denoted as $s(\theta_t)$ and the probability of an unmatched producer finding a vacant retailer is denoted as $q(\theta_t)$, where

$$s(\theta_t) = \frac{m_t}{v_t} = \chi\theta_t^{\alpha-1}, \tag{2}$$

$$q(\theta_t) = \frac{m_t}{u_t} = \chi\theta_t^{\alpha}. \tag{3}$$

We assume that $s(0) = 1$ and $q(\infty) = 1$. To simplify notation, we use (s_t, q_t) directly and omit the argument θ_t when there is no confusion. Each match is destroyed with an exogenous probability $\rho \in (0,1)$.

Once a producer and a retailer match, the producer produces Z^A units of product A for the retailer and a new price of product A is negotiated through the Nash bargaining protocol. In the model, we assume that the negotiated price changes by g from time t to time $t+1$ according to the contract during the duration of the match. There is no renegotiation of the price after the new price is determined. New prices are negotiated only when new matches are formed. This infrequent negotiation of prices directly follows Shimer (2004) and Hall (2005) in labor search models. For simplicity, the amount of product A transferred in each match is exogenously given by Z^A. Moreover, the cost of producing Z^A units of product A is X_t, where X_t can include any cost of production even though we do not specify the production function at this stage. Changes in X_t could be interpreted as potential cost-push shocks. The benefit for retailers of acquiring Z^A units of product A is given by Z_t^B, where Z_t^B is a random shock and depends on the sales revenue of the final good. We view Z_t^B as a demand shock to product A.

The free entry condition for a retailer is

$$\kappa = \beta s_t \mathbb{E}_t V_{t+1}\left(\tilde{P}_{t+1}^A\right), \tag{4}$$

where $\mathbb{E}_t$ is the expectations operator. We assume that each match entails a new product. Thus, this free entry condition decides the number of new products in the product market. Retailers decide to enter into a market when the profit from selling a new good with a new price is enough to cover the cost of entry. If the retailer is matched with a producer, production and trade will take place in the following period, where $\tilde{P}_{t+1}^A$ denotes the newly negotiated price of product A and $V_{t+1}(\cdot)$ denotes the value function for the retailer. Note that there is a one-period lag for production after a new match, as in the timeline of Trigari (2009).

The value function for the retailer with a contract price of $\tilde{P}_t^A$ is

$$V_t\left(\tilde{P}_t^A\right) = Z_t^B - Z^A\tilde{P}_t^A + \beta(1-\rho)\mathbb{E}_t V_{t+1}\left(g\tilde{P}_t^A\right), \tag{5}$$

where g captures changes in the price $\tilde{P}_t^A$ set at time t for time $t+1$. All matches that survive from time t to time $t+1$ are subject to the same price adjustment factor g. The term $Z_t^B - Z^A\tilde{P}_t^A$ is the flow benefit of being in a match and $\beta(1-\rho)\mathbb{E}_t V_{t+1}\left(g\tilde{P}_t^A\right)$

shows the continuation value of the match. The new price $\widetilde{P}_t^A$ for product A is set only by new matches. The adjustment of the contract price from time t to time $t+1$ is inherent in the contract.

Now consider the value functions for a producer. Let $J_t^1\left(\tilde{P}_t^A\right)$ denote the value function for a newly matched producer with a negotiated new price $\tilde{P}_t^A$ at time t and

$$J_t^1\left(\tilde{P}_t^A\right) = Z^A\tilde{P}_t^A - X_t + \beta\mathbb{E}_t\left[(1-\rho)\,J_{t+1}^1\left(g\tilde{P}_t^A\right) + \rho J_{t+1}^0\right]. \tag{6}$$

The flow benefit of the match is given by the term $Z^A\tilde{P}_t^A - X_t$. If the match survives at time $t+1$, the continuation value is $J_{t+1}^1\left(g\tilde{P}_t^A\right)$, where g again indicates the price adjustment within a match. If the match is destroyed at time $t+1$, the producer becomes unmatched with the value function J_{t+1}^0. The value of an unmatched producer is

$$J_t^0 = \beta\mathbb{E}_t\left[q_t J_{t+1}^1\left(\tilde{P}_{t+1}^A\right) + (1-q_t)J_{t+1}^0\right]. \tag{7}$$

The unmatched producer can go back to the product market in the same period and find a match with probability q_t. Production will take place in the following period and the value of the match is therefore $\mathbb{E}_t J_{t+1}^1\left(\tilde{P}_{t+1}^A\right)$. With the complementary probability $1-q_t$, the unmatched producer remains unmatched and has the continuation value J_{t+1}^0. Here, the benefit from having a new match is $J_t^1\left(\tilde{P}_t^A\right) - J_t^0$ for the producer.

In a match, the retailer and the producer bargain over the price $\tilde{P}_t^A$ of product A, taking into consideration that the price is not renegotiated during the duration of the match and the price can adjust by the factor g from time t to time $t+1$. The price $\tilde{P}_t^A$ solves

$$\max_{\tilde{P}_t^A}\left[V_t\left(\tilde{P}_t^A\right)\right]^{1-b}\left[J_t^1\left(\tilde{P}_t^A\right) - J_t^0\right]^b, \tag{8}$$

where b is the bargaining power for the producer. The solution $\tilde{P}_t^A$ is determined by

$$bV_t^A\left(\tilde{P}_t^A\right) = (1-b)\left[J_t\left(\tilde{P}_t^A\right) - J_t^0\right]. \tag{9}$$

Lastly, we describe the flow conditions and the aggregate price index. Following Trigari (2009), a newly separated producer can search again in the same period. The measure of unmatched producers is

$$u_t = 1 - (1-\rho)\,N_t, \tag{10}$$

where N_t denotes the measure of matches. The flow condition of u_t is therefore

$$u_{t+1} - u_t = \rho\,(1 - u_t) - q_t u_t. \tag{11}$$

It follows that

$$N_t = (1-\rho)\,N_{t-1} + q_{t-1}u_{t-1}. \tag{12}$$

As prices in the new matches are set through Nash bargaining and the old prices in survived matches adjust by the factor g from time t to time $t+1$, we use an aggregate price index P_t^A to denote the aggregate price in the economy at time t,

$$N_t P_t^A = (1-\rho)\,g N_{t-1} P_{t-1}^A + \chi\theta_{t-1}^{\alpha} u_{t-1} \tilde{P}_t^A. \tag{13}$$

The aggregate price index completes the description of the model, where (2), (3), (4), (5), (6), (7), (9), (10), (12), and (13) are used to solve the model.[16]

The inclusion of the price adjustment factor within a match is motivated by our second observation that prices decline after the first price in the Japanese data, as shown in Figure 2. We take this declining pattern as given and embed it into our model. Earlier studies rationalize this declining price pattern through the fashion effect or the product quality signaling effect. In our model, the producer and the retailer in a match negotiate the first price, taking into account of the passive price discounting after the first price. In the steady-state, an aggregate price index P^A from (13) leads to

$$P^A = \frac{\rho}{1-(1-\rho)\,g}\tilde{P}^A, \tag{14}$$

where $\tilde{P}^A$ is the steady-state price of product A in a new match. Given that $g \in (0,1]$, we have $P^A \leq \tilde{P}^A$. This can generate a difference between the average of the first prices and the average prices. In our model, entry decisions are endogenous and depend on the parameters and shocks. However, we treat exits as exogenous. This modeling choice is consistent with Japanese data, show a sufficiently larger standard deviation of product entry rates than that of product exit rates. The model can generate an endogenous number of products and can be used to examine how product entry is correlated with

[16] See Appendix B for more details of this model.

price and demand. This is a model with price discounting when $g < 1$. In the following, we consider two special cases: one with no price discounting, i.e., $g = 1$ and one with exogenous entry.

3.2 Model with Endogenous Entry and $g = 1$: No Price Change After a First Price

One special case of the model is to assume that the price is fixed after setting the first price. That is, $g = 1$ for all t. In our data, more than half of products do not experience price changes after entry. In this model, a new price is still set optimally only when a new match is endogenously created in the product market and there is variation of the number of products in the market so that an extensive margin effect exists for price changes.

3.3 Model with Exogenous Entry

Another special case is to assume that entry into the product market is exogenous. In this way, we assume that the number of products in the product market is constant in each period. Both the entry probability and exit probability are also exogenous and constant, so that in some sense we exclude the role of product market frictions.[17] In this model, there is no extensive margin effect on price changes because the number of products is constant. We also assume $g = 1$ for all t. This model has a similar structure to the traditional New Keynesian model and will have very similar dynamics in response to demand shocks as discussed below. It can serve as a benchmark to facilitate comparison with other models and help us understand the role of search frictions in the product market.

[17] See Appendix C for details of this model.

4 Inspecting the Three Models

Before we simulate our models quantitatively, we highlight the role of an endogenous number of products by log-linearizing the system of equations around a constant steady-state with zero inflation. Linearized price equations are convenient to reveal the features of price dynamics, in particular compared with the New Keynesian Phillips curve by Calvo (1983) and Yun (1996). To maintain a fair comparison with the New Keynesian Phillips curve, we use relative prices in the value functions in which individual prices set by producers and retailers are divided by the aggregate price.[18] We express the log-deviation of a variable (e.g., P_t) from its steady-state value ($\bar{P}$ or P) by placing a hat ($\hat{}$) over the lower case symbol ($\hat{p}_t$).[19]

For the model with exogenous entry, we have the following linearized price equation

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + b \frac{\rho \left[1 - \beta(1-\rho)\right]}{1-\rho} \frac{\bar{Z}^B}{Z^A} \hat{Z}_t^B, \tag{15}$$

where the inflation rate is defined as $\pi_t \equiv \hat{p}_t^A - \hat{p}_{t-1}^A$.[20] Current inflation simply depends on the expected future inflation and the demand shock $\hat{Z}_t^B$. The effect of search frictions in the product market appears only through ρ in the coefficient on the demand shock. The exit rate ρ works as a probability of re-setting prices as the spirit of the Calvo parameter because both the entry rate and the exit rate are constant in this model. The term $\bar{Z^B}/Z^A$ can be interpreted as the overall markup on product A.

With exogenous entry and exit rates, the model generates a constant number of products and a constant fraction of new products. The fraction of products that have a new price is also constant. There is no extensive margin effect on price changes in this model. Naturally, this model has very similar inflation dynamics to demand shocks as described by the New Keynesian Phillips curve with some differences in parameters.

When the exit rate ρ increases, inflation is more responsive to demand shocks. This is because new prices have a larger share in the aggregate price because old prices exit

[18] We provide details in Appendix D for a relative price model.

[19] A technical appendix for the log-linearization is available on request.

[20] To simplify the expressions, we assume that the cost shock X_t is zero and the steady-state aggregate price is one.

quicker. This model has another parameter related to the frictional product market, i.e., the bargaining power of producers. When b decreases, the inflation rate becomes less sensitive to demand shocks since retailers can take a larger share of the surplus and are likely to keep the price of input product A closer to its cost. Prices would change less in response to a demand shock.

In the model with endogenous entry and $g = 1$, we have the following linearized price equation.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \beta(1-b)\frac{\bar{q}}{1-\beta(1-\rho-\bar{q})}\frac{\rho\left[1-\beta(1-\rho)\right]}{1-\rho}\hat{\theta}_t$$
$$+b\frac{\rho\left[1-\beta(1-\rho)\right]}{1-\rho}\frac{\bar{Z}^B}{Z^A}\hat{Z}_t^B. \quad (16)$$

We can observe an explicit effect of product market frictions through market tightness $\hat{\theta}_t$. This generates a direct link between product cycles and prices. When the demand for products changes, the entry rate of retailers changes. Therefore, the number of products in the market also changes. More importantly, the number of new matches adjusts and this implies that the fraction of products with changing prices would adjust accordingly. These behaviors are summarized in market tightness. One way to interpret our results is that the model endogenizes the Calvo parameter through a search and matching product market. In this sense, we argue that there is an extensive margin effect associated with a price change. Market tightness is positively related to inflation and increases the price volatility.[21] Regarding the effects of parameters on inflation dynamics, the exit rate ρ, steady-state matching probability $\bar{q}$, and the bargaining power of producers b affect the response of the inflation rate to market tightness in (16).

In addition to (16), product market frictions captured by $\hat{\theta}_t$ accelerate/decelerate inflation dynamics as shown in the following equation,

$$\hat{\theta}_t = \beta(1-\rho-\frac{b}{1-\alpha}\bar{q})\mathbb{E}_t\hat{\theta}_{t+1} + (1-b)\frac{1-\beta(1-\rho)}{1-\alpha}\frac{\bar{Z}^B}{\bar{Z}^B - Z^A}\mathbb{E}_t\hat{Z}_{t+1}^B. \quad (17)$$

The market tightness $\hat{\theta}_t$ depends on the demand shock. Endogenizing entry decisions allows the market tightness to adjust, which further changes the inflation dynamics.

[21] In the general equilibrium model that we develop in Section 6, we incorporate preferences for the variety of goods through a price aggregator.

Here, (16) and (17) completely determine the dynamics of inflation and market tightness. When the exit rate ρ increases, market tightness is relatively more sensitive to demand shocks than to the expected future market tightness because of a shorter product cycle. The bargaining power of producers b, matching elasticity α, and steady-state matching probability $\bar{q}$ also affect the dynamics of market tightness.

For the model with endogenous entry and price discounting, we have the following linearized price equation.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{1-g}{(1-\rho)g} \left(\hat{m}_{t-1} - \hat{n}_t\right) - \beta \left(1-g\right) \mathbb{E}_t \left(\hat{m}_t - \hat{n}_{t+1}\right)$$
$$+(1-b)\beta\bar{q}\bar{S}\frac{1-\beta(1-\rho)g}{Z^A}\frac{\rho}{1-\rho}\left[\hat{\theta}_t + \frac{1-g}{g}\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(1-\rho\right)^j \hat{\theta}_{t+j}\right]$$
$$+b\frac{1-\beta(1-\rho)g}{g}\frac{\rho}{1-\rho}\frac{\bar{Z}^B}{Z^A}\hat{Z}_t^B,$$

where $M_{t-1} = q_{t-1}u_{t-1}$ denotes the number of new products at time t because production takes place one period after matching. The term $\hat{m}_{t-1} - \hat{n}_t$ expresses the share of new products in the total products, i.e., the entry rate. The effect of product entry is explicitly included in the model. Inflation can increase when the entry rate increases because the weight of existing products in the aggregate price decreases relatively and the weight of new prices increases. With price discounting, a higher fraction of new prices leads to a higher aggregate price and inflation. Moreover, the response of inflation to demand shocks increases as g decreases because retailers have incentives to set a higher first price due to price discounting after entry, as observed in the model before log-linearization. This effect directly increases the volatility of first prices. Price discounting provides a mechanism to raise the ratio of the standard deviation of first prices to that of average prices. The expected future market tightness works to amplify the inflation dynamics and to increase the persistence of the inflation rate.

5 Analysis using the Nikkei Data

Now, we show the performance of our model in Section 3 using the Nikkei data in Japan. We calibrate the models with exogenous entry, with endogenous entry and $g = 1$, and with endogenous entry and price discounting. Note that we use levels models for the simulations. The discount rate β is 0.99 as in conventional models and the exit rate ρ is 0.11 from our data as shown in Table 3. The bargaining power of firm b is 0.5, so that sellers and buyers hold an equal bargaining power.

In the price discounting model, we set $g = 0.95$ based on the differences between new prices and average prices from quarterly data as shown in Table 3. We consider a positive demand shock with a persistence parameter of $\rho_{Z^B} = 0.9$ estimated using the data, and set its standard deviation to match the standard deviations of average prices between the model with price discounting and data.

We have three parameters (χ, α, κ) that must be calibrated. Both χ and α are parameters in the matching function. There is less evidence in the literature guiding their values. We attempt to use our data to calibrate these parameters. In particular, we take the log transformation of (2) and estimate the values of χ and α by constructing measures of s_t and θ_t. To this end, we first construct measures of the matching probabilities for all retailers as follows.[22] The matching probability for a given product in a given period is calculated by the number of retail shops selling this product divided by the total number of retail shops. The underlying assumption behind this calculation is that all shops would like to carry this product. After calculating the matching probabilities for each product, we take the average of the matching probabilities as the matching probability for a given quarter. This allows us to construct a time series of retailers' matching probabilities for all products that correspond to $N_t/(N_t + v_t)$.

In the data, the JAN code allows us to calculate the time series of N_t. We count a match between a product and a retail shop as one using the 11-shops sample. This is the number of products at the shop-product level and is consistent with our matching

[22]More discussions about the matching probabilities and their implications for search frictions are in Section 7.1.

probability. Then we can infer the time series of v_t. Using (10) and (12), we calculate the time series of u_t and M_t. The values of u_t, v_t and M_t enable us to calculate the time series of s_t and θ_t. We regress $\log(s_t)$ on $\log(\theta_t)$ to estimate the values of $\chi = 0.2$ and $\alpha = 0.08$.[23] Finally, κ is calibrated by targeting the steady-state value of θ_t and $\kappa = 0.15$.

Table 4 contains our simulation results. The second column presents statistics from the data. The third to fifth columns include results from the model with exogenous entry, the model with endogenous entry and $g = 1$, and the model with endogenous entry and price discounting. The model with exogenous entry includes a constant entry rate and a constant number of products so that the correlations between the entry rate or the number of products and other variables are essentially zero. This model produces a constant fraction of new products that can have new prices. Therefore, it resembles the New Keynesian Phillips curve with Calvo price adjustment and demonstrates a positive correlation between demand and prices of 0.75 in our model and 0.84 in the data. The ratio of the average of new prices to the average price is 1 because there is no mechanism for new prices to deviate from existing prices on average. For the ratio of the standard deviation of new prices to the standard deviation of average prices, the model with exogenous entry explains 60 percent of the variation in the data.

The model with endogenous entry generates a positive correlation between entry and demand. This model, however, cannot generate a positive correlation between entry and prices because variations of new prices are insufficient to change average prices as explained below. In the model, the correlation between entry and demand (prices) is 0.32 (-0.05) compared with 0.12 (0.1) in the data. Regarding the number of products, the correlation between the number of products and demand (price) is 0.68 (0.78) compared with 0.81 (0.74) in the data. A positive demand shock raises the benefits for retailers to enter the product market. As more retailers enter the market, the total number of

[23] We assume that the maximum number of matches of N_t is given by $(3/2)N_t$ in each period, where $3/2$ is the ratio of the maximum number of matches over the average number of matches in data. The sample period is from 1988Q2 to 2017Q3. The estimated parameters are statistically significant at the 1 percent level.

matches increases and is positively correlated with demand and prices. Search frictions in the product market restrain the number of new matches and lower the correlation between entry and demand (price). These correlations depend on the degree of matching frictions, which is governed by the matching function. When we change parameters in the matching function, these results change significantly, which highlights the important role of matching frictions in explaining the data. Moreover, in each new match, the positive demand shock raises the total trading surplus, which leads to a higher new price. As a result, the average price increases. It follows that the correlation between prices and demand is also positive, which is 0.75 in the model and 0.84 in the data. The model is able to capture the key aspects of product cycles and prices. For the ratio of the standard deviation of new prices to the standard deviation of average prices, the model gives 1.37 compared with 2.34 in the data and explains 59 percent of the variation in the data. Because of product market frictions, the standard deviation of prices is 38 percent higher in the model with endogenous entry compared with the model with exogenous entry.

In the model with price discounting, we observe a positive correlation between entry and demand/prices. The correlation between the entry rate and demand (prices) is 0.32 (0.03) compared with 0.12 (0.1) in the data. The correlation between the number of products and demand (prices) is 0.68 (0.82) compared with 0.81 (0.74) in the data. The correlation between demand and prices is 0.82, which is very close to 0.84 in the data. The advantage of this model is that there is a mechanism for new prices to deviate from existing prices on average. With price discounting, retailers have incentives to set a higher first price because of price discounting after entry. This effect directly increases the volatility of first prices and is sufficient to change the average price so that entry has a weak positive correlation with price. The average of new prices can deviate from the average price, with the ratio of the average of new prices to the average price of 1.39, which is consistent with data. Regarding the ratio of the standard deviation of new prices to the standard deviation of average prices, the model gives 1.74 compared with 2.34 in the data. Thus, the model with price discounting explains 74 percent of the variation of the data. Product market frictions and price discounting raise the standard deviation of prices by 49 percent compared with the model with exogenous entry. It shows that the model with price discounting performs the best among the three models because it successfully captures all features of the data.

6 Quantitative General Equilibrium Model

The simple model is a partial equilibrium model where producers' cost of production X_t, quantity traded in each match Z^A, and retailers' benefit from trading Z_t^B are all exogenous. In this section, we extend the simple model of product cycles to a general equilibrium model by endogenizing X_t, Z^A, and Z_t^B. In addition to producers and retailers, we introduce a representative household and a central bank into the model.[24]

The representative household has love-of-variety preferences and purchases a variety of products from retailers. As usual, the household optimally makes intertemporal decisions about the demand for the aggregate consumption basket, amount of asset holdings, and labor supply. We assume that each retailer carries a distinct variety. Retailers set the price of each variety with a constant markup on the price of the product from producers following the standard monopolistic competition structure. To acquire products to sell to households, retailers search for producers in the frictional product market. The structure of the frictional product market remains the same as in the simple model, where the first price is set by sharing a trading surplus. To close the model, we assume that the central bank sets the return on assets, i.e., the interest rate, following a Taylor-type rule.

We again consider several versions of the model depending on our assumptions about entry and how subsequent prices evolve after the first price is set in a match. The first version of the general equilibrium model assumes that the number of product entries into the product market is exogenous and constant, and subsequent prices do not change after the first price. In the second version, we endogenize entry decisions by retailers and can

[24] See Appendix E for details of the general equilibrium model.

examine how endogenous product cycles affect prices. In the third version, we maintain the endogenous product cycles and assume price discounting after first prices, where the price discounting factor is exogenous. The three versions of our models correspond to the three versions that we consider in the partial equilibrium framework.

Lastly, we introduce the fourth version, which modifies the third version by allowing the price discounting factor to respond to the number of new products. That is, in the demand function for an individual product, we assume the following price discounting,

$$y^*_{t,t+j}(i) = \left[\frac{g_{t,t+j}h_t(i)}{H_{t+j}}\right]^{-\varepsilon} \frac{C_{t+j}}{N_{t+j}},$$

where $y^*_{t,t+j}(i)$ is the demand with a price discount of $g_{t,t+j}$ on $h_t(i)$ for product i at a retail shop i in period $t+j$, $h_t(i)$ is the price of product i with the first price set in period t, H_{t+j} is the aggregate price index in period $t+j$, ε is the parameter that governs the degree of substitution among different varieties of products, and

$$g_{t,t+j} = \prod_{d=t}^{t+j-1} \left(\frac{M_d}{\bar{M}}\right)^{-\xi},$$

where $g_{t,t} = 1$, $g_{t,t+j}$ is the price discounting factor for $j \geq 1$, and ξ is the degree of price discounting by the number of new products M_t. The number of products N_{t+j} in the expression of $y^*_{t,t+j}(i)$ decreases the demand for an existing product i when the number of new products increases as in Bilbiie et al. (2007). In addition to this direct effect, the number of new products has an indirect effect on price and demand through the price discounting factor. The price discounting factor indicates that product i's price declines when the number of new products increases. This captures a fashion effect because new products compete with existing products. Ueda et al. (2019) show that the fashion effect reduces the prices of existing products because the availability of new products lowers demand for existing products. In response to a positive demand shock, both price and demand for an existing product can decrease because of the increase in the number of new products. Conversely, both price and demand increase in response to a negative demand shock.

As for our calibration strategy, we borrow parameters from the simple model that correctly replicates features in the Nikkei data at the product level, as shown in Table 3. In addition to the parameters from the micro product data, we set the macro level parameters as shown in Table 5. We use conventional values for the Japanese economy following Sugo and Ueda (2008). We set the inverse of the elasticity of intertemporal substitution as $\sigma = 1.249$, the inverse of elasticity of labor as $\phi = 2.149$, and the product substitution parameter as $\varepsilon = 6$. For the monetary policy rule, we assume the Taylor-type rule as shown in Sugo and Ueda (2008). We give demand shocks with a standard deviation of $\sigma_C = 0.127$ and persistence of $\rho_C = 0.892$ in the IS equation as in Sugo and Ueda (2008). Regarding price changes after entry, we assume a negative shock of $g_{t-1,t}$ to replicate a declining price path from $t-1$ to t for existing products and calibrate the standard deviation of shocks $\sigma_g = 0.068$ and the AR(1) persistence $\rho_g = 0.894$ from our Nikkei data, where the AR(1) persistence is calculated using existing prices. This declining speed is set to replicate a 38 percent price decline observed in the micro product-level data over nine quarters, i.e., the average life-span of products. Here, the price level decreases by 38 percent following a one standard deviation shock. In the version with endogenous price discounting, we set $\xi = 5.71$ to produce a 38 percent price decline over nine quarters when the number of new matches increases by one standard deviation.[25]

The following analytical result in this general equilibrium model can be useful to understand the simulation outcomes. In particular, a price equation in the third version of our model with endogenous product cycles and price discounting is given by

$$\pi_t^H = \beta \mathbb{E}_t \pi_{t+1}^H + \kappa_\theta \hat{\theta}_t + \kappa_C \hat{C}_t - \kappa_N \hat{N}_t + \kappa_{g1} \hat{g}_{t-1,t} - \kappa_{g2} \mathbb{E}_t \hat{g}_{t,t+1}, \tag{18}$$

where π_t^H is the inflation rate in the general equilibrium model and (κ_θ, κ_C, κ_N, κ_{g1},

[25] To calibrate $\xi = 5.71$ in the fourth version of the model, we use the simulation results in the second version of the model. In the simulation of the second version of the model, one standard deviation of a new price is 0.574. We therefore need a price decline of 0.218 following a new price increase of one standard deviation to create a 38 percent price decline. This price decline is given by an increase in the number of new matches equal to one standard deviation, i.e., 0.017. Here, the price decline persistence equals 0.57, which is the persistence of the number of new matches.

κ_{g2}) are parameters. This is a Phillips curve with a search foundation. The derivation of the Phillips curve is in Appendix E. For reasonable parameter values including those in Table 5, these parameters in the Phillips curve are positive.

This Phillips curve is similar to the one derived from the simple model with endogenous product cycles and price discounting. Inflation depends on the market tightness and the demand for products. The price discounting factors $\hat{g}_{t-1,t}$ and $\hat{g}_{t,t+1}$ are price discounting factors from $t-1$ to t and from t to $t+1$, respectively. One difference from the simple model is that the number of products $\hat{N}_t$ negatively contributes to inflation because of love-of-variety preferences. Moreover, a fundamental difference from the simple model, regardless of the similarity of the price equation, is that this Phillips curve comes with other endogenous variables such as consumption, market tightness, number of products, and policy rate. Therefore, the general equilibrium model includes feedback effects among these variables, which are absent in the simple model.

Table 6 shows the simulation results. The presence of product market frictions increases the standard deviation of inflation to the same demand variation as shown in the third row. The feedback effects among the variables amplify the role of product market frictions. From the model with exogenous entry to the model with endogenous entry, the standard deviation of inflation increases by 23 percent. Moreover, the standard deviation of inflation increases by 35 percent in the model with endogenous entry and price discounting from the model with exogenous entry. The model with price discounting through the fashion effect also sufficiently increases the standard deviation of inflation by 44 percent.[26] Because of endogenous price discounting through M_t, the correlation between inflation and the number of new products decreases in this model relative to other versions of the model.

[26]When we exclude the effect of the number of products in the model with endogenous entry, the standard deviation of inflation does not change from the original model with endogenous entry.

7 Extensions

7.1 Search Frictions

We use a model with search and matching frictions in the product market to understand how product cycles and price cycles are correlated with business cycles. Search and matching frictions present a natural mechanism to generate product cycles. As discussed in the Introduction, a few papers find evidence that search and matching frictions exist in product markets. In this subsection, we attempt to provide some empirical support to justify the existence of such frictions. We first show observations and statistics related to search frictions and then demonstrate the performance of our model to match these statistics.

Observation 4: search frictions. The simple average matching probability for retailers is 28 **percent and the weighted average matching probability for retailers is** 52 **percent using sales as weights.**

Figure 6 shows matching probabilities for retailers in the product market over time. As discussed in Section 5, this probability is the retailers' matching probabilities for all products, which correspond to $N_t/(N_t + v_t)$ in our model. In addition to the simple average of the matching probabilities we use in our calibration, we also calculate the weighted average of the matching probabilities with weights given by sales.[27]

In the Nikkei data, we observe a product only when a producer sells this product to a retail shop and then a consumer buys this product at the retail shop. Here, in practice, the matching probability between a retailer and a consumer should be close to one at quarterly and annual frequencies because a retail shop is likely to stop purchasing a product if the retail shop cannot sell the product to a consumer during a quarter or a year. Moreover, if a producer stops selling a product to a retail shop, a consumer clearly cannot buy the product at this shop. Thus, we use the number of retail shops selling a

[27] In the case of a simple average, the matching probability is given by the average of the matching probabilities with equal weights across products.

product to approximate retailers' matching probabilities with producers.

When we use only 11 retail shops in Japan, there are both downward and upward biases in the matching probabilities because whether retail shops can match with producers or not depends on geographical distances. We mitigate such a bias in three ways. First, we use all the shop data in Nikkei POS scanner data. Then, we can mitigate bias in the matching probability by geographical distance. Second, we show two matching probabilities, i.e., a simple average and a weighted average by sales. By using sales as weights, we can mitigate bias by local and limited products that are counted by one match but show a very limited amount of sales.[28] Those products make our matching probabilities underestimate the true matching probabilities. Third, we calculate the matching probabilities for only the products that match with more than 10 percent of the shop, to exclude outliers.

As shown in Figure 6, the average matching probabilities are 28 percent for the simple average and 52 percent for the weighted average by sales on average at a quarterly frequency.[29] These numbers are sufficiently less than one. This observation implies that retailers and producers face frictions in trading food products and daily necessities sold in supermarkets in Japan.[30] Search frictions make slack between prices/demand and product entry as shown in the simulations previously.

[28] As mentioned in Ueda et al. (2019), there are many products with a limited label that are available only in a specific region and/or at a specific time in Japan.

[29] Even when we only include products that are sold at more than 20 percent of the shops to calculate the matching probabilities, the average matching probabilities are 42 percent for the simple average and 61 percent for the weighted average by sales.

[30] As shown in Figure A3, the matching probabilities change over a product's life-span. The matching probabilities increase and reach a peak after entry because of more effort to search among retailers and producers for new products using promotions and advertisements. Then, the matching probabilities gradually decrease because search efforts are likely to decrease for older products. These observations provide evidence that supports the presence of search frictions in product markets.

Observation 5: business cycle correlations. The correlation between retailers' matching probabilities and prices is -0.51. The correlation between retailers' matching probabilities and demand is -0.49.

Table 7 shows the correlations of the simple average matching probability with other variables.[31] The correlation between matching probability and price is -0.51 at a quarterly frequency and -0.45 at a yearly frequency. The correlation between the matching probability and demand is -0.49 at a quarterly frequency and -0.48 at a yearly frequency. These negative correlations imply that it is harder for retail shops to be matched with a product when price or demand is high in the economy. The correlation between the matching probability and the entry rate is -0.29 at a quarterly frequency and -0.48 at a yearly frequency. The correlation between the matching probability and the number of products is -0.62 at a quarterly frequency and -0.57 at a yearly frequency.

Table 8 shows the simulation results. The model with endogenous entry replicates the negative correlations among variables. The correlation between the matching probability and demand (price) is -0.73 (-0.51) compared with -0.49 (-0.51) in the data. In our model, when the price or demand for products increases, more retailers enter the market and the matching probability of retailers decreases. This is a negative externality associated with a frictional product market. Our analysis suggests that such a mechanism works behind the data. The correlation between the matching probability and the entry rate (the number of products) is -0.48 (-0.71) compared with -0.29 (-0.62) in the data.

In the model with price discounting, we also observe negative correlations among variables. The correlation between the matching probability and demand (price) is -0.73 (-0.58) compared with -0.49 (-0.51) in the data. The correlation between the matching probability and the entry rate (the number of products) is -0.48 (-0.71) compared with -0.29 (-0.62) in the data. Embedding price discounting into the endogenous

[31] We observe similar statistics as shown in Table 7 even when we use 11 retail shop samples and calculate the matching probabilities only for products that match with more than 20 percent of the shops, to mitigate lower and higher bias in the matching probability.

entry model does not change the implied negative correlations among variables. It is possible that endogenous entry with matching frictions is more important in explaining the negative correlations between the matching probability and other key variables than price discounting.

7.2 Quantity Discounting

In addition to noting the higher prices of new products, Ueda et al. (2019) emphasize that new products are associated with higher quantities of sales and that sales decline as products become older. They show that the sold quantity for a successor product is about 50 percent higher than that of the predecessor product on average in a matched sample.

To account for this declining sales pattern, we incorporate quantity discounting as well as price discounting into our simple model.[32] We assume 50 and 80 percent discounting in quantities sold after nine quarters. To calibrate the model with quantity discounting and price discounting in Appendix F, we set the discounting parameter f equal to 0.926 for 50 percent quantity discounting. We use the parameters shown in Table 3 and assume the same demand shock as in the price discounting model in Table 4.

Table 9 shows the simulation results. The third column in the table shows the model with 50 percent quantity discounting. The results do not change substantially from the model with only price discounting as shown in the fifth column of Table 4. In this case, the standard deviation of prices increases 49 percent from the model with price discounting.

Our model suggests that price discounting rather than quantity discounting plays a more important role in explaining the Nikkei data. Quantity discounting can significantly increase the price variation.

[32] Details of the model is shown in Appendix F.

8 Concluding Remark

We have built a new price model with a frictional product market. Product cycles naturally emerge by explicitly modeling product entries and exits. Endogenous product cycles are accompanied by price cycles, where first prices can be set in different ways from subsequent prices. Our model generates the New Keynesian Phillips curve as a special case and shows that product market frictions help explain price dynamics. When we calibrate our model using the product-level Nikkei POS data in Japan, our model performs well in explaining observations related to product cycles and price cycles. In the general equilibrium model, we find that endogenizing product entry can amplify the standard deviation of the inflation rate by 23 percent. Price discounting after first prices further increases this number to 35 percent.

References

[1] Abe, Nobuhiro, Yojiro Ito, Ko Munakata, Shinsuke Ohyama, and Kimiaki Shinozaki, 2016. "Pricing Patterns over Product Life-Cycle and Quality Growth at Product Turnover: Empirical Evidence from Japan," Bank of Japan Working Paper Series No. 16-E-5, Bank of Japan.

[2] Abe, Naohito, Toshiki Enda, Noriko Inakura, and Akiyuki Tonogi, 2017. "Effects of the Entry and Exit of Products on Price Indexes," RCESR Discussion Paper Series DP17-2, Research Center for Economic and Social Risks, Institute of Economic Research, Hitotsubashi University.

[3] Bai, Yan, Jose-Victor Rios-Rull and Kjetil Storesletten, 2017. "Demand Shocks as Technology Shocks," mimeo.

[4] Barrot, Jean-Noel and Julien Sauvagnat, 2016. "Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks," *Quarterly Journal of Economics* 131(3), pp. 1543-1592.

[5] Benabou, Roland, 1988. "Search, Price Setting and Inflation," *Review of Economic Studies* 55(3), pp. 353-376.

[6] Bilbiie, Florin, Fabio Ghironi, and Marc Melitz, 2007. "Endogenous Entry, Product Variety, and Business CyclesMonetary Policy and Business Cycles with Endogenous Entry and Product Variety," NBER Macroeconomics Annual 22, pp. 299-353.

[7] Broda, Christian, and David E. Weinstein, 2010. "Product Creation and Destruction: Evidence and Price Implications," *American Economic Review* 100(3), pp. 691-723.

[8] Calvo, G.A., 1983. "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12(3), pp. 983-998.

[9] Carvalho, Vasco, Makoto Nirei, Yukiko Saito, and Alireza Tahbaz-Salehi, 2021. "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake," *Quarterly Journal of Economics* 136(2), pp. 1255-1321.

[10] Christiano, Lawrence, Martin Eichenbaum, and Charles Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113(1), pp. 1-45.

[11] Diamond, Peter, 2010. Unemployment, Vacancies, Wages. Prize Lecture.

[12] Drozd, Lukasz and Jaromir Nosal, 2012. "Understanding International Prices: Customers as Capital," *American Economic Review* 102(1), pp. 364-395.

[13] Eaton, Jonathan, David Jinkins, James Tybout and Daniel Yi Xu, 2016. "Two-sided Search in International Markets," Mimeo.

[14] Gertler, Mark, and John Leahy, 2008. "A Phillips Curve with an Ss Foundation," *Journal of Political Economy* 116(3), pp. 533-572.

[15] Golosov, Mikhail, and Robert E. Lucas Jr, 2007. "Menu Costs and Phillips Curves," *Journal of Political Economy* 115(2), pp. 171-199.

[16] Hall, Robert, 2005. "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review* 95(1), pp. 50-65.

[17] Mankiw, Gregory and Ricardo Reis, 2002. "Sticky Information Versus Sticky Prices: A Proposal to replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics* 117(4), pp. 1295-1328.

[18] Melser, Daniel and Iqbal A. Syed, 2014. "Life Cycle Price Trends and Product Replacement: Implications for the Measurement of Inflation," Discussion Papers 40, School of Economics, The University of New South Wales.

[19] Michaillat, Pascal and Emmanuel Saez, 2015. "Aggregate Demand, Idle Time, and Unemployment," *Quarterly Journal of Economics* 130(2), pp. 507-569.

[20] Nakamura, Emi and Jon Steinsson, 2012. "Lost in Transit: Product Replacement Bias and Pricing to Market," *American Economic Review* 102(7), pp. 3277-3316.

[21] Nakamura, Emi and Dawit Zerom, 2010. "Accounting for Incomplete Pass-Through," *Review of Economic Studies* 77, pp. 1192-1230.

[22] Petrosky-Nadeau, Nicolas and Etienne Wasmer, 2015. "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions," *Journal of Monetary Economics* 72, pp. 97-113.

[23] Shimer, Robert, 2004. "The Consequences of Rigid Wages in Search Models," *Journal of the European Economic Association (Papers and Proceedings)* 2, pp. 469-479.

[24] Smets, Frank and Rafael Wouters, 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review* 97(3), pp. 586-606.

[25] Trigari, Antonella, 2009. "Equilibrium Unemployment, Job flows and Inflation Dynamics," *Journal of Money, Credit and Banking* 41(1), pp. 1-33.

[26] Ueda, Kozo, Kota Watanabe, and Tsutomu Watanabe, 2019. "Product Turnover and the Cost-of-Living Index: Quality versus Fashion Effects," *American Economic Journal: Macroeconomics* 11(2), pp. 310-347.

[27] Woodford, M., 2009. "Information-Constrained State-Dependent Pricing," *Journal of Monetary Economics* 56(S), pp. 100-124.

[28] Yun, T., 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37(2-3), pp. 345-370.

Table 1: Observations for Product Cycles and Price Cycles

	Average	Standard deviation
Entry rate	0.12	0.023
Exit rate	0.11	0.012
Average price	2.7	0.57
New price average/Average price	1.38	2.34

Note: Quarterly base. For the standard deviation of new price average/average price, we show the standard deviation of the average of new prices divided by the standard deviation of the average price. Regarding the average price, we normalize data to make the average as 2.7 which is the steady-state value in a calibrated model.

Table 2: Correlations among Variables

	Quarterly	Yearly
Corr(entry rate, price)	0.1	0.44
Corr(entry rate, demand)	0.12	0.5
Corr(number of products, price)	0.74	0.79
Corr(number of products, demand)	0.81	0.87
Corr(price, demand)	0.84	0.8

Note: Corr denotes the correlation between two variables.

Table 3: Calibrations for Nikkei Japanese Data

Parameters	Explanations	Values
β	Discount factor	0.99
ρ	Exit rate	0.11
g	Price discounting rate	0.95
ρ_{Z^B}	Shock persistence	0.9
α	Matching elasticity	0.08
b	Producer's bargaining power	0.5
χ	Matching efficiency	0.2
κ	Entry cost by retailers	0.15
$\bar{Z}^B$	Retailer's benefit	3.09
$\bar{X}$	Producer's cost	1.81
Z^A	Producer's production	1

Table 4: Simulations for the Nikkei Japanese Data

	Data	Exog. entry	Endo. entry	Price disc.
Corr(entry rate, price)	0.1	n.a	-0.05	0.03
Corr(entry rate, demand)	0.12	n.a	0.32	0.32
Corr(number of products, price)	0.74	n.a	0.78	0.82
Corr(number of products, demand)	0.81	n.a	0.68	0.68
Corr(price, demand)	0.84	0.75	0.75	0.82
Std(average price)	0.57	0.38	0.53	0.57
New price average/Average price	1.38	1	0.99	1.39
Std(new price)/Std(average price)	2.34	1.41	1.37	1.74

Note: Quarterly base. Corr denotes the correlation between two variables. Std denotes the standard deviation.

Table 5: Calibrations for General Equilibrium Model

Parameters	Explanations	Values
σ	Inverse of elasticity of substitution	1.249
ϕ	Inverse of elasticity of labor	2.149
ε	Products substitution	6
δ_π	Coefficient for inflation rate	0.606
δ_C	Coefficient for the output gap	0.11
$\delta_{\Delta\pi}$	Coefficient for a change of inflation rate	0.25
$\delta_{\Delta C}$	Coefficient for a change of the output gap	0.647
δ_i	Coefficient for interest rate lag	0.842
σ_C	Standard deviation of demand shock	0.127
ρ_C	Persistence of demand shock	0.892
σ_g	Standard deviation of price discounting shock	0.068
ρ_g	Persistence of price discounting shock	0.894
ξ	Degree of price discounting	5.71

Table 6: Simulations for General Equilibrium Model

	Exog. entry	Endo. entry	Price disc.	Price disc. by M
Std(inf)	0.061	0.071	0.094	0.095
Std(demand)	0.295	0.278	0.333	0.315
Std(inf)/Std(demand)	0.209	0.258	0.282	0.301

Note: Quarterly base. Std denotes the standard deviation and inf denotes the inflation rate.

Table 7: Correlations with Matching Probability

	Quarterly	Yearly
Corr(matching probability, price)	-0.51	-0.45
Corr(matching probability, demand)	-0.49	-0.48
Corr(matching probability, entry rate)	-0.29	-0.48
Corr(matching probability, number of products)	-0.62	-0.57

Note: Corr denotes the correlation between two variables.

Table 8: Simulations for Matching Friction

	Data	Endo. entry	Price disc.
Corr(matching probability, price)	-0.51	-0.51	-0.58
Corr(matching probability, demand)	-0.49	-0.73	-0.73
Corr(matching probability, entry rate)	-0.29	-0.48	-0.48
Corr(matching probability, number of products)	-0.62	-0.71	-0.71

Note: Quarterly base. Corr denotes the correlation between two variables.

Table 9: Simulations for Quantity Discounting

	Data	50 % demand discount
Corr(entry rate, price)	0.1	0.09
Corr(entry rate, demand)	0.12	0.32
Corr(number of products, price)	0.74	0.82
Corr(number of products, demand)	0.81	0.67
Corr(price, demand)	0.84	0.88
Std(average price)	0.57	0.85
New price average/Average price	1.38	1.23
Std(new price)/Std(average price)	2.34	1.46

Note: Quarterly base. Corr denotes the correlation between two variables. Std denotes the standard deviation.

Figure 1: Entry Rate and Exit Rate

Note: Yearly base.

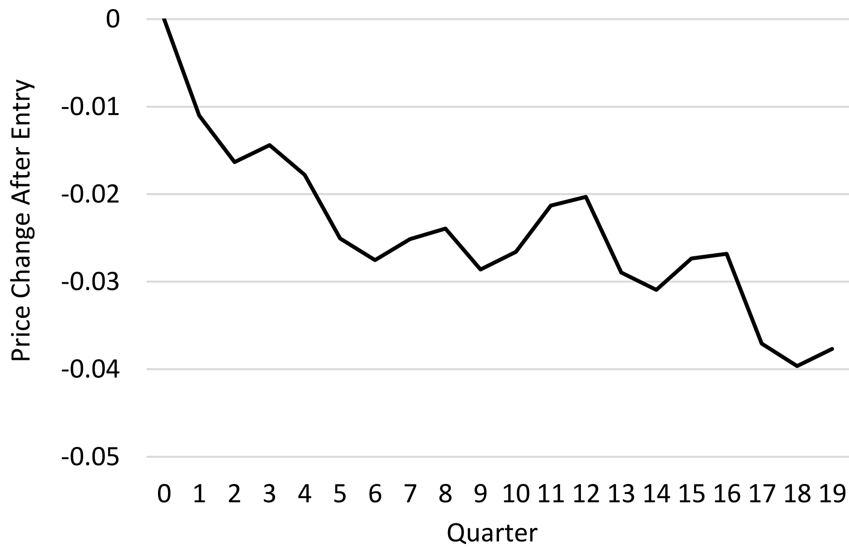

Figure 2: Price After Entry

Note: Quarterly base. A price change after entry is given by the growth rate of price from the first period. We use weights equal to the sales amount to calculate average prices. Note that products are restricted to those with life-span of 20 quarters or more and prices include temporary sales prices.

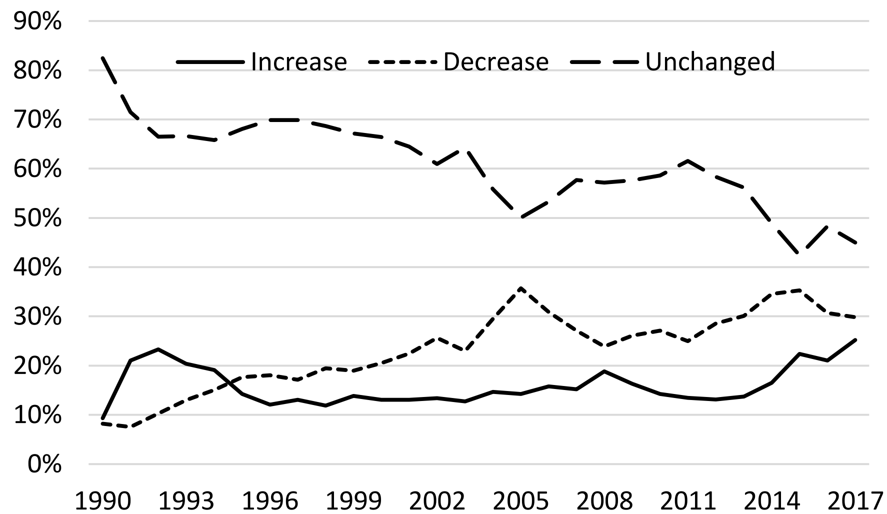

Figure 3: Fractions of No Price Change, Price Decrease, and Price Increase

Note: Yearly base. Percentages of products experiencing price increases, price decreases, and no price change.

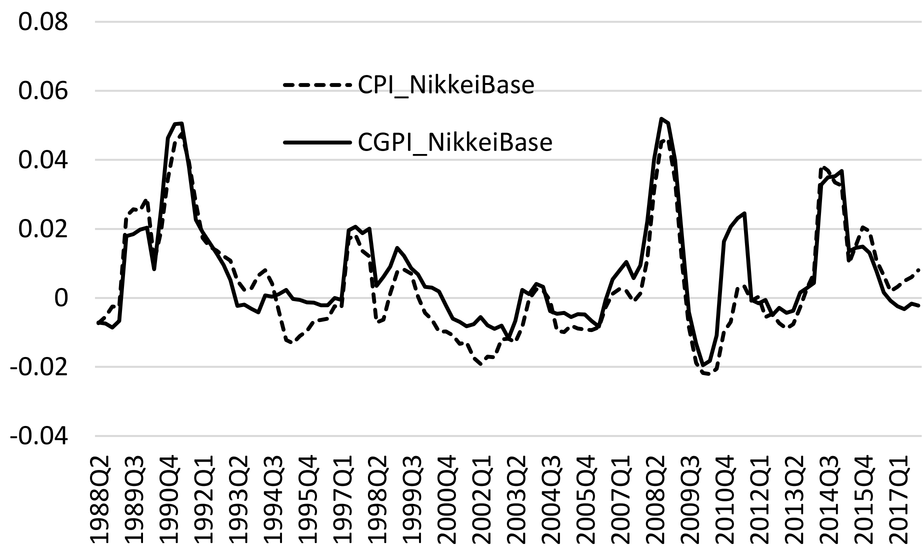

Figure 4: Consumer Price Index and Corporate Goods Price Index

Note: Quarterly base. We plot the first log differences of the indices. The Consumer Price Index (CPI) and the Corporate Goods Price Index (CGPI) in the figure correspond to product categories in our Nikkei data. The CPI is published by the Statistics Bureau of Japan and the CGPI is published by the Bank of Japan.

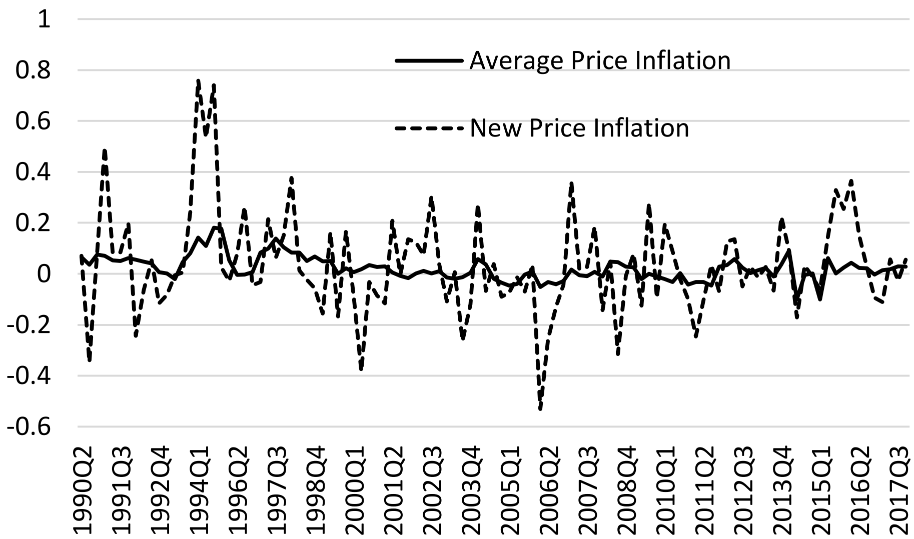

Figure 5: New Price Inflation and Average Price Inflation

Note: Quarterly base. Average price inflation denotes the year-to-year change of the average price. New price inflation denotes the year-to-year change of the average of new prices.

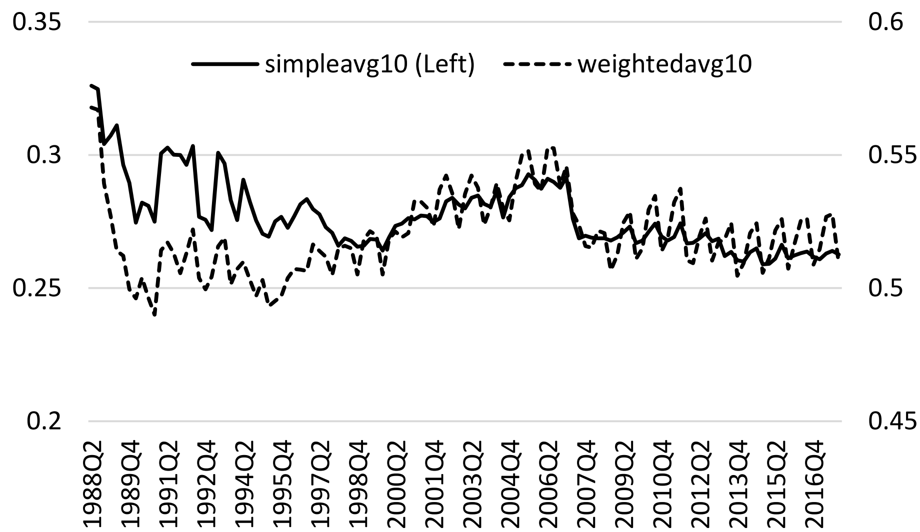

Figure 6: Matching Probability

Note: Quarterly base. The matching probabilities are calculated based on products that match with more than 10 percent of the shops. The simpleavg10 denotes the simple average probability and the weightedavg10 denotes the weighted average probability by sales values.

Appendix

We provide more details about the Nikkei data and derivations of our models in the Appendix.

A Details of Nikkei Data

A.1 Price

The Nikkei data contains the sales values and quantities sold for each product in each shop on a daily basis. By dividing the sales values by the quantities sold, we calculate the daily price for each product. These daily prices fluctuate because of special sales promotions, so we define a modal price in a given period as a regular price of each product in each shop. Based on these regular prices, we calculate an average price for all products, an average price for new products, and an average price for existing products. We use sales values as weights to calculate average prices.

To calculate the average prices, we use price levels. The first reason is that this is the average price that Japanese consumers face in shops to decide on purchases. The second reason is that price dispersion is not large because prices in the Nikkei data are for products in supermarkets where food products and daily necessities are sold. Figure A1 shows the price distribution for all prices in our 11-supermarket sample, where one price is defined as a yearly modal price of a product sold at a shop. The figure shows that about 70 percent of prices are between 100 yen and 999 yen. The median price and mean price are 284 yen and 622.3 yen, respectively. The minimum price and maximum price are one yen and 80,290 yen, respectively.

A.2 Entry Rate and Exit Rate

In calculating the entry (exit) rate, we define a new (discontinued) product as one for which a transaction is firstly (finally) recorded in a given period. Then, we obtain the number of new (discontinued) products in a given period, which is divided by the total number of products in a given period to calculate entry (exit) rates. Note that these rates are not weighted by sales. We interpret that a new product enters into the market when we observe the new product in at least one shop. We interpret that an existing product exits from the market when no shops sell the existing product. Thus, entry (exit) rates are at the product level.

A.3 Matching Probability

The matching probability is given by the number of shops selling a product divided by the total number of shops, for all shops in our sample, for each product. Notice that the matching probability is not the matching probability for new products s_t. It calculates the fraction of shops that sell a product. The product can be a new product or an existing product. In the case of a simple average, the retail shop's matching probability is given equal weight across products in aggregation. We use a sales-weighted average across products in the case of a weighted average.

Figure A3 shows the matching probabilities over product life-spans for the case of a simple average. Note that products are restricted to those with life-spans between one and two years, two and three years, three and four years, and four years or more in our calculations.

A.4 Product Categories

We show average prices, sales shares, and the number of individual products for 17 product categories in our 11-supermarket sample in Table A1. The average price for a category is obtained by the average of yearly modal prices of products in a category across shops for the sample period 1989-2017. A sales share is the sales amount of each category divided by the total sales amount for the sample period. Note that the sales amount includes sales with temporary promotion prices. The number of products denotes the total number of products sold in each category in the sample period.

B Model with Price Discounting

We would like to capture the feature from the data that prices decline after the first prices. A simple way to capture price discounting is to let price decrease at a constant rate. Suppose the first price that is set in a match is $\tilde{P}_t^A$ in period t. If the match survives in period $t+1$, the price of product A becomes $g\tilde{P}_t^A$ in this match, where $g \in (0,1]$. If the same match survives in period $t+2$, the price of product A declines to $g^2\tilde{P}_t^A$. Over time, we observe a declining price cycle.

Retailer's free entry condition is given by (4). The value of a matched retailer can be expanded as

$$\begin{aligned} V_t\left(P_t^A\right) &= Z_t^B - Z^A\tilde{P}_t^A + \beta\left(1-\rho\right)\mathbb{E}_t V_{t+1}\left(\tilde{P}_t^A\right) \\ &= Z_t^B - Z^A\tilde{P}_t^A + \beta\left(1-\rho\right)\mathbb{E}_t\left\{Z_{t+1}^B - Z^A g\tilde{P}_t^A + \beta\left(1-\rho\right)\mathbb{E}_{t+1}V_{t+2}\left(\tilde{P}_t^A\right)\right\}. \end{aligned}$$

Notice that $g\tilde{P}_t^A$ is the price of product A in period $t+1$. If we iterate $V_t\left(\tilde{P}_t^A\right)$ forward and use $V_{t+1}\left(\tilde{P}_{t+1}^A\right)$, we have

$$V_t\left(P_t^A\right) = Z_t^B - \frac{Z^A}{1-\beta g\left(1-\rho\right)}\tilde{P}_t^A + \frac{\beta\left(1-\rho\right)Z^A}{1-\beta g(1-\rho)}\mathbb{E}_t\tilde{P}_{t+1}^A + \beta\left(1-\rho\right)\mathbb{E}_t V_{t+1}\left(\tilde{P}_{t+1}^A\right).$$

For a newly matched producer and an unmatched producer, the value functions are given by (6) and (7). The benefit of having a match can be expanded as

$$\begin{aligned} & J_t^1\left(\tilde{P}_t^A\right) - J_t^0 \\ =\ & Z^A\tilde{P}_t^A - X_t + \beta\mathbb{E}_t\left\{(1-\rho)\left[J_{t+1}^1\left(\tilde{P}_t^A\right) - J_{t+1}^0\right] - q_t\left[J_{t+1}^1\left(\tilde{P}_{t+1}^A\right) - J_{t+1}^0\right]\right\} \\ =\ & Z^A\tilde{P}_t^A - X_t - \beta\mathbb{E}_t q_t\left[J_{t+1}^1\left(\tilde{P}_{t+1}^A\right) - J_{t+1}^0\right] \\ & + \left(1-\rho\right)\beta\mathbb{E}_t\left[\begin{array}{c} Z^A g\tilde{P}_t^A - X_{t+1} \\ +\beta\mathbb{E}_{t+1}\left[(1-\rho)\left[J_{t+2}^1\left(\tilde{P}_t^A\right) - J_{t+2}^0\right] - q_{t+1}\left[J_{t+2}^1\left(\tilde{P}_{t+2}^A\right) - J_{t+2}^0\right]\right] \end{array}\right]. \end{aligned}$$

By iterations, we finally have

$$\begin{aligned} J_t^1\left(\tilde{P}_t^A\right) - J_t^0 &= \frac{1}{1-\beta g\left(1-\rho\right)}Z^A\tilde{P}_t^A - \frac{\beta\left(1-\rho\right)}{1-\beta g(1-\rho)}Z^A\mathbb{E}_t\tilde{P}_{t+1}^A - X_t \\ & +\beta\mathbb{E}_t\left\{\left(1-\rho-q_t\right)\left[J_{t+1}\left(\tilde{P}_{t+1}^A\right) - J_{t+1}^0\right]\right\}. \end{aligned}$$

The matching probabilities are given by (2) and (3). The measure of unmatched producer and the evolution of the number of total matches are given by (10) and (12), respectively. The Nash bargaining problem is solved by

$$\begin{aligned}\frac{\partial V_t^A\left(\tilde{P}_t^A\right)}{\partial \tilde{P}_t^A} &= -\frac{Z^A}{1-\beta g\left(1-\rho\right)},\\ \frac{\partial\left[J_t^1\left(\tilde{P}_t^A\right)-J_t^0\right]}{\partial \tilde{P}_t^A} &= \frac{Z^A}{1-\beta g\left(1-\rho\right)}.\end{aligned}$$

Then, the F.O.C yields

$$bV_t^A\left(\tilde{P}_t^A\right)=\left(1-b\right)\left[J_t^1\left(\tilde{P}_t^A\right)-J_t^0\right].$$

Lastly, the aggregate price index for P_t^A and $\tilde{P}_t^A$ is defined by

$$N_tP_t^A=\left(1-\rho\right)N_{t-1}gP_{t-1}^A+\chi\theta_{t-1}^{\alpha}u_{t-1}\tilde{P}_t^A.$$

The steady-state is given by eliminating time in a model.

C Model with Exogenous Entry

We need to change a value function for a retailer. Instead of a free entry condition, we have the value of a new match for a retailer as

$$\overline{J}_t^1(\widetilde{P}_t^A)=Z_t^B-Z^A\tilde{P}_t^A+\beta\mathrm{E}_t\left[(1-\rho)\overline{J}_{t+1}^1(\tilde{P}_t^A)+\rho\overline{J}_{t+1}^0\right].$$

On the other hand, the value of a vacancy for a retailer is

$$\overline{J}_t^0=\beta\mathrm{E}_t\left[\overline{s}\overline{J}_{t+1}^1(\tilde{P}_{t+1}^A)+(1-\overline{s})\overline{J}_{t+1}^0\right].$$

These two equations imply that the surplus of a retailer from a new match is

$$\overline{J}_t^1(\widetilde{P}_t^A)-\overline{J}_t^0=\overline{S}_t=Z_t^B-Z^A\tilde{P}_t^A+\beta\mathrm{E}_t\left\{(1-\rho)\left[\overline{J}_{t+1}^1(\tilde{P}_t^A)-\overline{J}_{t+1}^0\right]-\overline{s}\left[\overline{J}_{t+1}^1(\tilde{P}_{t+1}^A)-\overline{J}_{t+1}^0\right]\right\},$$

where we have $\overline{q}=\overline{s}$ in a steady-state. In this case, product market variables, such as q_t, s_t, N_t, u_t, v_t, and θ_t, are constant since the number of products is constant even though other equations are the same as a model in Section 3.1.

D Relative Price Model

In a relative price model, the value function for a retailer with a contract of price $\tilde{P}_t^A$ is

$$V_t\left(\tilde{P}_t^A\right) = Z_t^B - Z^A \frac{\tilde{P}_t^A}{P_t^A} + \beta\left(1-\rho\right)\mathbb{E}_t V_{t+1}\left(g\tilde{P}_t^A\right).$$

The matched value functions for a producer is given by

$$J_t^1\left(\tilde{P}_t^A\right) = Z^A \frac{\tilde{P}_t^A}{P_t^A} - X_t + \beta\mathbb{E}_t\left[\left(1-\rho\right)J_{t+1}^1\left(g\tilde{P}_t^A\right) + \rho J_{t+1}^0\right].$$

Other parts of the model remain the same as in Section 3.1.

E General Equilibrium Model

A technical appendix for more details of a model and log-linearization is available on request.

E.1 Household

E.1.1 Cost Minimization

A representative household first solves a cost minimization problem for differentiated goods.

$$\int_0^{N_t} y_t^*\left(i\right) h_t\left(i\right) di$$

subject to a consumption bundle given by

$$C_t^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{1}{N_t}\right)^{\frac{1}{\varepsilon}} \int_0^{N_t} {y_t^*}^{\frac{\varepsilon-1}{\varepsilon}}\left(i\right) di,$$

where $y_t^*\left(i\right)$ and $h_t\left(i\right)$ are individual demand and price for a good i at retailers, respectively, C_t is an aggregate demand, and N_t is the number of goods (products).

For the consumption aggregator, the appropriate consumption-based price index H_t is given by

$$H_t^{1-\varepsilon} = \frac{1}{N_t}\int_0^{N_t} h_t^{1-\varepsilon}\left(i\right) di.$$

Then, we have demand functions for individual goods.

$$y_t^*(i) = \left[\frac{h_t(i)}{H_t}\right]^{-\varepsilon}\frac{C_t}{N_t}.$$

E.1.2 Intertemporal Behavior

We consider a representative household that derives utility from consumption and disutility from labour supply. The household maximizes the following welfare function:

$$U_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^{t+s} \left[U(C_{t+s}, \nu_{t+s}) - V\left(L_{t+s}, \nu_{t+s}\right) \right] \right\},$$

where $\mathbb{E}_t$ is an expectation conditional on the state of nature at period t, $U(\cdot)$ is an increasing and concave function in the consumption index C_t, $V(\cdot)$ is an increasing and convex function in total labour supply L_t, and ν_t is an exogenous disturbance of preference, where the steady-state value of ν_t is given by $\overline{\nu} = 1$. Note that the labor aggregator is distorted as a demand for goods,

$$L_t^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{1}{N_t} \right)^{\frac{1}{\varepsilon}} \int_0^{N_t} l_t^{\frac{\varepsilon-1}{\varepsilon}}(h) dh,$$

where $l_t(h)$ is labor supply to a producer h. Assuming that $y_t^*(h) = l_t(h)$ as explained below, we have

$$L_t^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{1}{N_t} \right)^{\frac{1}{\varepsilon}} \int_0^{N_t} l_t^{\frac{\varepsilon-1}{\varepsilon}}(h) dh = C_t^{\frac{\varepsilon-1}{\varepsilon}}.$$

$$\begin{aligned} C_t &= L_t \\ &= \left[\left(\frac{1}{N_t} \right)^{\frac{1}{\varepsilon}} \int_0^{N_t} y_t^{*\frac{\varepsilon-1}{\varepsilon}}(h) dh \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= Y_t^*, \end{aligned}$$

where we assume no resource used for search in a products market, just in the budget constraint as a lump sum tax. Note that an aggregate output holds the same distortion as the consumption bundle eventually.

The budget constraint of the household is given by

$$H_t C_t + \mathbb{E}_t X_{t,t+1} B_{t+1} + D_t \leq B_t + (1 + i_{t-1}) D_{t-1} + W_t L_t + \int_0^{N_t} \Pi_t^F(f) df + T_t,$$

where B_t is a set of risky assets, D_t is the amount of bank deposits, i_t is the nominal deposit rate (policy rate) set by the central bank from t to $t+1$, W_t is the nominal wage for labor supply L_t, $\int_0^1 \Pi_t^F(f)df$ is the nominal dividend from owning firms, T_t is a subsidy and $X_{t,t+1}$ is the stochastic discount factor between t and $t+1$. We assume a complete financial market for risky assets. Thus, we have a unique discount factor and can characterize the relationship between the deposit rate and the stochastic discount factor as follows:

$$\frac{1}{1+i_t} = \mathbb{E}_t X_{t,t+1}.$$

Given the optimal allocation of consumption expenditure across the differentiated goods, the household must choose the total amount of consumption, the optimal amount of risky assets to hold, and an optimal amount to deposit in each period to maximize the welfare function. The necessary and sufficient conditions are given by

$$U_C(C_t, \nu_t) = \beta(1+i_t)\mathbb{E}_t\left[U_C(C_{t+1}, \nu_{t+1})\frac{H_t}{H_{t+1}}\right].$$

The household provides labors. We have the following relation:

$$\frac{W_t}{H_t} = \frac{V_L\left(L_t, \nu_t\right)}{U_C(C_t, \nu_t)} = \frac{L_t^{\phi}}{C_t^{-\sigma}}.$$

E.2 Retailers

Retailers play two roles for a household and producers. Retailers buy products from producers in the frictional product market as in the simple model and sell differentiated goods to a household.

For a household, retailers solve

$$\begin{aligned} \max_{h_t(i)} \Pi &= \frac{h_t\left(i\right)}{H_t} y_t^*\left(i\right) - \frac{p_t\left(i\right)}{H_t} y_t\left(i\right) \\ &= \frac{h_t\left(i\right)}{H_t} y_t^*\left(i\right) - \frac{p_t\left(i\right)}{H_t} y_t^*\left(i\right), \end{aligned}$$

where we assume that retailers buy a product from producers and sell it for household as $y_t^*\left(i\right) = y_t\left(i\right)$, where $y_t\left(i\right)$ and $p_t\left(i\right)$ are individual demand and price for a product from producer i.

The F.O.C with respect to $h_t(i)$ gives

$$h_t(i) = \frac{\varepsilon}{\varepsilon - 1} p_t(i),$$

where $p_t(i)$ is given when deciding $h_t(i)$. Moreover, retailers first set $p_t(i)$ with producers and decide the amount of input $y_t^*(i)$ and so $y_t(i)$ after setting a price of $h_t(i)$. We have

$$y_t(i) = \frac{1}{N_t}\left[\frac{\varepsilon}{\varepsilon - 1}\right]^{-\varepsilon}\left[\frac{p_t(i)}{H_t}\right]^{-\varepsilon} C_t.$$

Note that we have different demand functions after time t when producers and so retailers do not change price, such as

$$y_{t,t+1}(i) = \frac{1}{N_t}\left[\frac{\varepsilon}{\varepsilon - 1}\frac{g_{t,t+1}p_t(i)}{H_t}\frac{H_t}{H_{t+1}}\right]^{-\varepsilon} C_{t+1},$$

where $g_{t,t+j}$ is a price shock from time t to $t+j$ to existing price, where $\overline{g} = 1$ and $g_{t,t} = 1$. Here, we use notation $y_{t,t+1}(i)$ from time t to time $t+1$ to describe a change in demand without a price change, where $y_{t,t}(i) = y_t(i)$. We use the same notations for other variables.

For producers, retailers solve an optimization problem for frictional products marker as in a simple model.

$$V_t(p_t(i)) = \frac{h_t(i)}{H_t} y_t^*(i) - \frac{p_t(i)}{H_t} y_t(i) + \mathbb{E}_t\left[\beta_{t,t+1}(1-\rho)V_{t+1}(g_{t,t+1}p_t(i))\right],$$

where $\beta_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t}$ and $\lambda_t = C_t^{-\sigma}$ is a marginal utility of consumption.

E.3 Producers

There is a measure 1 of producers in the economy. The value function is

$$\begin{aligned} J_t^1(p_t(i)) - J_t^0 &= \frac{p_t(i)}{H_t} y_t(i) - W_t^* l_t(i) \\ &+ \beta_{t,t+1}\mathbb{E}_t\left\{(1-\rho)\left[J_{t+1}^1(g_{t,t+1}p_t(i)) - J_{t+1}^0\right] - q_t\left[J_{t+1}^1(p_{t+1}(i)) - J_{t+1}^0\right]\right\}, \end{aligned}$$

where $y_t(i) = f(l_t(i)) = l_t(i)$ and $f(\cdot)$ is a production function of producer. We define $W_t^* \equiv \frac{W_t}{H_t}$ and $S_t \equiv J_t^1(p_t(i)) - J_t^0$.

E.4 Sharing Condition and Matching for Products

We assume the following sharing condition to set a price as

$$(1-b)\, S_t = bV_t.$$

The free entry condition for retailers is

$$s_t \mathbb{E}_t \left(\beta_{t,t+1} V_{t+1} \right) = \kappa.$$

The matching function in the products market is given by

$$m\left(u_t, v_t\right) = \chi u_t^{1-\alpha} v_t^{\alpha} \text{ where } \alpha \in (0,1)\,.$$

Here u_t represents the measure of producers that have not found a match with a retailer and v_t denotes the measure of available retailers that search in the products market. Define the market tightness as $\theta_t = v_t/u_t$. The matching probabilities are

$$\begin{aligned} s_t &= \frac{m_t}{v_t} = \chi \theta_t^{\alpha-1}, \\ q_t &= \frac{m_t}{u_t} = \chi \theta_t^{\alpha}. \end{aligned}$$

The flow condition for the measure of matches is

$$N_t = (1-\rho)\, N_{t-1} + q_{t-1} u_{t-1},$$

where u_t follows

$$u_t = 1 - (1-\rho)\, N_t.$$

E.5 Price Aggregation

$$\begin{aligned} H_t^{1-\varepsilon} &= \frac{1}{N_t} \int_0^{N_t} h_t^{1-\varepsilon}(i)\, di \\ &= \frac{q_{t-1} u_{t-1}}{N_t} h_t^{1-\varepsilon} + \left(1 - \frac{q_{t-1} u_{t-1}}{N_t}\right) g_{t-1,t}^{1-\varepsilon} H_{t-1}^{1-\varepsilon}, \end{aligned}$$

where a new price h_t is the same across price setters. We can decompose price of H_t into two parts, a new price and an old price since we assume that retailers set prices only when producers change prices. Thus, a new price is set only for a new products in this model.

E.6 Closed Economy

After log-linearizing a model around a constant steady-state, we have a closed economy consisting of five endogenous variables and five equations.

Form a relation in frictional products market, we have an equation regarding a market tightness.

$$\begin{aligned}\hat{\theta}_t = \beta \left\{ 1 - \rho - \frac{b}{1-b} \frac{\bar{q}}{(1-\alpha)\left\{ \frac{b}{1-b} + \left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] \frac{\varepsilon}{\varepsilon-1} \right\}} \right\} \mathbb{E}_t \hat{\theta}_{t+1} \qquad (19)\\
+ \frac{\sigma}{1-\alpha} \hat{C}_t \\
+ \left\{ \frac{\overline{C}\left(\frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{\bar{A}}\right) - \frac{\bar{W}^*\overline{C}}{\bar{A}}(\sigma+\phi) + \left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon}\right]\frac{\overline{C}}{\varepsilon-1}}{(1-\alpha)\bar{V}\left[\frac{b}{1-b} + \left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon}\right]\frac{\varepsilon}{\varepsilon-1}\right]} - \frac{\sigma}{1-\alpha} \right\} \mathbb{E}_t \hat{C}_{t+1} \\
- \frac{\overline{C}\left(\frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{\bar{A}}\right) + \left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon}\right]\frac{\overline{C}}{\varepsilon-1}}{(1-\alpha)\bar{V}\left[\frac{b}{1-b} + \left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon}\right]\frac{\varepsilon}{\varepsilon-1}\right]} \mathbb{E}_t \widehat{N}_{t+1}.\end{aligned}$$

From a price setting behavior, we have a Phillips curve with a search foundation.

$$\begin{aligned}\pi_t^H \;=\; & \beta \mathbb{E}_t \pi_{t+1}^H + \frac{\rho}{1-\rho} \frac{1-\beta(1-\rho)}{\overline{C}} \frac{1}{(1-b)\left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon}\right] + \frac{\varepsilon-1}{\varepsilon}b} (1-b)\beta\bar{q}\bar{S}\hat{\theta}_t \qquad (20)\\
& + \frac{\rho}{1-\rho}\left[\frac{b}{\varepsilon} - (1-b)\left(\frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{\bar{A}} - \frac{\bar{W}^*}{\bar{A}}\sigma - \frac{\bar{W}^*}{\bar{A}}\phi\right)\right] \frac{1-\beta(1-\rho)}{(1-b)\left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon}\right] + \frac{\varepsilon-1}{\varepsilon}b} \hat{C}_t \\
& - \frac{\rho}{1-\rho}\left[\frac{b}{\varepsilon} - (1-b)\left(\frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{\bar{A}}\right)\right] \frac{1-\beta(1-\rho)}{(1-b)\left[\frac{\bar{W}^*}{\bar{A}}\varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon}\right] + \frac{\varepsilon-1}{\varepsilon}b} \widehat{N}_t \\
& + \widehat{g}_{t-1,t} - \beta \mathbb{E}_t \widehat{g}_{t,t+1}.\end{aligned}$$

From a flow condition of products, we have

$$\hat{N}_t = (1-\rho)(1-\bar{q})\hat{N}_{t-1} + \rho\alpha\hat{\theta}_{t-1}. \qquad (21)$$

From the consumer side, we have the IS equation as

$$\hat{C}_t = \mathbb{E}_t \hat{C}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \mathbb{E}_t \pi_{t+1}^H\right) + Z_t, \qquad (22)$$

where Z_t is a demand shock.

The Taylor rule is given by

$$\hat{\imath}_t = \delta_\pi \pi^H_{t-1} + \delta_C \hat{C}_{t-1} + \delta_{\Delta\pi}\left(\pi^H_t - \pi^H_{t-1}\right) + \delta_{\Delta C}\left(\hat{C}_t - \hat{C}_{t-1}\right) + \delta_i \hat{\imath}_{t-1}. \tag{23}$$

Then, we have five endogenous variables of

$$\hat{N}_t, \hat{\theta}_t, \pi^H_t, \hat{\imath}_t, \hat{C}_t$$

and five equations above of (19), (20), (21), (22), and (23). We have one exogenous variable Z_t and have $\widehat{g}_{t,t+1}$ for a price change after an entry.

F Model with Quantity and Price Discountings

Now we develop a general model to capture both price discounting and demand discounting, where $g \in (0,1]$ represents the price discounting parameter and $f \in (0,1]$ represents the demand discounting parameter.

Consider the values functions of producer and retailer. Retailer's free entry condition does not change. The value of a matched retailer is

$$\begin{aligned}
V_t\left(P^A_t\right) &= Z^B_t - Z^A P^A_t + \beta(1-\rho)\mathbb{E}_t V_{t+1}\left(P^A_t\right) \\
&= Z^B_t - Z^A P^A_t + \beta(1-\rho)\mathbb{E}_t\left\{f Z^B_{t+1} - f Z^A g P^A_t + \beta(1-\rho)\mathbb{E}_{t+1}V_{t+2}\left(P^A_t\right)\right\}.
\end{aligned}$$

The value of a matched producer and an unmatched producer are given by (6) and (7). The benefit of having a match is therefore

$$\begin{aligned}
J^1_t\left(P^A_t\right) - J^0_t &= Z^A P^A_t - X_t + \beta\mathbb{E}_t\left\{(1-\rho)\left[J^1_{t+1}\left(P^A_t\right) - J^0_{t+1}\right] - q_t\left[J^1_{t+1}\left(P^A_{t+1}\right) - J^0_{t+1}\right]\right\} \\
&= Z^A P^A_t - X_t \\
&\quad + \beta\mathbb{E}_t\left\{\begin{array}{c}
(1-\rho)\left[\begin{array}{c}
f Z^A g P^A_t - f X_{t+1} + \\
\beta\mathbb{E}_t\left[\begin{array}{c}
(1-\rho)\left[J^1_{t+2}\left(P^A_t\right) - J^0_{t+2}\right] \\
-q_{t+1}\left[J^1_{t+2}\left(P^A_{t+2}\right) - J^0_{t+2}\right]
\end{array}\right]
\end{array}\right] \\
-q_t\left[J^1_{t+1}\left(P^A_{t+1}\right) - J^0_{t+1}\right]
\end{array}\right\}.
\end{aligned}$$

The aggregate price index is written as

$$N_t \tilde{Z}^A_t \tilde{P}^A_t = (1-\rho) N_{t-1} f \tilde{Z}^A_{t-1} g \tilde{P}^A_{t-1} + \chi\theta^\alpha_{t-1} u_{t-1} Z^A P^A_t,$$

where $\tilde{Z}^A$ can be found by

$$N_t \tilde{Z}_t^A = (1-\rho)\, N_{t-1} f \tilde{Z}_{t-1}^A + \chi \theta_{t-1}^{\alpha} u_{t-1} Z^A. \tag{24}$$

Note that demand for product A within each match declines at a constant rate over time. In the steady-state where $\tilde{Z}_{t-1}^A = \tilde{Z}_t^A = \tilde{Z}^A$ and $N_{t-1} = N_t = N$, we can combine (12) and (24) to derive

$$\tilde{Z}^A = \frac{\rho}{(1-\rho)\,(1-f)+\rho} Z^A.$$

It is clear that $\tilde{Z}^A \leq Z^A$ owing to demand discounting. Note that we have $P^A \leq \tilde{P}^A$ due to price discounting.

Table A1: Product Categories: Average Price, Sales Share, and the Number of Products in Each Category

Categories	Prices	Shares	Product Numbers
Processed meats and seafoods	324.4	0.073	42970
Dairy products and milks	243.1	0.083	34331
Beverages	382.8	0.081	43045
Seasonings	266.9	0.058	23281
Instant and frozen foods	215.6	0.088	53767
Canned foods	242.9	0.012	7602
Breads and cakes	196.5	0.055	68160
Confectioneries	234.6	0.085	164642
Alcoholic beverages	1100.2	0.064	35156
Baby foods, cereals, and eggs	1913.1	0.074	37143
Beans and other agricultural products	237.7	0.062	34648
Other foods	287.8	0.082	62767
Body/Oral care products	641.7	0.024	29947
Detergents and cosmetics	1365	0.075	115425
Stationeries	331.3	0.007	50939
Pet foods and sanitary products	385.4	0.008	16031
Other daily necessities	665.5	0.071	72757

Note: Price is Japanese Yen.

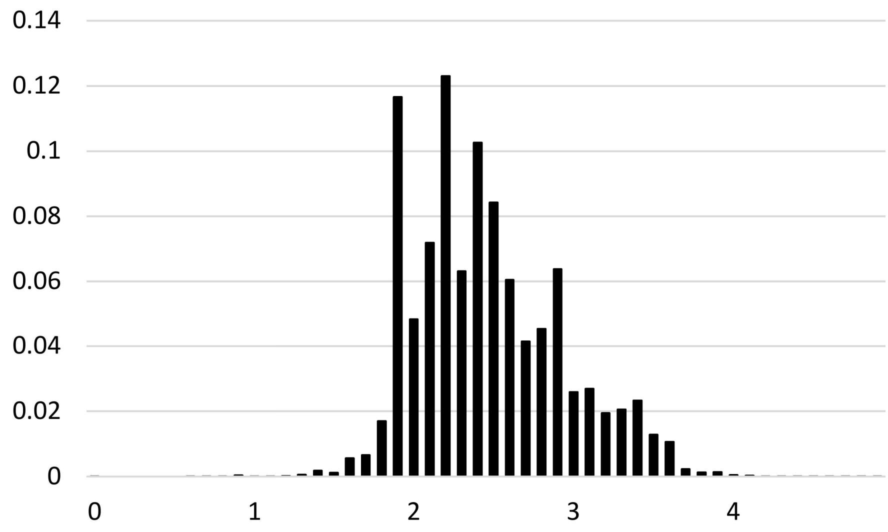

Figure A1: Price Distribution

Note: A horizontal axis is Log 10 price.

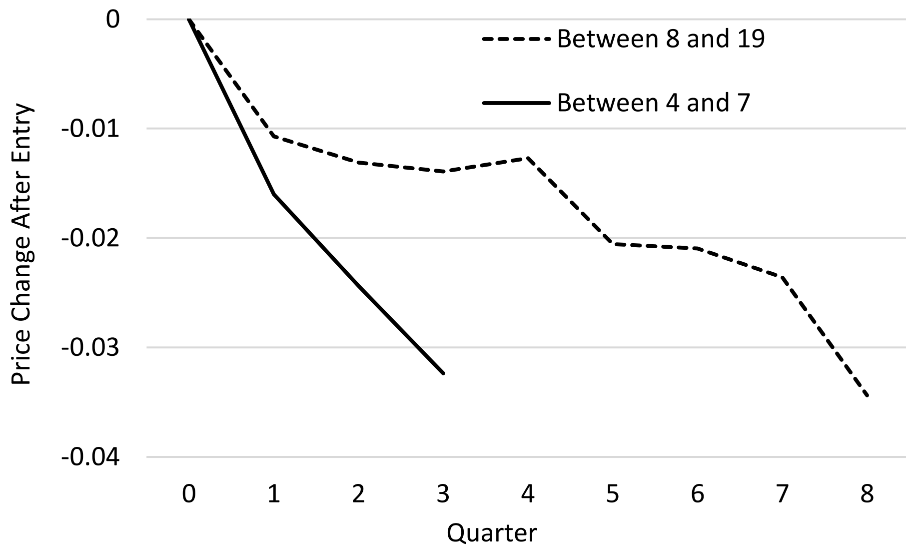

Figure A2: Price After Entry: Shorter Life-span Products

Note: Quarterly base. A price change after entry is given by the growth rate of price from the first period. We use weights equal to the sales amount to calculate average prices. Note that products are restricted to those with life-span between 4 and 7 and between 8 and 19 quarters, respectively. Prices include temporary sales prices.

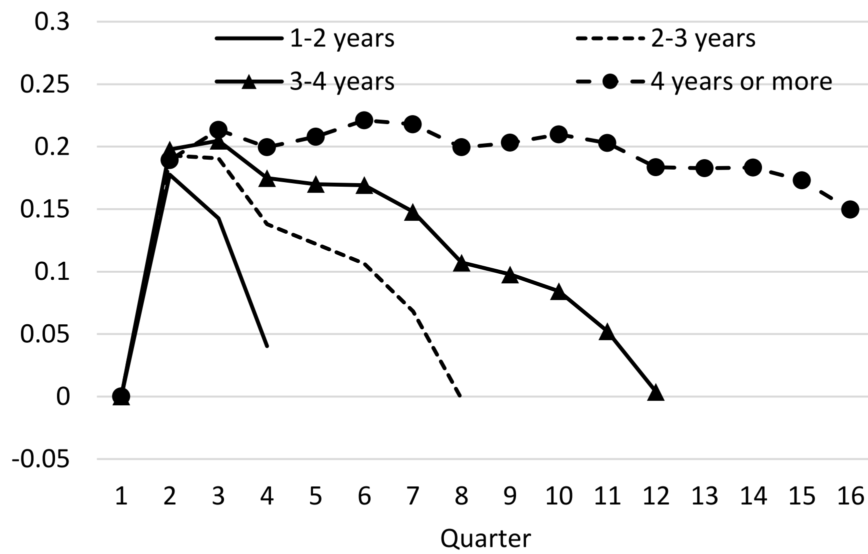

Figure A3: Matching Probability over Life-span

Note: Quarterly base and the simple average probability. Log difference from the first period. All shop samples and the matching probability only for products that match with more than 10 percent of the shops. Products with life-span of one year or more and less than two years, two years or more and less than three years, three years or more and less than four years, and four years or more.