Duopolistic Competition and Monetary Policy

Kozo Ueda

February 2022

TCER

Abstract

This study constructs a tractable duopoly model with price stickiness to consider
the strategic pricing of duopolistic firms and its implications for monetary policy.
Dynamic strategic complementarity, in which an increase in a firm's price increases
the optimal price set by the rival firm in the following periods, increases steady-
state price and the real effect of monetary policy. However, when temporary sales
arise as a mixed strategy, the real effect of monetary policy decreases
considerably.

Kozo Ueda
TCER
and
Waseda University
School of Political Science and Economics
1-6-1 Nishi-Waseda Shinjuku-ku, Tokyo
kozo.ueda@waseda.jp

Duopolistic Competition and Monetary Policy

Kozo Ueda*

February 1, 2022

Abstract

This study constructs a tractable duopoly model with price stickiness to consider the strategic pricing of duopolistic firms and its implications for monetary policy. Dynamic strategic complementarity, in which an increase in a firm's price increases the optimal price set by the rival firm in the following periods, increases steady-state price and the real effect of monetary policy. However, when temporary sales arise as a mixed strategy, the real effect of monetary policy decreases considerably.

Keywords: duopoly; monetary policy; strategic complementarities; New Keynesian model

JEL classification: D43, E32, E52

1 Introduction

Models based on monopolistic competition provide a simple framework to analyze firms' price-setting behavior, which enables us to study the effects of monetary policy in tractable macroeconomic models (see Dixit and Stiglitz 1977, Blanchard and Kiyotaki 1987). However, aside from certain exceptions, these models focus on cases in which the number of firms is infinite, and they do not incorporate the strategic behaviors of oligopolistic firms. Firms in the model do not need to consider how their price will influence their rival firms' prices and how a rival firm's price influences their optimal price. However, in reality, firms compete with a finite number of rival firms, and they closely monitor the pricing of only a limited number of rival firms (particularly market leaders). These observations motivate us to consider the role of the strategic pricing of oligopolistic firms in macroeconomic models.

This study aims to investigate how implications for monetary policy change when we incorporate the strategic pricing behaviors of oligopolistic firms into the model. The first objective of this study is to demonstrate analytically and numerically how demand influences dynamic (intertemporal) strategic complementarity and, in turn, the effect of monetary policy. We construct a macroeconomic model with Calvo-type price stickiness by incorporating oligopolistic competition. Demand is described by an arbitrary invertible demand system, which encompasses the constant elasticity of substitution (CES) preferences and Hotelling's (1929) address model (see Pettengill 1979, Dixit and Stiglitz 1979, Anderson, de Palma, and Thisse 1992 for the similarities and differences between the CES and Hotelling models).

The second objective of this study is to consider how the implications of temporary sales (price discounts) for the effect of monetary policy change when we incorporate the strategic pricing behaviors of oligopolistic firms. It is widely known that temporary sales are held frequently and that price stickiness decreases considerably once we include sales in the measurement of price stickiness (see, e.g., Bils and Klenow 2004 for the United States and Sudo, Ueda, and Watanabe 2014 for Japan). This implies that the use of temporary sales is an important source of price flexibility for firms that face costs of regular price adjustments, which might decrease the real effect of monetary policy. Further, temporary sales may be determined by strategic motives and are therefore potentially important in understanding the response of prices in oligopolistic markets. We consider a mixed strategy equilibrium, where the higher and lower prices constitute the regular and sale prices, respectively.

Our main results are as follows. First, we show analytically and numerically how demand influences dynamic (intertemporal) strategic complementarity.[1] We solve the Markov equilibrium in our dynamic game using log-linearization around the steady state. The pricing strategy of every firm is formulated based on a rival's past price, its own past price, and a shock in the current period. When cross elasticity is positive, a firm's own price positively influences the price set by the rival firm in the following periods. This dynamic strategic complementarity effect depends on demand parameters. Specifically, we show that the degree of dynamic strategic complementarity increases as cross elasticity increases, the absolute value of own superelasticity decreases, or cross superelasticity increases. This implies that an increase in cross superelasticity, rather than the absolute value of own superelasticity, is needed to increase dynamic strategic complementarity.

Second, we find that an increase in dynamic strategic complementarity is associated with an increase in price markup under sticky prices and an increase in the real effect of monetary policy. Because of dynamic strategic complementarity, the expectation that the rival firm may not revise its price on a given day under price stickiness discourages a firm from revising its price aggressively on that given day, which decreases the nominal effect of monetary policy (i.e., price response) and increases the real effect of monetary policy (i.e., output response).

Finally, we find that the real effect of monetary policy decreases considerably under duopoly if there is a mixed strategy equilibrium (i.e., if temporary sales exist). We show that a mixed strategy equilibrium can arise when own demand elasticity depends on a rival's price, unlike in the case of CES preferences. Particularly, when we incorporate the Hotelling model with heterogeneous consumer heterogeneity, a mixed strategy equilibrium arises such that temporary sales exemplify strategic complementarity for the interaction between setting the higher price and the lower price, whereas dynamic strategic complementarity decreases for the interaction between setting the higher prices by two firms. We find that this decreases the real effect of monetary policy considerably.

The three studies most similar to ours are those by Faia (2012), Mongey (2017), and Wang and Werning (2020), who investigate monetary policy under oligopolistic competition.[2]

---

[1]See Jun and Vives (2004) and Mongey (2017) for the terminology of *static* and *dynamic* (intertemporal) strategic effects.

[2]Faia (2012), the first in this research area, considers optimal monetary policy when the number of firms is finite. However, Faia (2012) incorporates the effect of finiteness only partially: each firm takes into

Particularly, Wang and Werning's (2020) work is the closest to this study. They consider strategic pricing behavior under nominal rigidity and find that higher market concentration amplifies the real effect of monetary policy. They demonstrate an empirically useful closed-form relation for the degree of dynamic strategic complementarity using observable markup. The main contribution of this study relative to that of Wang and Werning (2020) is that first, we obtain a closed-form solution for the degree of dynamic strategic complementarity in a general model by using structural parameters associated with the demand system. In this way, we demonstrate how demand parameters consisting of own elasticity, cross elasticity, and three kinds of superelasticity influence the degree of dynamic strategic complementarity. Particularly, we show that an increase in superelasticity does not necessarily increase dynamic strategic complementarity. To increase dynamic strategic complementarity, an increase in cross, not own, superelasticity is needed. Further, in our model, monetary policy shocks are stochastic, which enables us to analyze how firms' pricing responds to a monetary policy shock and how strategic pricing behavior changes implications for the effects of monetary policy.

Further, in our model, the monetary policy shock is stochastic. In this way, we demonstrate how demand parameters consisting of own elasticity, cross elasticity, and three kinds of superelasticity influence the degree of dynamic strategic complementarity and the response to a monetary policy shock. Particularly, we show that an increase in superelasticity does not necessarily increase dynamic strategic complementarity. To increase dynamic strategic complementarity, an increase in cross, not own, superelasticity is needed.

The second contribution of this study relative to that of Wang and Werning (2020) is that we investigate the role of temporary price sales in considering the effects of monetary policy. Empirically, it is well known that the degree of temporary sales varies across products

account the effect of its price on the aggregate price but not the strategic effect of its price on the prices of other firms. Mongey (2017) introduces not only cross- but also within-sector elasticity of substitution to incorporate duopoly: two firms exist in each sector, and they compete strategically by optimally changing prices in the context of a menu cost model of price adjustment with idiosyncratic shocks. Mongey (2017) shows that a dynamic strategic complementarity decreases the responses of price adjustment and increases the real effect of monetary policy, which is consistent with the findings of our study. One may also refer to Nirei and Scheinkman (2020), who construct a comparable model, in which competition among a finite number of firms generates inflation volatility.

(Nakamura and Steinsson 2008), regions (Coibion, Gorodnichenko, Hong 2015), over the business cycle (Kryvtsov and Vincent 2021), and over time (Sudo et al. 2018). Theoretically, some studies argue that the implications of sales for the real effect of monetary policy are not important (Guimaraes and Sheedy 2011, Kehoe and Midrigan 2015, Anderson et al. 2017), while others deem them important (Sudo et al. 2018 and Kryvtsov and Vincent 2021). Particularly, Guimaraes and Sheedy (2011) construct a New Keynesian model with heterogeneous elasticity of substitution and show that sales exist in equilibrium but the real effect of monetary policy is more or less unchanged. Our result is markedly different, showing that the real effect of monetary policy decreases considerably. The most important modeling difference is that the literature, including Guimaraes and Sheedy (2011), relies on models with monopolistic competition (no strategic motives), whereas our model is based on duopolistic competition and incorporates strategic pricing behaviors. We show that a mixed strategy equilibrium does not arise when own demand and its elasticity are independent of the rival's price, which suggests the importance of a departure from monopolistic competition baesd on the CES preferences to explain temporary sales as a mixed strategy equilibrium.

Among a number of contributions to the development of New Keynesian models, our study is related to works that stress the importance of strategic complementarity and real rigidity (see, e.g., Ball and Romer 1990, Kimball 1995, Woodford 2003, Christiano, Eichenbaum, and Evans 2005, Levin, Lopez-Salido, and Yun 2007, Angeletos and La'o 2009, Aoki, Ichiue, and Okuda 2019, and L'Huillier 2020). For example, Angeletos and La'o (2009) show that the combination of noisy information and strategic complementarity yields lasting effects of nominal shocks on inflation and real output. L'Huillier (2020) considers a strategic interaction between a monopolistic firm and uninformed consumers. Our study provides a new insight into the source of strategic complementarity from the perspective of an oligopoly.

This contribution builds on the extant literature studying the relations between strategic pricing and sticky prices. Fershtman and Kamien (1987) consider a duopolistic competition model, as we do, under the assumption that prices do not adjust instantaneously to the price indicated by a demand function. They study how the sticky price assumption changes Nash equilibrium strategies. Their work is different from this study in that their model is based on Cournot competition, and thus, firms are not price setters and instead optimize their output. Price changes are governed by an exogenous reduced-form equation. Accordingly,

the author's research contribution does not directly identify implications for the macroeconomy. For further research on this topic, see Maskin and Tirole (1988), Slade (1999), Bhaskar (2002), Fehr and Tyran (2008), and Chen, Korpeoglu, and Spear (2017). These studies investigate whether price becomes sticky endogenously in a market where firms compete strategically.

The remainder of this paper is structured as follows. Section 2 presents the basic model and discusses its implications. Section 3 considers temporary sales as a mixed strategy equilibrium and investigates monetary policy effects. Section 4 concludes.

2 Duopolistic Competition Model with Nominal Stickiness

2.1 Setup

The head of a household maximizes the following preference: $U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - (L_t + \tau D_t) \right]$, where aggregate consumption C_t is given by $\log C = \int_0^1 \log c^j dj$, L_t represents labor supply, and shopping distance D_t is given by $D = \int_0^1 d^j dj$.[3] Here, c^j represents consumption for product line $j \in [0, 1]$, parameter $\beta \in (0, 1)$ is the subjective discount factor, and parameter $\tau > 0$ represents the transport cost incurred per unit of distance. Except for the Hotelling (1929) address model, one may ignore the transport cost and shopping distance.

The budget constraint is given by $M_t + B_t + P_t C_t \leq M_{t-1} + R_{t-1} B_{t-1} + W_t L_t + \Pi_t + T_t$, where M_t, B_t, P_t, R_t, W_t, Π_t, and T_t represent money supply, nominal bonds, aggregate price, nominal interest rate, nominal wages, dividends from firms, and lump-sum transfer, respectively. We assume that nominal spending must be equal to the money supply: $P_t C_t = M_t$. The first-order condition leads to $W_t = M_t = P_t C_t$.[4]

In each product line $j \in [0, 1]$, there exist n firms denoted by i. If there are two firms, we denote them as A or B ($i = A, B$). The quantity demanded for firm i's goods in period t, x_t^i, is

---

[3]We assume a log utility in consumption and linear disutility of labor supply as in Nakamura and Steinsson (2010) and Midrigan (2011). See also Hansen (1985) and Rogerson (1988).

[4]One may assume that the monetary authority targets a path for nominal output to ensure $P_t C_t = M_t$. Because $M_t = W_t$, money supply shocks are equivalent to changes in nominal marginal costs. See Nakamura and Steinsson (2010) and Midrigan (2011).

described by an arbitrary invertible demand system $x_t^i = x^i(p_t^i, p_t^{-i}; M_t) = x^i(p_t^i/M_t, p_t^{-i}/M_t)$, where p_t^i and p_t^{-i} represent the prices set by firm i and its rivals, respectively. Demand invertibility is a mild technical requirement, encompassing the demand system characterized by CES preferences, quasi kinked demand (Kimball 1995), and the Hotelling (1929) address model. To produce one unit of a product, firms require one unit of labor, which costs W_t.

The goods market is cleared as $Y_t(= L_t) = C_t$. Money supply is exogenous and given by $\log(M_t/M_{t-1}) = \varepsilon_t = \rho\varepsilon_{t-1} + \mu_t$, where μ_t is an i.i.d. shock to money supply. Money supply has zero trend growth.

Given firm symmetry, demand is shared equally among the firms in each product line, which leads to $x^i \equiv M_t/(nP_t) = 1/(np)$ in a steady state. We define own elasticity $\Psi^i \equiv \frac{\partial \log x^i(p^i/M, p^{-i}/M)}{\partial \log(p^i/M)}$, cross elasticity $\Psi^{-i} \equiv \frac{\partial \log x^i(p^i/M, p^{-i}/M)}{\partial \log(p^{-i}/M)}$, and three kinds of superelasticity $\Psi^{i,i} \equiv \frac{\partial \Psi^i}{\partial \log(p^i/M)}$, $\Psi^{-i,-i} \equiv \frac{\partial \Psi^{-i}}{\partial \log(p^{-i}/M)}$, and $\Psi^{i,-i} \equiv \frac{\partial \Psi^{-i}}{\partial \log(p^i/M)} = \frac{\partial \Psi^i}{\partial \log(p^{-i}/M)}$, all of which emerge as important for the consideration of monetary non-neutrality. Constant returns to scale imply $(n-1)\Psi^{-i} = -(1+\Psi^i)$. We discuss the main results in the subsequent discussion and present the detailed derivations in Appendices A and B. We consider Ψ's in the following two special cases.

CES and Oligopolistic Competition For each product line j, consumption is aggregated following the CES form of aggregation: $c_t^j = \left\{\sum_{i=1}^{n} (1/n)^{1/\sigma} (x_t^i)^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}$, where σ represents the elasticity of substitution. This yields demand $x_t^i = 1/n\,(p_t^i/P_t)^{-\sigma} M_t/P_t$ and $P_t = \left\{\sum_{i=1}^{n} 1/n\,(p_t^i)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$. Then, we obtain $\Psi^i = -\{(n-1)\sigma + 1\}/n$, $\Psi^{-i} = (\sigma-1)/n$, $\Psi^{i,i} = \Psi^{-i,-i} = -(n-1)\{(1-\sigma)/n\}^2$, and $\Psi^{i,-i} = \{(1-\sigma)/n\}^2$. The cross elasticity (Ψ^{-i}) and own superelasticity $(\Psi^{i,i}$ or $\Psi^{i,i}/|\Psi^i|)$ are different from zero unless $\sigma = 1$ or $n \gg 1$. Specifically, $\Psi^{-i} > 0$ and $\Psi^{i,i} < 0$ when $\sigma > 1$. Introducing quasi kinked demand further lowers superelasticity. The absolute value of own elasticity $|\Psi^i|$ increases as n increases (decreases) when $\sigma > 1$ ($\sigma < 1$), which converges to σ for the monopolistic competition model ($n \to \infty$). By contrast, the absolute value of cross elasticity $|\Psi^{-i}|$ decreases as n increases (decreases) when $\sigma > 1$ ($\sigma < 1$), which converges to zero for $n \to \infty$.

Hotelling Address Model and Duopolistic Competition A household comprises an infinite number of consumers who are uniformly located along the interval $[0, 1]$.[5] A consumer located at $x \in [0, 1]$ is at a distance x from firm A and $1 - x$ from firm B. The consumer's net surplus is written as $u^i = \log c^i - \tau d^i = \log M - \log p^i - \tau d^i$, where d^i represents the distance the consumer travels to firm i. Although we call τ the transport cost throughout the paper, this parameter also represents consumers' choosiness, that is, how much they dislike buying from their less preferred firms. When τ is high, the consumers are loyal to their preferred firms. When τ is low, they care about the prices of the two firms, acting as a bargain hunter.

In this case, demand is given by $x_t^i = \left(\frac{1}{2} - \frac{\log(p_t^i/M_t) - \log(p_t^{-i}/M_t)}{2\tau} \right) \frac{M_t}{p_t^i}$ when $p_t^i \simeq p_t^{-i}$, which yields $\Psi^i = -(1 + \tau)/\tau$, $\Psi^{-i} = 1/\tau$, $\Psi^{i,i} = \Psi^{-i,-i} = -1/\tau^2$, and $\Psi^{i,-i} = 1/\tau^2$.

All Ψ's have the same values as those based on the CES and duopolistic competition model ($n = 2$) when $\sigma - 1 = 2/\tau$. This is consistent with the finding by Anderson, de Palma, and Thisse (1992). However, there is a key difference: demand depends on prices and is kinked in the Hotelling address model. When the deviation of p_t^i from p_t^{-i} is large and positive (negative), demand becomes zero (saturated) as $x_t^i = 0$ (M_t/p_t^i). This leads to a mixed strategy when τ is heterogeneous, as discussed in Section 3.

2.2 Steady State without Price Stickiness

We consider the steady state in the case of no price stickiness. Firm i's profit is given by $\Pi_t^i = (p_t^i - W_t)x^i(p_t^i/M_t, p_t^{-i}/M_t)$. The first-order condition with respect to p_t^i yields

$$p = \Psi^i/(\Psi^i + 1), \tag{1}$$

where we assume that the steady state is given by $W = M = 1$.

Around the steady state, the slope of the best response price is given by $\partial \log p^i / \partial \log p^{-i} = \Psi^{i,-i}/\{\Psi^i(1 + \Psi^i)\}$, which is positive in the CES oligopolistic competition model with $\sigma > 1$ and finite n and the Hotelling duopolistic competition model. This suggests static strategic complementarity. By contrast, the slope is negative in the CES oligopolistic competition model when $\sigma < 1$ and n is finite. No strategic complementarity or substitutability exists in the CES monopolistic competition model ($n \to \infty$) because $\Psi^{i,-i} = 0$.

---

[5]The Hotelling (1929) part of the model in this study is based on the model presented by Armstrong (2006). Accordingly, we replicate Armstrong's results in the model without sticky prices.

2.3 Pricing under Price Stickiness

Hereafter, we consider the case of duopolistic competition $n = 2$. We assume Calvo-type price stickiness. Both firms A and B can reset their prices with a probability of $1 - \theta \in (0, 1)$.[6] In Appendix C, we confirm the robustness of our results by introducing Rotemberg-type price stickiness. We limit our analysis by assuming that the Markov perfect equilibrium concept applies. Each firm's action (i.e., price setting decision) depends on a state consisting only of the following three variables: its price in the previous period, the rival firm's price in the previous period, and a shock to money supply. We exclude collusive pricing, although the folk theorem suggests that dynamic settings can generate multiple collusive equilibria. The Calvo-type signal of price resetting is uncorrelated across product lines $j \in [0, 1]$.

When firm i has a chance to set its price at t, it sets $\bar{p}_t^i$ to maximize

$$\max \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \left[\left(\bar{p}_t^i - W_{t+k} \right) \theta^{k+1} x^i (\bar{p}_t^i / M_{t+k}, p_{t-1}^{-i} / M_{t+k}) \right] \cdot \frac{\Lambda_{t+k}}{\Lambda_t} \frac{P_t}{P_{t+k}}$$
$$+ \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \left[\left(\bar{p}_t^i - W_{t+k} \right) \sum_{k'=0}^{k} (1 - \theta) \theta^{k-k'} x^i (\bar{p}_t^i / M_{t+k}, p_{t+k'}^{-i} / M_{t+k}) \right] \cdot \frac{\Lambda_{t+k}}{\Lambda_t} \frac{P_t}{P_{t+k}}, \tag{2}$$

where Λ_t represents the stochastic discount factor given by C_t^{-1}. Solving this optimization problem is more complex than solving a similar problem in a standard New Keynesian model based on monopolistic competition, because we have to explicitly consider the path of the prices set by the rival firm. Note that the rival firm's price p_{t+k}^{-i} equals $\bar{p}_{t+k}^i$ with the probability of $1 - \theta$; $\bar{p}_{t+k-1}^i$ with the probability of $\theta(1 - \theta)$; $\cdots$; $\bar{p}_t^i$ with the probability of $\theta^k(1 - \theta)$; and p_{t-1}^i with the probability of θ^{k+1} for $k \geq 0$.

Firm i has to consider how its reset price at t influences the rival firm's reset price at $t+k'$, which is given by $\partial \log \bar{p}_{t+k'}^{-i} / \partial \log \bar{p}_t^i$. Given the Markov perfect equilibrium, the log-linearized policy function for the optimal reset price is expressed in the following forms:

$$p_t^{i*} = \Gamma \hat{p}_{t-1}^i + \Gamma^* \hat{p}_{t-1}^{-i} + \Gamma^\varepsilon \varepsilon_t, \tag{3}$$

$$\partial \log \bar{p}_{t+k}^{-i} / \partial \log \bar{p}_t^i = \Gamma^* \text{ for } k \geq 1, \tag{4}$$

---

[6]In this type of model, firms have to optimize their reset price by taking into account the fact that their reset price today influences the optimal reset price in the future. Below, we consider the effects of this only to the extent that a firm's reset price today influences the rival firm's reset price in the following periods. The effect of a firm's reset price today on its own reset price in the following periods does not matter for its own pricing decision today, as the marginal change of firm value to a change in its own reset price in the following periods is zero.

where $\bar{p}_t^i \equiv pM_te^{\bar{p}_t^{i*}}$, $p_t^i \equiv pM_te^{\hat{p}_t^i}$. Firms A and B set their prices simultaneously so that neither firm knows whether the rival resets its price in the current period. Therefore, the optimal reset price is written as a function of prices not in the current period but in the previous period. Furthermore, firm A's value is influenced by firm B's reset price in the current period if resetting occurs, and firm B's reset price is influenced by firm A's price in the previous period. Therefore, firm A's optimal reset price depends on its own price in the previous period. Further, the optimal reset price depends on a stochastic aggregate shock in the current period.

The coefficients Γ and Γ^* show the elasticity of the optimal price in the current period with respect to a change in own price and the rival's price, respectively, in the previous period. Particularly, Γ^* is an important parameter that shows the degree of dynamic strategic complementarity if it is positive. The coefficient $(\Gamma^* = \partial \log \bar{p}_{t+k}^{-i}/\partial \log \bar{p}_t^i)$ is independent of the period elapsed after price revision k and aggregate state ε_{t+k} because we consider a case in which the price of firm i is unchanged at $\bar{p}_t^i$ from t and a log-linearized economy around a steady state, respectively. The coefficient Γ^ε shows the elasticity of the optimal price with respect to the money supply shock. A smaller Γ^ε means that the price reacts less strongly to the money supply shock, amplifying the real effect of monetary policy.

The first-order condition for the optimal $\bar{p}_t^i$ enables us to solve p, Γ, Γ^*, and Γ^ε by using the method of undetermined coefficients from the coefficients of 1, $\hat{p}_{t-1}^i$, $\hat{p}_{t-1}^{-i}$, and ε_t. We provide a detailed description in Proposition 1 in Appendix A In the main text, we focus on steady state p and the degree of dynamic strategic complementarity Γ^*.

Lemma 1 *The steady-state price under price stickiness equals*

$$p = 1 - \left\{ 1 + \Psi^i + \frac{\theta\beta(1-\theta)}{1-\theta^2\beta}\Psi^{-i}\Gamma^* \right\}^{-1}. \tag{5}$$

All proofs are presented in the Appendix. This lemma shows that unless Γ^* is zero, the steady state under nominal rigidity is different from that without nominal rigidity. Firms take into account the effect of their price on the rival firm's price in the following periods. Specifically, if Γ^* is positive (dynamic strategic complementarity), an increase in firm A's price increases firm B's price in the following periods. This effect increases the steady-state price level. The above equation also shows that the steady-state price level becomes identical

to that in the scenario without nominal rigidity in the limit of $\theta \to 0$. When $\Gamma^* = 0$, then $p = \Psi^i/(\Psi^i + 1)$. Specifically, in the CES and monopolistic competition model, $p = \sigma/(\sigma - 1)$.

This result is essentially the same as in Proposition 1 in Wang and Werning (2020).[7] At the first glance, one may interpret Proposition 1 in Wang and Werning (2020) as the equation that shows how Γ^* is determined by the demand elasticity and *observed* steady-state markup p. This expression is used for an empirical purpose, which we will discuss in the next section. As Wang and Werning (2020) note, both p and Γ^* are endogenous, depending on demand parameters Ψ's.

While the above lemma shows how steady-state price p is determined by Γ^*, the next lemma shows how Γ^* is determined by demand.

Lemma 2 *When $p \simeq /\Psi^i(\Psi^i + 1)$, the degree of dynamic strategic complementarity Γ^* satisfies*

$$(\Psi^{i,-i} - \Psi^{-i})\left(\frac{\theta\beta}{1-\theta\beta} - \frac{\theta^2\beta}{1-\theta^2\beta}\right)\Gamma^{*2} + (\Psi^{-i}\Psi^{-i} + \Psi^{-i,-i})\mathbb{\Gamma}_{k1}^{-i}\Gamma^* + \Psi^{i,-i}\left(\frac{\theta}{1-\theta^2\beta} + \mathbb{\Gamma}_{k0}^{-i}\right)$$
$$= \frac{1}{1-\theta\beta}\left(\Psi^i(\Psi^i + 1) - \Psi^{i,i}\right)\Gamma^*, \quad (6)$$

where $\mathbb{\Gamma}_{k0}^{-i}$ is the second row and the second column of the following matrix:

$$(1-\theta)\Gamma\left[I - (\mathbb{\Gamma}/\theta)\right]^{-1}\left[\frac{1}{1-\theta^2\beta}I - \mathbb{\Gamma}/\theta\left[I - (\theta\beta\mathbb{\Gamma})\right]^{-1}\right]$$

$$and \ \mathbb{\Gamma} = \begin{pmatrix} \Gamma & \Gamma^* & \Gamma^\varepsilon \\ \Gamma^* & \Gamma & \Gamma^\varepsilon \\ 0 & 0 & \rho \end{pmatrix}.$$

Two results emerge from Lemma 2. First, when cross elasticity is zero ($\Psi^{-i} = 0$), no dynamic strategic complementarity exists, that is, $\Gamma^* = 0$. Second, except for special cases such as $\Psi^{-i} = 0$, the solution of Γ^* is not expressed in a closed form because the $\mathbb{\Gamma}_{k0}^{-i}$ term in the above equation depends on Γ and Γ^ε. However, Γ^* can be approximated to a closed form when $|\mathbb{\Gamma}_{k0}^{-i}|$ is sufficiently small, as is shown in the following corollary.

Corollary 1 *Assume $p \simeq /\Psi^i(\Psi^i + 1)$, $|\mathbb{\Gamma}_{k0}^{-i}| \ll 1$, $\beta \simeq 1$, $\Psi^i = -1 - \Psi^{-i}$, $\Psi^{-i,-i} = -(\Psi^{-i})^2$, $\Psi^{i,i} = -(\Psi^{-i})^2 - \gamma^{i,i}\Psi^{-i}$, and $\Psi^{i,-i} = (\Psi^{-i})^2 + \gamma^{i,-i}\Psi^{-i}$, where $|\gamma^{i,i}, \gamma^{i,-i}| \ll 1$. Then, dynamic strategic complementarity Γ^* is written as*

$$\Gamma^* = \frac{1 + \gamma^{i,i} + 2\Psi^{-i} - \sqrt{(1 + \gamma^{i,i} + 2\Psi^{-i})^2 - 4(\Psi^{-i} + \gamma^{i,-i} - 1)\left(\frac{\theta}{1+\theta}\right)^2(\Psi^{-i} + \gamma^{i,-i})}}{2(\Psi^{-i} - 1)\frac{\theta}{1+\theta}}. \quad (7)$$

---

[7]Equality holds when $(\lambda + \rho)/\lambda = \theta\beta(1 - \theta)/(1 - \theta^2\beta)$ and $(n - 1)\Psi^{-i} = -(1 + \Psi^i)$, where λ and ρ represent the Calvo and discount parameters, respectively, in continuous time (equivalent to θ and β).

Further, $\Gamma^* > 0$, $\partial\Gamma^*/\partial\Psi^{-i} > 0$, $\partial\Gamma^*/\partial\gamma^{i,i} < 0$ *and* $\partial\Gamma^*/\partial\gamma^{i,-i} > 0$ *when* $\Psi^{-i} > 0$, $\Psi^{-i} \neq 1$, *and* $\theta/(1+\theta) \simeq 1/2$. *When* $\Psi^{-i} < 0$ *and* $|\Psi^{-i}| \ll 1$, $\Gamma^* < 0$.

Note that we have $\Psi^i = -1 - \Psi^{-i}$, $\Psi^{i,i} = \Psi^{-i,-i} = -\Psi^{i,-i} = -(\Psi^{-i})^2$ in the duopolistic CES/Hotelling models. We allow a slight deviation for superelasticity $\Psi^{i,i}$ and $\Psi^{i,-i}$ to investigate their effects on Γ^*.

This corollary indicates the following properties: First, a positive cross elasticity yields dynamic strategic complementarity, whereas a negative cross elasticity (when the size is not too large) yields dynamic strategic substitutability. Second, dynamic strategic complementarity increases as cross elasticity increases. Third, dynamic strategic complementarity decreases as the absolute value of own superelasticity increases. Fourth, dynamic strategic complementarity increases as cross superelasticity increases. It should be noted that quasi kinked demand is often interpreted as a decrease in $\Psi^{i,i} < 0$ (i.e., an increase in $|\Psi^{i,i}|$), which is often considered to cause an increase in dynamic strategic complementarity. However, the third result we presented here is the opposite. An increase in cross superelasticity, rather than own superelasticity, is needed to increase dynamic strategic complementarity.

Corollary 1 is in line with Corollary 1 in Wang and Werning (2020), which shows the condition for the steady-state price (p) and the degree of dynamic strategic complementarity (Γ^*). Our method of log-linearization is equivalent to their approximation of a locally linear equilibrium ($m = 2$), where the policy function for the optimal price ($g(p)$) is linear (i.e., $g^{(k)}(p) = 0$ for $k \geq m$), and demand elasticity matters up to the second order (superelasticity). One difference is that the optimal reset price in our model depends on a firm's own price in the previous period, a rival's price in the previous period, and the aggregate shock in the current period, whereas that in Wang and Werning (2020) depends only on a rival's price in the current period because there is no aggregate shock and time is continuous in their study. Consequently, the response to a aggregate shock is explicitly expressed in our study as Γ^ε. The second difference is that we derive a closed-form solution to Γ^*, albeit an approximation. Third, we show that an increase in superelasticity does not necessarily increase dynamic strategic complementarity, although Wang and Werning (2020) demonstrate the examples numerically such that an increase in superelasticity increases dynamic strategic complementarity. Our analysis shows that an increase in own superelasticity actually decreases dynamic strategic complementarity.

2.4 Numerical Illustrations

We solve p, Γ, Γ^*,and Γ^ε numerically without resorting to the approximation given by equation (6) or (7). A time unit is a quarter. We calculate demand parameters Ψ's using transport cost $\tau = 0.125$ based on the Hotelling model such that the steady-state markup without price stickiness is consistent with that based on the CES monopolistic competition model with $\sigma = 9$, as assumed by Gali (2015). This value of τ is equivalent to $\sigma = 17$ in the CES duopolistic competition model. Price stickiness is set at $\theta = 0.75$, which implies that price revisions occur once per year. We also use $\rho = 0.85$ and $\beta = 0.99$.

Policy Function Figure 1 shows policy functions for the optimal reset price, specifically, coefficients Γ^* and Γ^ε in equation (3). We change one of the five parameter values that characterize demand, $\Psi^i, \Psi^{-i}, \Psi^{i,i}/|\Psi^i|, \Psi^{-i,-i}/|\Psi^{-i}|, \Psi^{i,-i}/\sqrt{/|\Psi^i\Psi^{-i}|}$, while keeping the other four parameter values fixed.

This figure suggests that Γ^* is positive (i.e., dynamic strategic complementarity exists) and that Γ^* increases as own elasticity $|\Psi^i|$ decreases, cross elasticity Ψ^{-i} increases, own superelasticity $|\Psi^{i,i}/\Psi^i|$ decreases, or cross superelasticity $\Psi^{i,-i}/\sqrt{/|\Psi^i\Psi^{-i}|}$ increases. This result is consistent with Corollary 1.

Figure 1 also indicates that the response to aggregate shock, Γ^ε, tends to move in the opposite direction to Γ^*. This implies that dynamic strategic complementarity increases price sluggishness and the real effect of monetary policy.

Although it is useful to show how each of the demand parameters Ψ's influences policy functions Γ^* and Γ^ε, demand parameters are not independent of one another. Thus, we consider how a single parameter, given by transport cost τ based on the Hotelling model, influences policy functions by affecting Ψ's. Recall that the model is equivalent to the CES duopolistic competition model with the elasticity of the substitution of $\sigma = 1 + 2/\tau$.

Figure 2 shows policy functions; for comparison, we plot herein the policy functions in the CES monopolistic competition model ($n \to \infty$) with $\sigma = 9$. The figure shows that both Γ and Γ^* are positive. This suggests that a firm revises its price upward when its previous price was high (i.e., $\Gamma > 0$) or its rival's previous price was high (i.e., $\Gamma^* > 0$). Quantitatively, the size of Γ^* is around 10 times larger than the size of Γ, showing that a firm's own past price is much less important than its rival firm's price. While Γ and Γ^* increase, Γ^ε decreases relative

to that in the CES monopolistic competition model. The price responds to an aggregate shock to a lesser extent, which increases the real effect of monetary policy.

Figure 3 shows how the number of firms n influences policy functions. Our analysis assumes a duopolistic competition because otherwise, the timing of other firms' price resetting becomes heterogeneous under Calvo-type price stickiness. If one assumes that all other firms reset their prices simultaneously with a probability of $1-\theta$, then the following results hold for any $n \geq 2$. On the basis of the CES oligopolistic competition model with $\sigma = 1 + 2/\tau = 17$, we change n, which is a proxy for market concentration, from 2 to 100. The figure shows that Γ and Γ^* decrease as n increases, whereas Γ^ε increases with n. The degree of dynamic strategic complementarity and the real effect of monetary policy are the largest when n is 2.

Figure 4 presents policy functions, where the horizontal axis represents price stickiness θ. The figure shows that both Γ and Γ^* are positive unless $\theta = 0$ and that they depend on the value of θ. Specifically, Γ^* increases monotonically as θ increases, showing that an increase in price stickiness increases the degree of dynamic strategic complementarity. By contrast, the response of Γ to θ is not monotonic: Γ increases with θ when θ is small; however, when θ is around 0.75, Γ decreases with θ. The size of Γ^ε increases with θ, suggesting that price stickiness increases the price response to an aggregate shock.

Steady-State Price The dynamic strategic complementarity owing to a positive Γ^* leads to a higher markup in the steady state under sticky prices, as illustrated in equation (5). Figure 5 shows the steady-state price under sticky prices. The steady-state price is plotted as a ratio to the price under flexible prices given by equation (1). In the three panels, we show the steady-state price by changing the parameter values of transport cost τ, the number of firms n, and the degree of price stickiness θ. Transport cost τ influences the steady-state price in a non-monotonic manner. There are two forces. On the one hand, a high τ increases the steady-state price under flexible prices. Specifically, when τ is zero, firms earn no markup, and thus, price stickiness does not matter for the ratio of the steady-state price under sticky prices to that under flexible prices. On the other hand, a high τ weakens dynamic strategic complementarity (i.e., Γ^* decreases as τ increases). As a result of equation (5), this decreases the steady-state price under sticky prices. Consequently, there is a certain τ that maximizes the ratio of the steady-state price under sticky prices relative to that under flexible prices.

The steady-state price increases as the number of firms n decreases to two. This does not stem from an increase in the steady-state markup under flexible prices as the figure shows a ratio of the price under sticky prices to the price under flexible prices. As n decreases, dynamic strategic complementarity Γ^* increases, which increases the steady-state price under sticky prices from equation (5).

For the influence of the degree of price stickiness θ, there is a certain θ that maximizes the ratio of the steady-state price under sticky prices to that under flexible prices. When θ is not too high, an increase in θ magnifies the importance of reacting to $\hat{p}_{t-1}^B$ for firm A because firm B is more likely to keep its price unchanged. This strengthens dynamic strategic complementarity and, in turn, increases the steady-state price under sticky prices. However, when θ is very large, firm A's price today is less likely to influence firm B's price tomorrow because firm B is less likely to reset its price. This weakens the effect of dynamic strategic complementarity and, in turn, decreases the steady-state price under sticky prices.

Impulse Responses Figure 6 shows the impulse response functions to a positive money supply shock ($\mu_t = 1$ at $t = 1$) for aggregate inflation rate π_t and output $\hat{Y}_t$. Whereas the nominal effect of monetary policy on inflation is smaller, the real effect of monetary policy in this model is larger than that in the New Keynesian model by approximately 50%. This arises from the dynamic strategic complementarity of price setting, as explained above. Firms react to the monetary policy shock by taking their rivals' pricing into account strategically, which reduces the size of price adjustments (i.e., Γ^ε decreases). This increases aggregate price stickiness, which increases the real effect of monetary policy.

3   Mixed-Strategy Pricing under Duopolistic Competition and Nominal Stickiness

In this section, we investigate the possibility of accounting for temporary sales (price discounts) as a mixed strategy and then the effect of monetary policy in the presence of temporary sales. See Appendix D for details.

We assume two firms ($i = A, B$) and consider an arbitrary invertible demand system $x_t^i = x^i(p_t^i/M_t, p_t^{-i}/M_t)$ for $i = A, B$. We allow demand elasticities to depend on prices

so that firms may set two possible prices, namely, higher p_H (with the probability of $1 - s$) and lower p_L (with the probability of s), as a mixed strategy. Firm profit is given by $\Pi_t^i = (p_t^i - W_t)x^i(p_t^i/M_t, p_t^{-i}/M_t)$, and expected demand from choosing p_t is given by $(1 - s)x(p_t/M_t, p_H/M_t) + sx(p_t/M_t, p_L/M_t)$. We define own and cross elasticities, which depend on a firm's own price and the rival's price, as $\Psi^i(p^i/M, p^{-i}/M) \equiv \frac{\partial \log x^i(p^i/M, p^{-i}/M)}{\partial \log(p^i/M)}$ and $\Psi^{-i}(p^i/M, p^{-i}/M) \equiv \frac{\partial \log x^i(p^i/M, p^{-i}/M)}{\partial \log(p^{-i}/M)}$, respectively. Whether elasticities depend on the rival's price is particularly important for the mixed strategy.

3.1 Steady State without Price Stickiness

Pure Strategy First, we consider a steady state without price stickiness. One possible option for a pricing strategy is a pure strategy. The prices of firms A and B are symmetric, $p^* = p^A = p^B$, which satisfies $p^* = \Psi^i(p^*, p^*)/(\Psi^i(p^*, p^*) + 1)$.

Under some conditions, a firm may have an incentive to deviate and choose p^d. For example, in the Hotelling model, given firm B's price p^*, firm A may be able to increase its profit by giving up revenues from price-sensitive bargain hunters and charging a higher price. The condition for the pure strategy equilibrium to hold is written as $\Pi^i(p^*, p^*) > \Pi^i(p^d, p^*)$ for any p^d.

Mixed Strategy If there exists a mixed strategy equilibrium, the first-order conditions with respect to p_H and p_L should be zero for $p_H > p_L$; that is, $\partial \Pi^i / \partial p^H = \partial \Pi^i / \partial p^H = 0$, yielding

$$0 = (1 - s)x^{HH}\left\{1 + (p_H - 1)\frac{\Psi^i(p_H, p_H)}{p_H}\right\} + sx^{HL}\left\{1 + (p_H - 1)\frac{\Psi^i(p_H, p_L)}{p_H}\right\} \tag{8}$$

$$0 = (1 - s)x^{LH}\left\{1 + (p_L - 1)\frac{\Psi^i(p_L, p_H)}{p_L}\right\} + sx^{LL}\left\{1 + (p_L - 1)\frac{\Psi^i(p_L, p_L)}{p_L}\right\}, \tag{9}$$

where $x^{ij} \equiv x(p_i, p_j)$ for $i, j = H$ or L. Furthermore, expected profits choosing the higher price, $\Pi_H(p_H, p_L, s) \equiv \Pi(p_H | p_H, p_L, s)$ and the lower price $\Pi_L(p_H, p_L, s) \equiv \Pi(p_L | p_H, p_L, s)$ should be equal:

$$\Pi_H(p_H, p_L, s) \equiv (p_H - 1)\left\{(1 - s)x^{HH} + sx^{HL}\right\}$$
$$= (p_L - 1)\left\{(1 - s)x^{LH} + sx^{LL}\right\} \equiv \Pi_L(p_H, p_L, s). \tag{10}$$

These three equations yield steady-state p_H, p_L, and s. We obtain the following corollaries.

Corollary 2 *When own demand elasticity is independent of prices, a mixed strategy equilibrium does not arise.*

When $\Psi^i(p_H, p_H) = \Psi^i(p_H, p_L)$, $\Psi^i(p_L, p_H) = \Psi^i(p_L, p_L)$, $x^{HL} = x^{HH}$, and $x^{LH} = x^{LL}$, a mixed strategy equilibrium does not arise either.

This corollary suggests that the CES preferences do not explain temporary sales as a mixed strategy equilibrium. Even in the presence of consumer heterogeneity, a mixed strategy equilibrium does not arise as long as aggregate demand elasticity (the sum of consumer demands) is constant. Furthermore, even if own demand elasticity depends on prices, a mixed strategy equilibrium does not arise when own demand and its elasticity are independent of the rival's price. Kinked demand that changes the shape of demand from CES is not enough to produce a mixed strategy; we need a strategic demand system.

The next corollary shows that a mixed strategy may entail two kinds of strategic complementarity.

Corollary 3 *Assume that a mixed strategy equilibrium exists. Then, $p_H(p_L, s)$ obtained from equation (8) satisfies $\partial p_H / \partial p_L > 0$ unless $\Psi^{i,-i}(p_H, p_L)$ is too large.*

Assume $x^{HH} > x^{HL}$ and $x^{LH} > x^{LL}$. Then, $\frac{\partial \Pi_H(p_H, p_L, s)}{\partial s} < 0$ and $\frac{\partial \Pi_L(p_H, p_L, s)}{\partial s} < 0$. If $x^{LH} - x^{HH} \gtreqless x^{LL} - x^{HL}$, then $\frac{\partial \Pi_H(p_H, p_L, s)}{\partial s} - \frac{\partial \Pi_L(p_H, p_L, s)}{\partial s} \lesseqgtr 0$.

The first line of the corollary suggests that the higher price exhibits strategic complementarity with the lower price. For the second line of the corollary, the assumption of $x^{HH} > x^{HL}$ and $x^{LH} > x^{LL}$ is standard. This corollary shows that an increase in the frequency of sales decreases the firm profit, irrespective of whether the firms choose the higher price. More importantly, it shows that as the frequency of sales increases, the profit from choosing the higher price decreases more than that from choosing the lower price when $x^{LH} - x^{HH} > x^{LL} - x^{HL}$. Thus, this increases the incentive to choose the lower price, which implies strategic complementarity in temporary sales. The condition $x^{LH} - x^{HH} > x^{LL} - x^{HL}$ means that an increase in demand by choosing the lower price given that the rival sets the higher price is greater than an increase in demand by choosing the lower price given that the rival sets the lower price. When the condition is reversed (i.e., $x^{LH} - x^{HH} < x^{LL} - x^{HL}$), the profit from choosing the lower price decreases more than that from choosing the higher price. Thus, there is strategic substitutability in temporary sales.

It should be noted that these two kinds of strategic complementarity concern the interaction between setting the higher price on the one hand and setting the lower price or the frequency of temporary sales on the other. There is the third kind of strategic complementarity, which concerns the interaction between two firms' higher prices.

Hotelling Address Model In the case of the Hotelling address model with heterogeneous τ, a mixed strategy equilibrium may arise. Specifically, we incorporate consumers' heterogeneity in terms of not only their location (x) but also their transport cost (τ): some consumers may have access to a car and be more mobile, whereas others may not be, for example, because they are elderly, unhealthy, or busy working (see Armstrong 2006). Note that transport cost τ also represents a consumer's choosiness. Thus, the heterogeneity of τ also indicates how some consumers are loyal to a particular firm (brand) (i.e., price-insensitive), whereas others are bargain hunters (i.e., price-sensitive). Then, the former customers have higher τ than the latter. Specifically, τ takes τ_L with the probability of α or τ_H otherwise ($0 < \tau_L < \tau_H$). This probability is independent of consumer location. Firms A and B cannot observe consumers' transport cost, but they know the distribution characterized by τ_H, τ_L, and α correctly.[8] Appendix D.2 explains the details on pure and mixed strategy equilibria and the welfare implications.

We have the following corollary as a special case of Corollary 2.

Corollary 4 *Assume that a mixed strategy equilibrium exists in the Hotelling address model with heterogeneous transport costs. Then, $p_H(p_L, s)$ obtained from equation (8) satisfies $\partial p_H / \partial p_L > 0$. Moreover, $\frac{\partial \Pi_H(p_H, p_L, s)}{\partial s} - \frac{\partial \Pi_L(p_H, p_L, s)}{\partial s} < 0$.*

Temporary sales exhibit two kinds of strategic complementarity in the Hotelling model. First, optimal p_H increases when p_L increases. Second, if firm A expects firm B to hold sales with a higher probability, then it is better for firm A to choose the lower price, leading to a further increase in the frequency of sales. Note that the condition $x^{LH} - x^{HH} > x^{LL} - x^{HL}$ is satisfied. That is, when firm B sets the higher price, firm A can increase its profits to a large

---

[8]If firms A and B can observe consumers' transport cost one by one, then they set their price differently: $p = (1 + \tau_L/2) W$ for consumers with τ_L and $p = (1 + \tau_H/2) W$ for consumers with τ_H. In other words, firms charge a higher price for price-insensitive loyal consumers and a lower price for price-sensitive bargain hunters.

extent by setting the lower price, because all price-sensitive consumers come to purchase from firm A. By contrast, when firm B sets the lower price, firm A can increase its profits to a lesser extent by setting the lower price because half of price-sensitive consumers continue to purchase from firm B.

This second property is markedly different from that reported by Guimaraes and Sheedy (2011). They argue that sales show strategic substitutability; that is, if other firms choose a higher price rather than a lower price more frequently, a firm will be better off if it chooses a lower price rather than a higher price more frequently.

To understand the difference in the strategic property, we can point out two differences. First, Guimaraes and Sheedy (2011) assume that there exist two types of consumers whose elasticities of substitution differ but are constant. As we showed above, the CES preference does not yield a mixed strategy. Strictly speaking, two price equilibria in their study are not mixed strategy equilibria as in our model. In their model, each firm can set multiple prices in each period, choosing a higher price for a certain fraction of products and a lower price for the other. Second, in their model, firms are monopolistic rather than duopolistic. Thus, firms do not make strategic pricing.

3.2 Pricing under Price Stickiness

In the next step, we add Calvo-type price stickiness. Both firms A and B compete in multiple periods and can reset their prices at the probability of $1 - \theta \in (0, 1)$.

Pure Strategy   First, we consider a case in which the pure strategy equilibrium holds. Importantly, the condition for the pure strategy equilibrium to hold, which was shown in the previous subsection, is relaxed. In the dynamic setup where two firms compete repeatedly, the incentive to deviate from the pure strategy decreases. Nevertheless, we can obtain the implications of price stickiness, which are similar to those in Section 2.3. We obtain the following lemma.

Lemma 3 *Suppose that the pure strategy equilibrium holds. The steady-state price under price stickiness p equals*

$$p = 1 - \left\{ 1 + \Psi^i(p, p) + \frac{\theta\beta(1 - \theta)}{1 - \theta^2\beta} \Psi^{-i}(p, p)\Gamma^* \right\}^{-1}. \tag{11}$$

Mixed Strategy Next, we consider the case of a mixed strategy. The literature often states that regular prices are sticky while sale prices are highly flexible. As in Guimaraes and Sheedy (2011), we assume that the higher price is subject to Calvo-type price stickiness and that the lower price is perfectly flexible. Specifically, we assume that the lower price $p_{L,t}$ is set at $p_L W_t$, where p_L denotes the steady-state lower price when $W_0 = 1$. In other words, the lower price is indexed to the aggregate wage level (money supply) fully. This assumption is motivated by the empirical fact that firms tend to adjust the frequency of sales rather than their size to respond to shocks (Sudo et al. 2018, Kryvtsov and Vincent 2021). However, it should be noted that this price is not necessarily optimal since it does not consider price history. With the probability of $1 - \theta$, firms can revise the higher price. Firms may also choose to set the lower price, and even in this case, we assume that the rival firm can observe the higher price.

For the mixed strategy equilibrium to hold, the two choices (higher price and lower price) must yield the same payoff. However, this is very restrictive, particularly when we impose the equality of the payoff both when firms can revise their higher price and when they cannot. If the payoff from not revising the higher price is the same as that from choosing the lower price, the payoff from revising the higher price is likely to exceed that from choosing the lower price because not revising the higher price is suboptimal.

In this study, we assume that the Calvo-type lottery determines either the higher price or the frequency of sales as the variable that firms can reoptimize. With the probability of $1 - \theta$, firms can revise the higher price, and in this case, firms cannot optimize the frequency of sales, which is kept at a steady-state level (s). The equality of payoffs does not hold unless the economy is in a steady state. With the probability of θ, firms cannot revise the higher price, but they can optimize the frequency of sales s_t^n. The first-order condition with respect to the higher price is not ensured to hold. As documented by Zbaracki et al. (2004), there exist managerial costs (information gathering, decision-making, and communication costs) and customer costs (communication and negotiation costs), which prevent firms from optimizing prices. We assume that they prevent firms from optimizing both the higher price and the frequency of sales simultaneously. However, it is important to check the robustness of our results to this assumption. We will examine how a different assumption on the frequency of sales changes our results.

Based on an arbitrary invertible demand system, we obtain the following lemma, which is related to Lemma 1 in the case of pure strategy.

Lemma 4 *Suppose that the mixed strategy equilibrium holds. The steady-state price under price stickiness p_H satisfies*

$$\frac{p_H - 1}{p_H} \left\{ s x_{HL} \Psi^i_{HL} + (1-s)/2\Psi^i_{HH} + (1-s)/2 \frac{\theta\beta(1-\theta)}{1-\theta^2\beta} \Psi^{-i}_{HH} \Gamma^* + \theta^2 \beta s \Lambda^{n*} \left(x^i_{HL} - 1/2 \right) \right\}$$
$$= -\left(s x_{HL} + (1-s)/2 \right), \tag{12}$$

where $\Gamma^* \equiv \partial log \bar{p}^{-i}_{H,t+k}/\partial log \bar{p}^i_{H,t}$ and $\Lambda^{n*} \equiv \partial log s^n_{t+k}/\partial log \bar{p}^i_{H,t}$ for $k \geq 1$.

Remarks on Empirical Work The above equation can be transformed into

$$\Gamma^* = -\left\{ \left(\frac{p_H}{p_H - 1} + \Psi^i_{HH} \right) + \frac{s}{1-s} 2 p_H x_{HL} \left(\frac{p_H}{p_H - 1} + \Psi^i_{HL} \right) + \frac{s}{1-s} \theta^2 \beta \Lambda^{n*} \left(2 p_H x^i_{HL} - 1 \right) \right\} \left\{ \frac{\theta\beta(1-\theta)}{1-\theta^2\beta} \Psi^{-i}_{HH} \right\}^{-1}. \tag{13}$$

Four observations can be made from this equation. First, this equation confirms Wang and Werning's (2020) remark that regressing a measure of non-neutrality of money (Γ^*) on average markups (p_H), controlling for concentration, should yield a positive coefficient even in the presence of temporary sales. Unless Λ^{n*} is negative and large, the coefficient on p_H is positive.

Second, this equation implies the existence of a bias in the evaluation of dynamic strategic complementarity Γ^* by ignoring temporary sales. Suppose that one incorrectly evaluates Γ^* (denoted by Γ^*_{pure}) from equation (11) based on the pure strategy equilibrium that ignores temporary sales. Further, we assume that one uses only the higher price p_H as a proxy for p. Then, the biased Γ^*_{pure} is evaluated as

$$\Gamma^*_{pure} = -\left(\frac{p_H}{p_H - 1} + \Psi^i_{HH} \right) \left\{ \frac{\theta\beta(1-\theta)}{1-\theta^2\beta} \Psi^{-i}_{HH} \right\}^{-1}. \tag{14}$$

Two values of Γ^* and Γ^*_{pure} deviate from the second and third terms in the numerator for the equation of Γ^*. Given a mixed strategy, the deviation increases as the frequency of sales s increases or the gap $p_H/(p_H - 1) + \Psi^i_{HL}$ between the markup and own elasticity when a firm sets the higher price and the rival sets the lower price is large.

Third, suppose $|\Lambda^{n*}|$ is small, that is, the frequency of sales is not highly responsive to the rival firm's price. Then, extending Wang and Werning's (2020) remark, we can argue that regressing a measure of non-neutrality of money (Γ^*) on average markups (p_H), the frequency of sales (s), and their cross term ($p_H s$), while controlling for concentration, should yield a positive, ambiguous, and positive coefficient, respectively. The coefficient on s is

positive (negative) if $p_H/(p_H - 1) + \Psi_{HL}^i$ is positive (negative), that is, when the absolute own elasticity is small (large).

Finally, equation (13) shows that the degree of dynamic strategic complementarity may decrease in the presence of temporary sales. In the Hotelling model where a mixed strategy arises, a large heterogeneity in transport costs such as $\tau_L \ll \tau_H$ needs to be assumed, which leads to $\Psi_{HH}^i < \Psi_{HL}^i < 0$ and $\Psi_{HH}^{-i} > \Psi_{HL}^{-i} > 0$. This inequality can be explained because when firm B sets the higher price, both price-sensitive and insensitive consumers purchase from firm A by setting the higher price (i.e., demand elastic), whereas only price insensitive consumers purchase from firm A when firm A sets the higher price and firm B sets the lower price (i.e., demand inelastic). Owing to this inequality, demand becomes more inelastic when there is a possibility of temporary sales than when there is only a pure strategy equilibrium, which decreases the degree of dynamic strategic complementarity.

To calculate the mixed strategy equilibrium, we need to the solve steady state under sticky prices, p_H, p_L, s, and the policy function up to the first order characterized by, for example, Γ^*, Γ^ε, and Λ^{n*}. We provide a detailed description of the mixed strategy equilibrium based on the Hotelling model as Proposition 2 in Appendix D.4.

3.3   Numerical Illustrations

For the simulation, we use the parameter values explained in Section 2.4. The only exception is the transport cost. Instead of the homogeneous value of $\tau = 0.125$, we use various combinations for the values of τ_H, τ_L, and α.

Pure and Mixed Strategy Region under Flexible Prices   We show numerical results under flexible prices (i.e., $\theta \to 0$). First, Figure 7 illustrates the parameter region in which the equilibrium is characterized by either the pure or mixed strategy. We fix τ_L at either 0.01 or 0.05. The figure shows that the pure strategy equilibrium is more likely to arise as the difference between τ_L and τ_H becomes smaller (i.e., τ_L is larger and τ_H is smaller) or the probability of τ_L becomes higher. The mixed strategy equilibrium is more likely to arise in the opposite case. The line with circles shows the combination of τ_H and α, which is required to keep the harmonic mean of τ at 0.125. This line intersects with the boundary at the point dividing the pure and mixed strategy equilibria. This suggests that if a harmonic

mean-preserving difference for τ_H and τ_L exceeds a certain level, the equilibrium changes from the pure strategy equilibrium to the mixed strategy equilibrium.

Second, Figure 8 shows how the equilibrium price changes when τ_H changes. Here, we fix τ_L at 0.01 and adjust α so that the harmonic mean of τ is unchanged at 0.125. When τ_H is low (close to τ_L), the pure strategy constitutes the equilibrium, and the equilibrium price is independent of τ_H because the harmonic mean of τ is unchanged. When τ_H is high, the mixed strategy constitutes the equilibrium, generating two possible equilibrium prices. The figure shows that compared with the pure strategy, the mixed strategy leads to higher equilibrium prices. Interestingly, the equilibrium price is higher even for the lower price of the two. As τ_H increases (i.e., the harmonic mean-preserving difference increases), the two equilibrium prices increase. Meanwhile, the probability of sales (i.e., the probability of choosing the lower price rather than the higher price) decreases. It should also be noted that in this model, the size of the sale discount, $(p_H - p_L)/p_H$, is approximately 15%, whereas the frequency of sales is around 60%. Compared with the data (e.g., see Sudo et al. 2018), the former is almost the same, but the latter is considerably higher. The frequency of sales decreases if we allow the own demand elasticity to be smaller than that based on the Hotelling model when both firms A and B set the price around the lower price, which we will discuss in the next analysis.

Impulse Responses Next, we simulate the model under price stickiness ($\theta = 0.75$). We set τ_L and τ_H at 0.01 and 1, respectively, whereas α is chosen to make the harmonic mean of τ equal 0.125 (i.e., $\alpha = 0.07$). In this case, the mixed strategy serves as the equilibrium. We numerically calculate policy functions with respect to pricing and then calculate the impulse responses to a positive money supply shock ($\mu_t = 1$ at $t = 1$).

The solid line with the cross in Figure 9 shows the simulation results. All the variables are log-linearized from their steady-state values.[9] The figure shows that a positive money supply shock increases the inflation rate and output. The higher price is negative, suggesting that price stickiness prevents some firms from adjusting their price upward. Meanwhile, these firms optimally adjust the frequency of sales; specifically, they decrease it.

---

[9]The lower left-hand panel shows the log-linearized aggregate higher price. Specifically, by denoting the log-linearized optimal reset higher price by p_t^*, we can express the aggregate higher price in period t as $\hat{p}_t = \theta(\hat{p}_{t-1} - \varepsilon_t) + (1 - \theta)p_t^*$.

For comparison, we plot the impulse responses when τ is homogeneous at 0.125 as the line. In this case, equilibrium is characterized by the pure strategy. Although the inflation rate and output exhibit the same qualitative pattern, the magnitude of the change in output is considerably different. Namely, the output under the mixed strategy increases by less than one-tenth.

The real effect of monetary policy decreases for the following three reasons. First, this effect decreases because the degree of dynamic strategic complementarity for setting the higher price Γ^* decreases. Demand becomes more inelastic when there is a possibility of temporary sales than when there is only a pure strategy equilibrium, which decreases the degree of dynamic strategic complementarity. Numerically, we actually obtain $\Gamma^* = -0.09$, which suggests dynamic strategic substitutability, rather than complementarity, although assuming a pure strategy incorrectly leads to $\Gamma^*_{pure} = 1.11$.

Second, instead of decreasing, the degree of dynamic strategic complementarity increases for the interaction of the setting of the higher and lower prices. The lower left-hand panel shows that although the aggregate higher price under the mixed strategy is negative, the extent to which it deviates from the steady state is smaller than the extent to which the aggregate price deviates under the pure strategy. Under the mixed strategy, the lower (i.e., sale) price is revised upward fully in response to the positive money supply shock. Combined with the first type of strategic complementarity effect shown in Corollary 4, this induces firms to increase their higher price more when they can reset it. Furthermore, owing to the second type of strategic complementarity effect shown in Corollary 4, the greater increase in the lower price than in the higher price increases the expected profit from setting the lower price. To maintain profit equality, the frequency of sales needs to decrease. This decrease in the frequency of sales increases the aggregate price. Therefore, nominal prices are adjusted upward more strongly, which weakens the real effect of monetary policy.

Third, sales are assumed to be perfectly flexible, and the frequency of sales is high in our model. For robustness, we set the own demand elasticity when both firms A and B set the price around p_L at -50, which is smaller than $-\mathbb{E}[1/\tau] - 1 = -9$ based on the Hotelling model (see Appendix D.1). Then, the frequency of sales decreases from 60% to 15%, which is comparable to the value based on the data. Note that this change in own demand elasticity affects only the first-order condition with respect to p_L and not the first-order condition with

respect to p_H or the equation for the profit indifference between choosing p_H and p_L. The dashed line in Figure 9 shows that the real effect of monetary policy remains substantially small, although the decrease in the frequency of sales increases the real effect of monetary policy relative to the benchmark case of a mixed strategy when the frequency of sales is 60%.

Recall that we assume a certain constraint that prevents firms from optimizing both the higher price and the frequency of sales simultaneously. Firms optimize the frequency of sales only when they cannot revise their higher price. To investigate the quantitative importance of this assumption, in Figure 9, we also demonstrate simulation results when the frequency of sales is kept constant. Technically, we assume that the equality of profits from choosing the higher price and lower price does not necessarily hold except in the steady state. The figure shows that although the frequency of sales does not decrease in response to the shock, the other three variables (i.e., the inflation rate, output, and higher price) hardly change.

This result is markedly different from that reported by Guimaraes and Sheedy (2011). In their model, the existence of temporary sales hardly changes the real effect of monetary policy. Sales exhibit strategic substitutability; that is, if other firms choose a higher price rather than a lower price more frequently, a firm will be better off if it chooses a lower price rather than a higher price more frequently. This leads to a result wherein the real effect of monetary policy hardly changes despite sales.

4   Concluding Remarks

In this study, we provided a tractable macroeconomic model incorporating duopolistic competition and price stickiness. We found that implications for monetary policy change when we incorporate the strategic pricing behaviors of oligopolistic firms. Firms' pricing entails a dynamic (intertemporal) strategic complementarity. The optimal reset price positively depends on the price set by a rival firm in the previous period. This property increases the real effect of monetary policy slightly relative to that in a standard monopolistic competition model in which a strategic complementarity is absent. However, when a mixed strategy equilibrium arises, the real effect of monetary policy is weakened considerably.

It will be interesting to see how a model extension can help connect recent developments in firm dynamics (e.g., an increase in markup and a decrease in business dynamism) with

those in inflation dynamics (e.g., a decrease in the inflation rate in developed countries and the disappearing Phillips curve).

References

[1] Anderson, Simon P., Andre de Palma, and Jacques-Francois Thisse (1992) "Discrete Choice Theory of Product Differentiation." Cambridge: MIT Press.

[2] Anderson, Eric, Benjamin A. Malin, Emi Nakamura, Duncan Simester, and Jon Steinsson (2017) "Informational Rigidities and the Stickiness of Temporary Sales." *Journal of Monetary Economics*, 90, 64–83.

[3] Angeletos, George-Marios, and Jennifer La'O (2009) "Incomplete Information, Higher-order Beliefs and Price Inertia." *Journal of Monetary Economics*, 56S, 19–37.

[4] Aoki, Kosuke, Hibiki Ichiue, and Tatsushi Okuda (2019) "Consumers' Price Beliefs, Central Bank Communication, and Inflation Dynamics." Bank of Japan Working Paper Series 19-E-14.

[5] Armstrong, Mark (2006) "Recent Developments in the Economics of Price Discrimination." In R. Blundell, W. Newey, and T. Persson (Eds.), *Advances in Economics and Econometrics: Theory and Applications*, Ninth World Congress (Econometric Society Monographs, pp. 97-141). Cambridge: Cambridge University Press.

[6] Ball, Laurence, and David Romer (1990) "Real Rigidities and the Non-Neutrality of Money." *Review of Economic Studies*, 57(2), 183–203.

[7] Bhaskar, Venkataraman (2002) "On Endogenously Staggered Prices." *Review of Economic Studies*, 69, 97–116.

[8] Bils, Mark, and Peter J. Klenow (2004) "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy*, 112, 947–985.

[9] Blanchard, Olivier Jean, and Nobuhiro Kiyotaki (1987) "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review*, 77(4), 647–666.

[10] Chen, Guo, C. Gizem Korpeoglu, and Stephen E. Spear (2017) "Price Stickiness and Markup Variations in Market Games." *Journal of Mathematical Economics*, 72, 95–103.

[11] Christiano, Lawrence, Martin Eichenbaum, and Charles Evans (2005) "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1), 1–45.

[12] Coibion, Olivier, Yuriy Gorodnichenko, and Gee Hee Hong (2015) "The Cyclicality of Sales, Regular and Effective Prices: Business Cycle and Policy Implications." *American Economic Review*, 105, 993–1029.

[13] Dixit, Avinash, and Joseph Stiglitz (1977) "Monopolistic Competition and Optimum Product Diversity." *American Economic Review*, 67(3), 297–308.

[14] Dixit, Avinash, and Joseph Stiglitz (1979) "Monopolistic Competition and Optimum Product Diversity: Reply." *American Economic Review*, 69(5), 961–963.

[15] Faia, Ester (2012) "Oligopolistic Competition and Optimal Monetary Policy." *Journal of Economic Dynamics and Control*, 36(11), 1760–1774.

[16] Fehr, Ernst, and Jean-Robert Tyran (2008) "Limited Rationality and Strategic Interaction: The Impact of the Strategic Environment on Nominal Inertia." *Econometrica*, 76(2), 353–394.

[17] Fershtman, Chaim, and Morton I. Kamien (1987) "Dynamic Duopolistic Competition with Sticky Prices." *Econometrica*, 55(5), 1151–1164.

[18] Gali, Jordi (2015) *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications.* Princeton: Princeton University Press, 2nd edition.

[19] Guimaraes, Bernardo, and Kevin Sheedy (2011) "Sales and Monetary Policy." *American Economic Review*, 101, 844–876.

[20] Hansen, Gary D. (1985) "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*, 16, 309–327.

[21] Hotelling, Harold (1929) "Stability in Competition." *Economic Journal*, 39(153), 41–57.

[22] Jun, Byoung, and Xavier Vives (2004) "Strategic Incentives in Dynamic Duopoly." *Journal of Economic Theory*, 116(2), 249–281.

[23] Kehoe, Patrick J., and Virgiliu Midrigan (2015) "Prices Are Sticky After All." *Journal of Monetary Economics*, 75, 35–53.

[24] Kimball, Miles S. (1995) "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 27(4), 1241–1277.

[25] Kryvtsov, Oleksiy, and Nicolas Vincent (2021) "The Cyclicality of Sales and Aggregate Price Flexibility." *Review of Economic Studies*, 88(1), 334–377.

[26] Levin, Andrew T., Jose David Lopez-Salido, and Tack Yun (2007) "Strategic Complementarities and Optimal Monetary Policy." Kiel Working Papers 1355.

[27] L'Huillier, Jean-Paul (2020) "Consumers Imperfect Information and Price Rigidities." *American Economic Journal: Macroeconomics*, 12(2), 94–123.

[28] Maskin, Erin, and Jean Tirole (1988) "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs." *Econometrica*, 56(3), 549–569.

[29] Midrigan, Virgiliu (2011) "Menu Costs, Multi-product Firms, and Aggregate Fluctuations." *Econometrica*, 79(4), 1139–1180.

[30] Mongey, Simon (2017) "Market Structure and Monetary Non-neutrality." Federal Reserve Bank of Minneapolis Staff Report 558.

[31] Nakamura, Emi, and Jon Steinsson (2008) "Five Facts about Prices: A Reevaluation of Menu Cost Models." *Quarterly Journal of Economics*, 123(4), 1415–1464.

[32] Nakamura, Emi, and Jon Steinsson (2010) "Monetary Non-neutrality in a Multisector Menu Cost Model." *Quarterly Journal of Economics*, 125(3), 961–1013.

[33] Nirei, Makoto, and Jose A. Scheinkman (2020) "Menu Costs and the Volatility of Inflation." Presented at Econometric Society World Congress 2020.

[34] Pettengill, John S. (1979) "Monopolistic Competition and Optimum Product Diversity: Comment." *American Economic Review*, 69(5), 957–960.

[35] Rogerson, Richard (1988) "Indivisble Labor, Lotteries and Equilibrium." *Journal of Monetary Economics*, 21, 3–16.

[36] Slade, Margaret E. (1999) "Sticky Prices in a Dynamic Oligopoly: An Investigation of (s,S) Thresholds." *International Journal of Industrial Organization*, 17(4), 477–511.

[37] Sudo, Nao, Kozo Ueda, and Kota Watanabe (2014) "Micro Price Dynamics during Japan's Lost Decades." *Asian Economic Policy Review*, 9, 46–64.

[38] Sudo, Nao, Kozo Ueda, Kota Watanabe, and Tsutomu Watanabe (2018) "Working Less and Bargain Hunting More: Macroimplications of Sales during Japan's Lost Decades." *Journal of Money, Credit and Banking*, 50(2–3), 449–478.

[39] Wang, Olivier, and Ivan Werning (2020) "Dynamic Oligopoly and Price Stickiness." NBER Working Paper 27536.

[40] Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton: Princeton University Press.

[41] Zbaracki, Mark J., Mark Ritson, Daniel Levy, Shantanu Dutta, and Mark Bergen (2004) "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets." *Review of Economics and Statistics*, 86(2), 514–533.
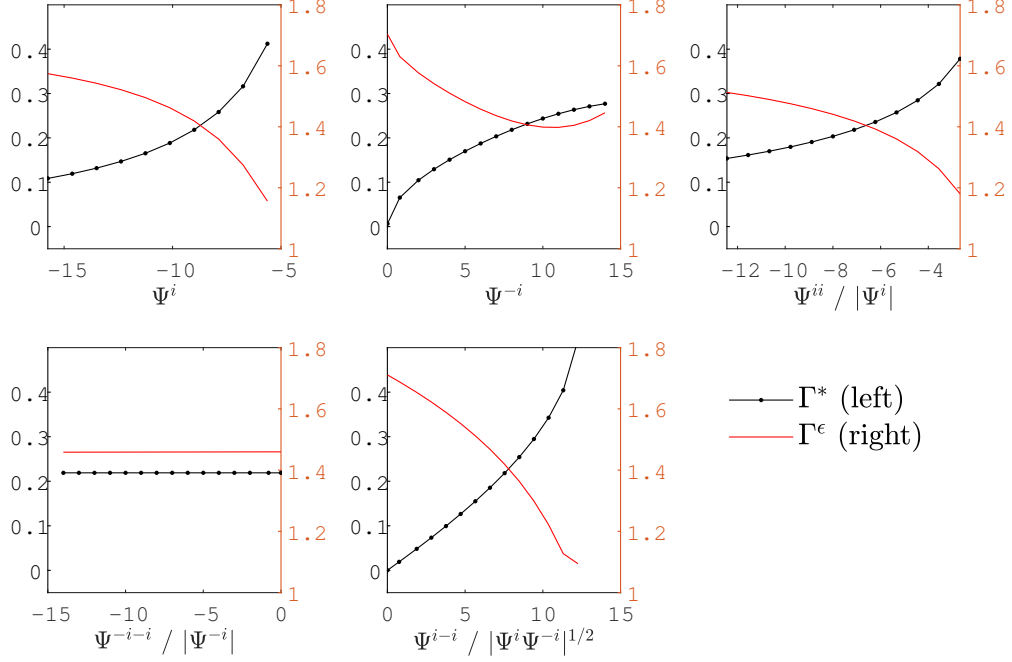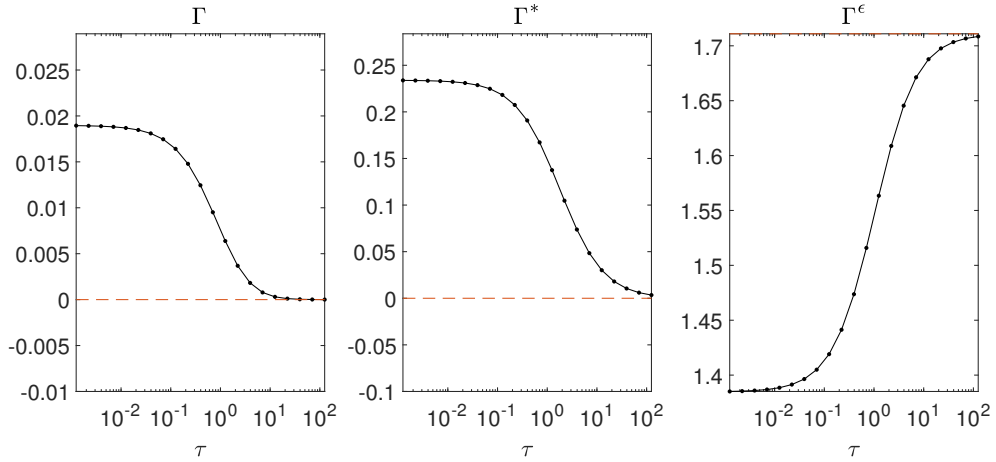
Figure 1: Policy Functions under Price Stickiness: Dependence on Demand Elasticities

Note: The figure shows the coefficients of policy functions for the optimal reset price by firm A (specifically Γ^* and Γ^ε) given by $p_t^{A*} = \Gamma \hat{p}_{t-1}^A + \Gamma^* \hat{p}_{t-1}^B + \Gamma^\varepsilon \varepsilon_t$ when we change one of the five parameter values, $\Psi^i, \Psi^{-i}, \Psi^{i,i}/|\Psi^i|, \Psi^{-i,-i}/|\Psi^{-i}|, \Psi^{i,-i}/\sqrt{/|\Psi^i\Psi^{-i}|}$ while keeping the other four parameter values fixed.

Figure 2: Policy Functions under Price Stickiness: Dependence on Transport Costs

Note: The figure shows the coefficients of policy functions for the optimal reset price by firm A given by $p_t^{A*} = \Gamma \hat{p}_{t-1}^A + \Gamma^* \hat{p}_{t-1}^B + \Gamma^\varepsilon \varepsilon_t$. The horizontal axis represents the transport cost based on the Hotelling model (τ, log scale), which is equivalent to the elasticity of substitution with $\sigma = 1 + 2/\tau$ based on the CES duopolistic competition model.
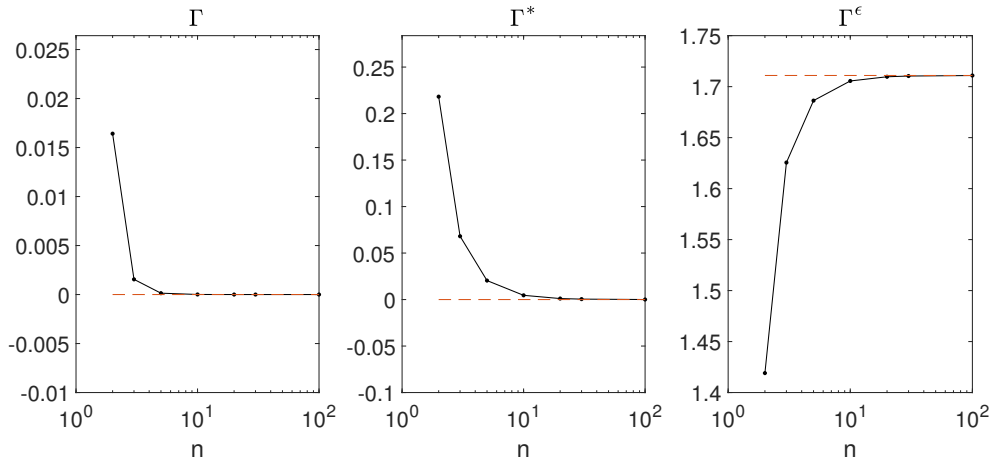
Figure 3: Policy Functions under Price Stickiness: Dependence on the Number of Firms

Note: The figure shows the coefficients of policy functions for the optimal reset price by firm A given by $p_t^{A*} = \Gamma \hat{p}_{t-1}^A + \Gamma^* \hat{p}_{t-1}^B + \Gamma^\varepsilon \varepsilon_t$. The horizontal axis represents the number of firms in an industry (n, log scale) based on the CES oligopolistic competition model.
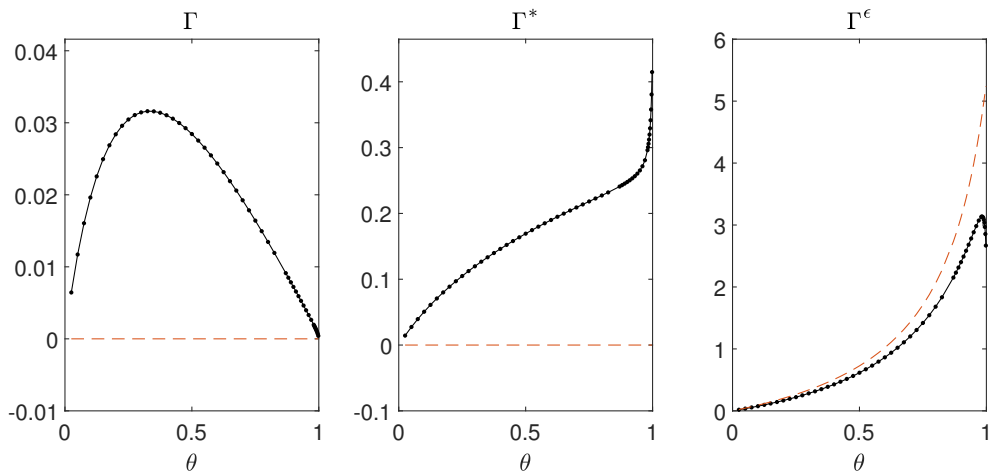

Figure 4: Policy Functions under Price Stickiness: Dependence on Price Stickiness

Note: The figure shows the coefficients of policy functions for the optimal reset price by firm A given by $p_t^{A*} = \Gamma \hat{p}_{t-1}^A + \Gamma^* \hat{p}_{t-1}^B + \Gamma^\varepsilon \varepsilon_t$. The horizontal axis represents price stickiness (θ).
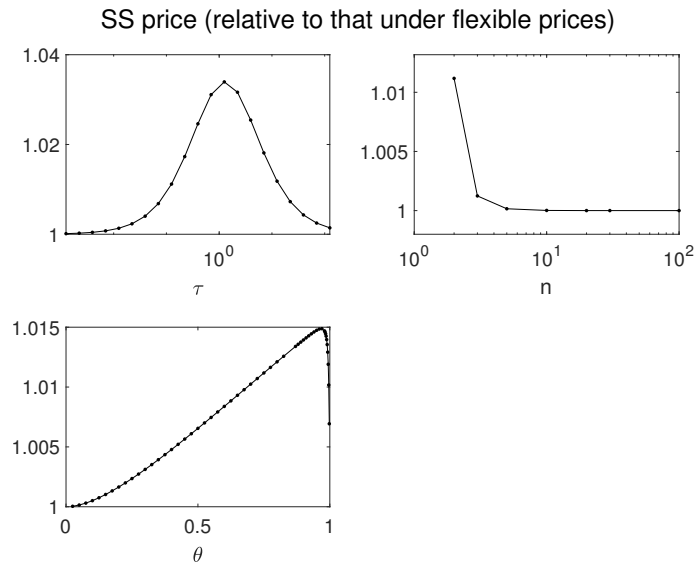
SS price (relative to that under flexible prices)

Figure 5: Steady-State Price under Price Stickiness

Note: The vertical axis represents the ratio of the steady-state (SS) price under sticky prices to that under flexible prices.
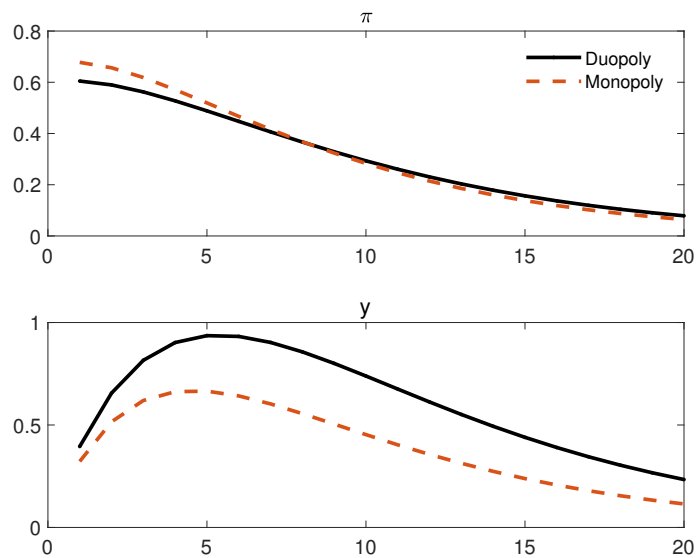

Figure 6: Impulse Responses to Money Supply Shock

Note: The horizontal axis represents quarters after a positive money supply shock occurs at $t = 1$.
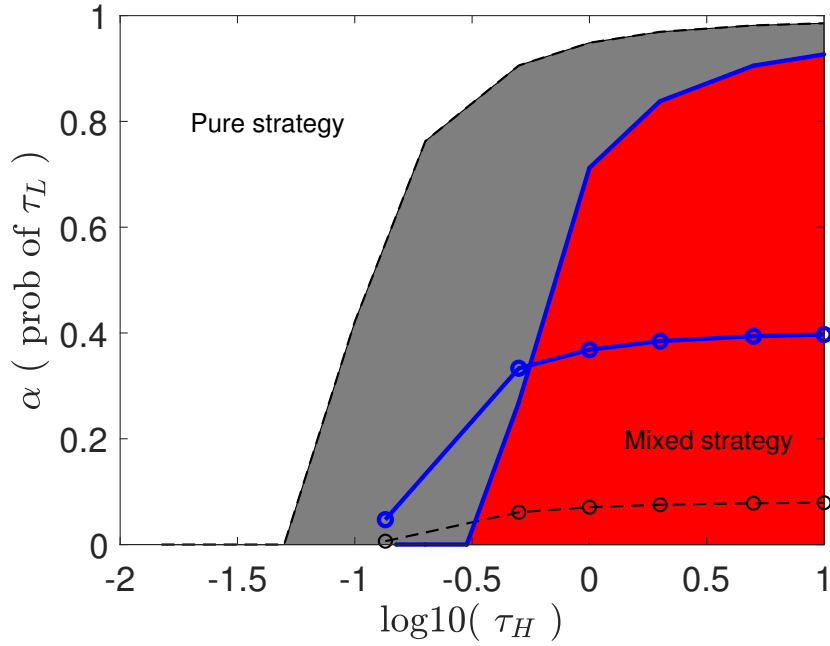
Figure 7: Equilibrium Region under Consumer Heterogeneity and Flexible Prices

Note: The thin dashed line (in black) and the thick solid line (in blue) represent the boundary between the pure and mixed strategies when the parameter τ_L is set at 0.01 and 0.05, respectively. The thick solid line with circles (in blue) and the thin dashed line with circles (in black) indicate the combination of τ_H and α that keeps the harmonic mean of τ at 0.125, respectively.
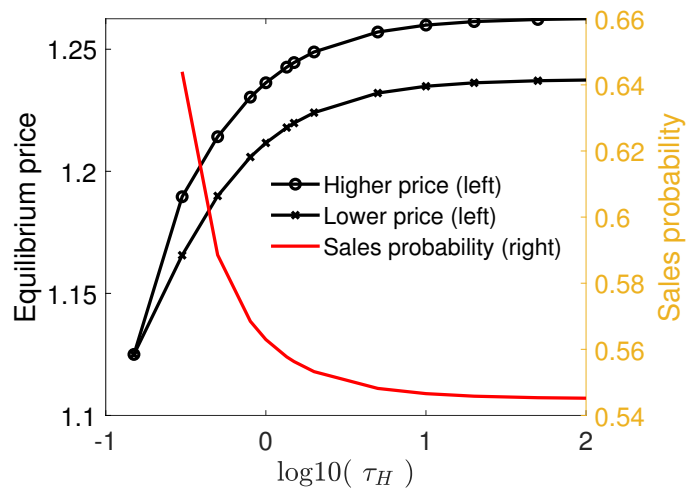

Figure 8: Equilibrium Price under Consumer Heterogeneity and Flexible Prices

Note: The parameter τ_L is set at 0.01, and α is chosen to keep the harmonic mean of τ at 0.125.
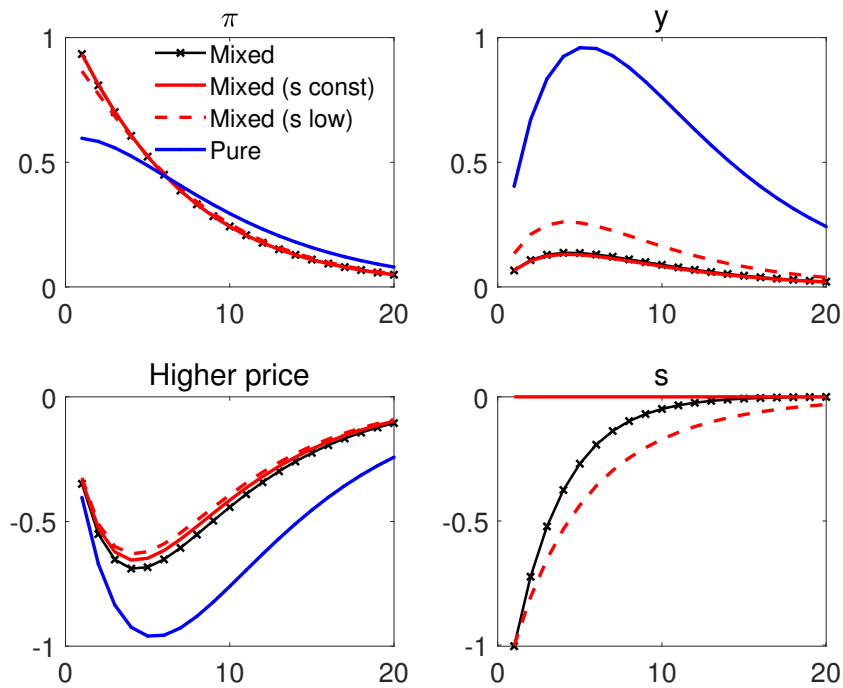
Figure 9: Impulse Responses to Money Supply Shock

Note: For the pure strategy, τ is homogeneous and equals 0.125. For the mixed strategy, τ_L and τ_H equal 0.01 and 1, respectively, whereas parameter α is chosen to make the harmonic mean of τ equal 0.125 (i.e., $\alpha = 0.07$). Mixed (s low) represents the case in which the frequency of sales (s) is around 15%, whereas Mixed (s const) represents the case in which s is kept constant.