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Climate change and monetary policy

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Abstract

Motivated by recent climate actions of central banks and supervisors, this study develops an overlapping generations model of the environment and money and explores a role of monetary policy on climate problems. It is shown that a stationary monetary equilibrium exists uniquely but be suboptimal so that this study explores optimal policies. When a policymaker can control money growth rates only, any monetary policy cannot attain an optimal allocation but a certain positive money growth rate can be the second-best policy. In contrast, when a policymaker can choose tax instruments in addition to money growth rates, there exists a continuum of optimal combinations of money growth rates and tax instruments, which implement an optimal allocation as a stationary monetary equilibrium allocation. These results suggest that, to resolve climate problems, monetary and fiscal authorities need to coordinate with each other.

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Abstract: Motivated by recent climate actions of central banks and supervisors, this study develops an overlapping generations model of the environment and money and explores a role of monetary policy on climate problems. It is shown that a stationary monetary equilibrium exists uniquely but be suboptimal so that this study explores optimal policies. When a policymaker can control money growth rates only, any monetary policy cannot attain an optimal allocation but a certain positive money growth rate can be the second-best policy. In contrast, when a policymaker can choose tax instruments in addition to money growth rates, there exists a continuum of optimal combinations of money growth rates and tax instruments, which implement an optimal allocation as a stationary monetary equilibrium allocation. These results suggest that, to resolve climate problems, monetary and fiscal authorities need to coordinate with each other.

Keywords: Climate change; Monetary policy; Golden rule optimality; Friedman rule; Overlapping generations model.

JEL Classification Number: E40; E50; Q50.

1 Introduction

Since the establishing of the Network of Central Banks and Supervisors for Greening the Financial System (NGFS) at the Paris "One Planet Summit" in December 2017, central banks and supervisors in a lot of countries have directed significant attention towards developing an understanding of the implications of climate change for the economy and monetary policies. In fact, NGFS (2020) reported that all central banks (26 central banks representing 51 countries), which participated its survey, consider climate change to be a challenge its impact on central banks' operational frameworks. As mentioned by Brunetti et al. (2021), however, 'analysis and research is at an early stage.' Therefore, it is worthwhile to develop an economic model to argue on climate change and monetary policy.

Since the environment, including climate change, affects not only a certain generation but also many generations in a long period, this study develops an overlapping generations (OLG) model with the environment and money. The model is constructed by embedding money holdings by agents in a variant of the OLG model of the environment developed by John and Pecchenino (1994). Similar to John and Pecchenino, the environmental quality is modeled by a long-lasting accumulable public good. In the model, money with a Clower constraint circulates as a means to intergenerational trade. This study also introduces the lump-sum money transfers and tax-instruments as possible policies. Then, this study adopts the golden rule optimality as an efficiency criterion of stationary allocations and explores the set of optimal policies.

In the model, this study observes the conflicts between market activities and environmental quality, that is, an equilibrium with valued money is not necessarily golden rule optimal. So, we explore optimal policies. It is shown that, when the policymaker can control money growth rates only, the golden rule optimal allocation cannot be attained. In such a case, we should consider a second-best money growth rate, which maximizes equilibrium welfare. We then find that the second-best money growth rate exists and is positive. This implies suboptimality of the Friedman rule (Friedman, 1969), which is a policy such that the nominal interest rates goes to zero, is shown to be suboptimal. In contrast, when the policymaker can choose tax instruments in addition to controlling money growth rates, it is shown that there exists a continuum of combination of money growth rates and tax instruments implementing a golden rule optimal allocation as a stationary monetary equilibrium allocation. In this case, the Friedman rule with certain tax instruments can be optimal as one of special cases.

As mentioned in the next section, previous studies explored mainly optimal 'tax' policies (John et al., 1995; Ono, 1996; Ono and Maeda, 2002; Jouvet, Michael, and Vidal, 2000; Jouvet, Pestieau, and Ponthiere, 2010; Dao and Dávila, 2014) or market creations (Jouvet, Michael, and Rotillon, 2005) to remedy environmental externalities. Therefore, a role of monetary policies in environmental issues is overlooked in the literature. This study contributes to the literature by developing the OLG model of the environment and money. This study also contributes to the literature by exploring a role of monetary policies in the environmental issues and by finding that a policymaker should choose a suitable combination of monetary policy and tax instruments to achieve optimal allocations.

The remainder of this paper is organized as follows: Section 2 presents some of the relevant results from the existing literature. In Section 3, we present ingredients of the model. In Section 4, we define a stationary monetary equilibrium. Section 5 argues on existence of stationary monetary equilibrium. Section 6 then presents the main results. In Subsection 6.1, we define and characterize golden rule optimality. In Subsection 6.2, we argue the equilibrium efficiency when the policymaker controls money growth rates only. In Subsection 6.3, we explore optimal policies. Section 7 concludes this study with some remarks. Proofs of main propositions are given in the Appendix.

2 Related Literature

Even in economics, there is a lot of studies that tried to resolve conflicts between economic activities and the environment that includes climate change. The environment affects not only a certain generation but also many generations in a long period. To incorporate the effects of the environmental quality on many generations, John and Pecchenino (1994) first develops the OLG model of the environment by embedding the environment quality, which is modeled by a long-lasting accumulable public good, in a variant of the OLG model of capital accumulation (Diamond, 1965). They then explored implications of the rate of substitution between economic activities and the environment resource and its role on environmental sustainability and found that a competitive equilibrium cannot attain Pareto efficiency.

After John and Pecchenino (1994), there is the ever-growing literature considering environmental issues in OLG frameworks. For example, John et al. (1995), Ono (1996), Ono and Maeda (2002) studied about optimal tax-subsidy systems in the framework of John and Pecchenino (1994). Jouvet, Michael, and Vidal (2000) showed that, in the presence of altruism, proportional taxes should be used in order to neutralize the external effects. Jouvet, Michael, and Rotillon (2005) introduced a market for permits to a variant of John and Pecchenino (1994) and showed that all permits should be auctioned. Ono (2007) studied about the effect of the environmental tax reform on economic growth and welfare in the OLG model with endogenous growth, unemployment, and pollution. Prieur (2009) showed that the emergence of the environmental Kuznets curve is no longer the rule when the assimilation capacity of nature is limited and vanishes beyond a critical level of pollution. Jouvet, Pestieau, and Ponthiere (2010) considered an OLG model with endogenous longevity and showed that the decentralization of the social optimum requires a tax not only on capital income but also on health expenditures. Bosi and Desmarchelier (2013) considered an OLG model with endogenous fertility and pollution externalities and showed that a raise in the cost of rearing children increases (decreases) consumption and decreases (increases) pollution under dominant income (substitution) effects. Dao and Dávila (2014) assumed that both consumption and production generates pollution and studied about optimal tax and transfer policies. Ponthiere (2016) characterized the optimal level of pollution when pollution deteriorates survival conditions. Fodha, Seegmuller, and Yamagami (2018) studied about the environmental tax reform in an OLG model with pollution and the government's debt constraint. Constant and Davin (2019) considered an open OLG economy and examined how the underlying costs can spread from a vulnerable to a non-vulnerable country through international trade. Cisco and Gatto (2021) provided several calibration results for a variant of John and Pecchenino (1994).

All of these studies, however, paid no attention to the relation between the environment and monetary policies.[1] Recent climate actions of central banks (see also Dennis, 2022), of course, are not captured at all in these studies. One of exception is the study by Ohtaki (2023), which showed the saddle-point property of monetary steady state in a pure-endowment OLG model with the environment and money. Although Ohtaki (2023) also showed that the optimal monetary policy, which maximizes the equilibrium welfare, can achieve optimal allocation, such a result should be reexamined in a model with intertemporal production technology. In fact, an optimal allocation can be achieved by an appropriate monetary policy only in a pure-endowment model but cannot in our model with production technology. This study contributes to the literature by explore implications of monetary policies on the environment in an OLG model with intertemporal

[1] Bloise, Currarini, and Kikidis (2002) explored optimal inflation rates in a pure-endowment OLG model with accumulated public goods, whereas they paid no attention to environmental problems.

production technology.

3 Ingredients of the Model

An overlapping generations model of the environment and money is considered. Time runs discretely from $t = 1$ to infinity. At each date, two types of commodities are available. One is a perishable commodity, called the consumption good, and the other is an accumulable commodity, called the environmental quality.

The consumption good at each date t is produced by a simple production technology which yields $F(k_t, \ell_t) = \rho k_t + \omega \ell_t$ units of the consumption good at date t when the capital stock and the labor at the date are k_t and ℓ_t, where $\rho \geq 0$ and $\omega > 0$ are marginal productivities of capital and labor, respectively, and the domain of $F(\bullet, \ell_t)$ is assumed to be equal to $[0, \omega]$.[2] We denote by f the per-capita production function, i.e., $f(k) := F(k, 1)$ for each $k \in [0, \omega]$. The depreciation rate is assumed to be zero, i.e., there exists no depreciation of capital stock. On the other hand, similar to John and Pecchenino (1994), the environmental qualities $(E_t)_{t \geq 1}$ follows the law of motion:

$$(1) \quad (\forall t \geq 1) \quad E_{t+1} = (1 - \alpha)E_t - \beta c_t + \gamma z_t,$$

where $\alpha \in (0, 1)$ and $\beta, \gamma > 0$. In the last equation, $z_t \geq 0$ and $c_t \geq 0$ represents the amount of environmental investment, i.e., the maintenance for the environment quality, initiated at date t and the consumption externalities at date t, respectively. The capital stock k_1 and environmental quality E_1 at the initial date are treated as given.

At each date $t \geq 1$, one new agent is born and lives for two periods. The agent is endowed with one unit of the labor, which will be inelastically provided in the market, in the first period of her life and nothing in the second period. However, she ranks pairs of the environmental quality and the consumption good in the second period, $(E_{t+1}, c_{t+1}) \in \Re_+ \times \Re_+$, according to a lifetime utility function $u : \Re_+ \times \Re_+ \to \Re$, where u is strictly monotone increasing, strictly concave, and twice continuously differentiable on the interior of its domain and satisfies that, for each $(E, c) \in \Re_{++} \times \Re_{++}$, $u_1(E, c) > 0$, $u_2(E, c) > 0$, $u_{11}(E, c) < 0$, $u_{22}(E, c) < 0$, $u_{12}(E, c) = u_{21}(E, c) \geq 0$, $\lim_{x \downarrow 0} u_1(x, c) = \infty$, and $\lim_{x \downarrow 0} u_2(E, x) = \infty$.[3] At the initial date, there also exists a one-period-

[2] This type of linear production technologies is called a *storage technology* and is often adopted in the literature. See, for example, Demange and Laroque (1999) and Haslag and Martin (2007).

[3] Two goods are said to be *Edgeworth complements* [*substitutes*] if the marginal utility of one increases [decreases] as the quantity of the other increases. The assumption that $u_{12} = u_{21} \geq 0$ implies therefore that the environment quality and the consumption good are Edgeworth complements.

lives agent, called the *initial old*, whose utility is given by $u^0(E_1, c_1) = E_1 + c_1$. The initial old is endowed with the initial capital stock, k_1.

A *feasible allocation* of this economy is a triplet $(E_t, c_t, k_t)_{t\geq 1}$ of the environmental qualities E_t, the consumption c_t, and the capital stocks k_t with $k_t \in [0, \omega]$ such that there exists a nonnegative sequence of environmental investments, $(z_t)_{t\geq 1}$, satisfying the law of motion of environmental quality (1) and the equation that

$$(2) \quad (\forall t \geq 1) \quad c_t + k_{t+1} - k_t + z_t = f(k_t),$$

which means that the sum of consumption, capital investment, and environmental investment must be equal to the total output at each date. Summing up the last two equations, we can obtain a resource constraint

$$(3) \quad (\forall t \geq 1) \quad \frac{\beta + \gamma}{\gamma} c_t + [k_{t+1} - k_t] + \frac{1}{\gamma}[E_{t+1} - (1 - \alpha)E_t] = f(k_t).$$

Note that $(E_t, c_t, k_t)_{t\geq 1}$ is a feasible allocation if and only if Eq.(3) and the inequalities, $f(k_t) - c_t - [k_{t+1} - k_t] \geq 0$ for each date $t \geq 1$, hold.[4] A feasible allocation is *stationary* if it is independent of dates t. It is often identified with a triplet of nonnegative numbers, (E, c, k), such that

$$(4) \quad \frac{\beta + \gamma}{\gamma} c + \frac{\alpha}{\gamma} E = f(k),$$

provided that $f(k) - c \geq 0$.

4 Definition of Monetary Equilibrium

This section defines a monetary equilibrium. First, we introduce a policymaker who behaves as both a central bank and a government. As a central bank, the policymaker issues an outside asset yielding no dividend, money. The stock of money at each date t is denoted by M_t and satisfies that $M_{t+1} = (1 + \theta)M_t$, where θ is the net rate of growth of money and chosen by the policymaker. The newly issued money at date t, $\Delta_t := M_t - M_{t-1}$, is distributed as lump-sum money transfer to the old agent at the date. As a government, the policymaker has three policy instruments at each date t: (i) a lump-sum subsidy/tax on the young agent's consumption, τ_t^y, (ii) a lump-sum subsidy/tax on the old agent's consumption, τ_t^o, and (iii) a subsidy/tax on the maintenance for the environmental quality at the rate τ_t^E. These tax instruments, τ_t^y, τ_t^o, and

[4]The latter inequalities ensure nonnegativity of the maintenance, z_t.

τ_t^E cannot be set independently of one another. They are related through the policymaker's budget constraint:

$$(5) \quad \tau_t^y + \tau_t^o + \tau_t^E z_t = 0,$$

when the individual environmental investment is z_t. We call $(\theta, \tau^y, \tau^o, \tau^E) = (\theta, (\tau_t^y, \tau_t^o, \tau_t^E)_{t \geq 1})$ a policy. A policy is *stationary* if it is independent of date t.

We then consider agents' economic activities. At each date t, a firm is established by the old agent at that date. Its production technology is given by F defined in the previous section. The firm chooses a per-capita capital stock, k_t, in order to maximize its per-capita profit at date t, $f(k_t) - r_t k_t - w_t$, given by the real interest rate, r_t, and the real wage rate, w_t, at date t, respectively. As usual, r_t and w_t are determined according to $r_t = \rho$ and $w_t = \omega$.

Given the behavior of firms, spot markets of the consumption good and money are held at each date t. We denote by P_t and q_t the nominal price and the real balance of money at date t, respectively. Of course, it must holds that $q_t = M_t/P_t$. Let π_{t+1} be the inflation rate, i.e., $\pi_{t+1} := P_{t+1}/P_t - 1$. Furthermore, define the nominal interest rate by $i_{t+1} := (1 + r_{t+1})(1 + \pi_{t+1}) - 1$.

At each date t, the agent born at that date faces sequential budget constraints in spot markets at dates t and $t+1$. At date t, she allocates her after-tax income, $\omega + \tau_t^y$, to the capital investment k_{t+1}, money holding m_t, and the individual maintenance z_t, i.e.,

$$(6) \quad k_{t+1} + \frac{m_t}{P_t} + (1 - \tau_t^E) z_t = \omega + \tau_t^y.$$

At date $t+1$, she faces the constraint on the environmental quality (1) given E_t and c_t and the budget constraint on the second-period consumption,

$$(7) \quad c_{t+1} = (1 + \rho) k_{t+1} + \frac{m_t + \Delta_{t+1}}{P_{t+1}} + \tau_{t+1}^o.$$

In order to rationalize the holding of money, a Clower constraint formulated by Hahn and Solow (1995) is also imposed:

$$(8) \quad \lambda c_{t+1} \leq \frac{m_t + \Delta_{t+1}}{P_{t+1}},$$

where $\lambda \in [0, 1]$ denotes the proportion of the consumption that must be financed from money holdings.[5] The case that $\lambda = 0$ means that the Clower constraint can be ignored. On the other hand, the case that $\lambda = 1$ means "cash in advance." Except for some specifications, we assume that $0 < \lambda \leq 1$.

[5]See, for example, Eq.(2.2.3) at page 14 in Hahn and Solow (1995).

A *monetary equilibrium given a policy* $(\theta, \tau^y, \tau^o, \tau^E)$ is then defined by $(q^e, E^e, c^e, k^e) = (q_t^e, E_t^e, c_t^e, k_t^e)_{t\geq 1}$ such that there exist a sequence $m^e = (m_t^e)_{t\geq 1}$ and a nonnegative sequence $z^e = (z_t^e)_{t\geq 1}$ satisfying that: for any date $t \geq 1$,

E1. Individual Optimization:
$(E_{t+1}^e, c_{t+1}^e, k_{t+1}^e, m_t^e, z_t^e)$ maximizes $u(E_{t+1}, c_{t+1})$ subject to Eqs.(5), (1), (6), and (7) given c_t^e, E_t^e, and $P_s^e := M_s/q_s^e$ for $s = t, t+1$,

E2. The Policymaker's Budget Constraint:
it holds that $\tau_t^y + \tau_t^o + \tau^E z_t^e = 0$, and

E3. Market Clearing Condition for Money:
$m_t^e = M_t$ hold.

Moreover, it is *stationary* (given a stationary policy) if it is independent of dates t. This is a standard definition of monetary equilibrium: the condition E1 is the utility-maximizing problem with sequential budget constraints, the condition E2 requires that the policymaker's budget constraint consists with the market activities, and the condition E3 stands for the market clearing condition for money.

In a stationary monetary equilibrium, one can easily find that

$$1 + \pi_{t+1} = \frac{P_{t+1}}{P_t} = (1+\theta)\frac{M_t/P_t}{M_{t+1}/P_{t+1}} = (1+\theta)\frac{q_t}{q_{t+1}} = 1 + \theta,$$

where the last equality follows from the fact that $q_t = q_{t+1}$ at each date t in any stationary monetary equilibrium. Then, the nominal interest rate in stationary monetary equilibrium is calculated as

$$i_{t+1} = (1 + r_{t+1})(1 + \pi_{t+1}) - 1 = (1 + \rho)(1 + \theta) - 1 =: i.$$

This implies that the nominal interest rate in (any) stationary monetary equilibrium is determined by the exogenous variables, ρ and θ, and has a one-to-one relation with the money growth rate θ chosen by the policymaker. We can therefore consider that the policymaker chooses i instead of θ. In this stationary environment, the *Friedman rule* is corresponding to the choice that $i = 0$, or equivalently, the money growth rate $\theta^f := -\rho/(1 + \rho) \in]-1, 0]$ with $\theta^f = 0$ if $\rho = 0$. Note that $i = 0$ if and only if $\theta = \theta^f$ and i increases when θ does.

Here, we provide two remarks on monetary equilibrium. The first remark is on the market clearing condition for the consumption good.

Remark 1 At a monetary equilibrium given a policy, the market clearing condition for the consumption good also holds. In order to observe this fact, consider the budget constraints of the young and the old agents at the same date t:

$$k_{t+1} + \frac{m_t}{P_t^e} + (1 - \tau_t^E) z_t = \omega + \tau_t^y$$

and

$$c_t = (1+\rho) k_t + \frac{m_{t-1} + \Delta_t}{P_t^e} + \tau_t^o.$$

With the money market clearing condition $m_s = M_s$ for $s = t-1, t$, these equations imply that

$$c_t + k_{t+1} - k_t + z_t = \omega + \rho k_t + \tau_t^y + \tau_t^o + \tau_t^E z_t = \omega + \rho k_t,$$

where the last equality follows from the policymaker's budget constraint (4). Combining this with Eq.(1), $E_{t+1} = (1-\alpha)E_t - \beta c_t + \gamma z_t$, it holds that

$$\frac{\beta + \gamma}{\gamma} c_t + [k_{t+1} - k_t] + \frac{1}{\gamma} [E_{t+1} - (1-\alpha) E_t] = \omega + \rho k_t,$$

which is the market clearing condition for the consumption good at date t.

The second remark explains a role of the Clower constraint.

Remark 2 Combining Eqs.(6) and (7), we can obtain the young agent's lifetime budget constraint that

$$\frac{1}{1+\rho} c_{t+1} + (1 - \tau_t^E) z_t = \omega + \tau_t^y + \frac{\tau_{t+1}^o}{1+\rho} + \frac{1}{1+\rho} \frac{\Delta_{t+1}}{P_{t+1}} + \left(\frac{1}{1+\rho} \frac{P_t}{P_{t+1}} - 1 \right) \frac{m_t}{P_t}.$$

The component in the brackets of the last term of the previous equation, $(P_t/P_{t+1})/(1+\rho) - 1$, can be rewritten as $[1 - (1+r_{t+1})(1+\pi_{t+1})]/[(1+r_{t+1})(1+\pi_{t+1})] = -i_{t+1}/(1+i_{t+1})$. When it is positive, agents wish to hold money boundlessly, which cannot occur in a monetary equilibrium. Therefore, it should be nonpositive, in any monetary equilibrium. Especially, when it is negative, agents wish to decrease their holdings of money. In such a situation, the Clower constraint (7) plays an important role. It prevents the money holdings from being zero. Furtheremore, if $i := (1+\theta)(1+\rho) - 1 = 0$ in a stationary enviroment, investments in capital and money are completely substitute in the sense that they yields the same rate of return and therefore stationary monetary equilibrium might be indeterminate. On the other hand, when $i > 0$, we can avoid such indeterminacy (See Proposition 1 in the next section). Therefore, we will identify a stationary monetary equilibrium given $i = 0$ as a limit of stationary monetary equilibrium given i as $i \downarrow 0$.

5 Existence of Stationary Monetary Equilibrium

This section explores a stationary monetary equilibrium. First, we consider individual optimization problems. As argued in Remark 2, we assume that $i_{t+1}/(1+i_{t+1}) \geq 0$ at each date. Then, the Clower constraint (8) must hold with equality. Using this fact, the combination of Eqs.(6) and (7) implies the young agent's lifetime budget constraint such that

$$\begin{aligned}
& \omega + \tau_t^y + \frac{\tau_{t+1}^o}{1+\rho} + \frac{\Delta_{t+1}}{P_t} \\
= \ & \frac{c_{t+1}}{1+\rho} + \frac{m_t}{P_t} + \frac{\Delta_{t+1}}{P_t} - \frac{1}{1+\rho}\frac{m_t + \Delta_{t+1}}{P_{t+1}} + (1-\tau_t^E)z_t \\
= \ & \frac{c_{t+1}}{1+\rho} + \frac{P_{t+1}}{P_t}\left(\lambda c_{t+1} - \frac{\Delta_{t+1}}{P_{t+1}}\right) + \frac{\Delta_{t+1}}{P_t} - \frac{\lambda c_{t+1}}{1+\rho} + (1-\tau_t^E)z_t \\
= \ & \frac{1+\lambda(1+i_{t+1}) - \lambda}{1+\rho}c_{t+1} + (1-\tau_t^E)z_t \\
= \ & \frac{1+\lambda i_{t+1}}{1+\rho}c_{t+1} + \frac{1-\tau_t^E}{\gamma}[E_{t+1} - (1-\alpha)E_t + \beta c_t],
\end{aligned}$$

where we use Eq.(1) in the last equality. Then, the first order condition of each agent's optimization problem is given by the above equation and

$$\frac{u_2(E_{t+1}, c_{t+1})}{u_1(E_{t+1}, c_{t+1})} = \frac{1+\lambda i_{t+1}}{1-\tau_t^E}\frac{\gamma}{1+\rho}. \tag{9}$$

We next consider monetary equilibrium. Because the money market clearing condition hold at any monetary equilibrium, it follows that $\lambda c_{t+1} = M_{t+1}/P_{t+1} = q_{t+1}$ from the Clower constraint (8) and that

$$\frac{\Delta_{t+1}}{P_t} = \frac{P_{t+1}}{P_t}\frac{M_{t+1}}{P_{t+1}}\left(1 - \frac{1}{1+\theta}\right) = (1+\pi_{t+1})\frac{\theta}{1+\theta}\lambda c_{t+1}.$$

In addition to the money market clearing condition, the policymaker's budget constraint holds at any monetary equilibrium. Hence, a monetary equilibrium (q, E, c, k) is characterized by Eq.(9),

$$\frac{1+\frac{\lambda}{1+\theta}(i_{t+1}-\theta)}{1+\rho}c_{t+1} + \frac{1}{\gamma}[E_{t+1} - (1-\alpha)E_t + \beta c_t] = \omega - \tau_t^o + \frac{\tau_{t+1}^o}{1+\rho},$$

$q_{t+1} = M_{t+1}/P_{t+1} = \lambda c_{t+1}$, $k_{t+1} = [(1-\lambda)c_{t+1} - \tau_{t+1}^o]/(1+\rho)$, and $i_{t+1} = (1+\rho)(1+\theta)q_t/q_{t+1}$. Therefore, we can say that a *stationary* monetary equilibrium given constant $(\theta, \tau^y, \tau^o, \tau^E)$ is characterized by (q, E, c, k), which is constant over dates, such that

$$\frac{u_2(E, c)}{u_1(E, c)} = \frac{1+\lambda i}{1-\tau^E}\frac{\gamma}{1+\rho}, \tag{10}$$

(11) $$\frac{\beta+\gamma}{\gamma}c+\frac{\alpha}{\gamma}E=\omega+\rho k,$$

(12) $$q=\lambda c, \text{ and}$$

(13) $$k=[(1-\lambda)c-\tau^{o}]/(1+\rho),$$

where $i := (1+\rho)(1+\theta)$.

Given these characterizations, we are now ready to state existence and uniqueness of stationary monetary equilibrium:

Proposition 1 *For any $\theta \geq \theta^f$, any $\tau^o < \omega(1+\rho)/\rho$, and any $\tau^E \in [0,1[$, a stationary monetary equilibrium (q^e, E^e, c^e, k^e) given the stationary policy $(\theta, \tau^y, \tau^o, \tau^E)$, being $c^e, E^e > 0$, exists and is unique, where $\tau^y = -(1-\tau^E)\tau^o - \tau^E(\omega - k^e - q^e)$.*

Because the unique stationary monetary equilibrium given stationary policy exists, we can precisely argue on optimal policies.

6 Optimal Policy

6.1 Golden Rule Optimality

We now turn to discuss the optimal monetary policy. First, we should define and argue the first-best situation. We adopt golden rule optimality as a criterion of optimality. Given Eq.(4), an *interior stationary feasible allocation* is defined by $(E, c, k) \in \Re^3_{++}$ satisfying that

$$\frac{\beta+\gamma}{\gamma}c+\frac{\alpha}{\gamma}E=\omega+\rho k$$

and $k \in [0, \omega]$, where E, c, and k are the environmental quality, the second-period consumption, and the capital stock respectively. Note in the last equation that the per-unit cost, measured by the consumption good, of producing the environmental quality is equal to $(\beta+\gamma)/\alpha$. It is *golden rule optimal* if it satisfies that $U(E, c) \geq U(\tilde{E}, \tilde{c})$ for each feasible stationary allocation $(\tilde{E}, \tilde{c}, \tilde{k})$. Then, a golden rule optimal allocation can be characterized as follows:

Proposition 2 *An interior stationary feasible allocation (E^*, c^*, k^*) is golden rule optimal if and only if it satisfies that $k^* = \omega$,*

(14) $$\frac{u_2(E^*, c^*)}{u_1(E^*, c^*)}=\frac{\beta+\gamma}{\alpha},$$

and

$$(15)\quad \frac{\beta+\gamma}{\gamma}c^* + \frac{\alpha}{\gamma}E^* = (1+\rho)\omega.$$

Furthermore, it exists uniquely.

Note that, similar to John and Pecchenino (1994), the environment quality in our model plays a role as the public good. Therefore, Eq.(14), which means that the marginal rate of substitution is equal to the marginal cost of producing the environmental good, can be then interpreted as an intertemporal variation of the Samuelson condition for the optimal provision of the public good.

6.2 Optimum Quantity of Money

We then consider the case of "pure" monetary policy, in which the policymaker tries to improve welfare by controlling the money growth rate only. Thus, suppose that $\tau^y = \tau^o = \tau^E = 0$ throughout this subsection. In this situation, a stationary monetary equilibrium given θ, (q, E, c, k), is characterized by

$$(16)\quad \frac{u_2(E,c)}{u_1(E,c)} = (1+\lambda i)\frac{\gamma}{1+\rho}$$

and Eqs.(11),(12), and (13). A *first-best* choice of money growth rate for the policymaker is such that a stationary monetary equilibrium given the rate generates a golden rule optimal allocation. Define $z := (\alpha E + \beta c)/\gamma > 0$. Then, Eqs.(11), (12), and (13) imply that

$$\begin{aligned} \omega &= (1-\lambda)c - \rho k + \lambda c + \frac{\alpha E + \beta c}{\gamma} \\ &= k + q + z. \end{aligned}$$

Because q and z are positive in a stationary monetary equilibrium, k must be less than ω. Applying Proposition 2, it follows that any stationary monetary equilibrium cannot attain golden rule optimal allocation, provided that $\rho > 0$. In short, there exists no first-best choice of money growth rate.

The policymaker therefore should find a second-best choice. Because the unique stationary monetary equilibrium given $\theta > 0$ exists as shown in Proposition 1, its consumption and the environment quality can be written as $\bar{c}(\theta)$ and $\bar{E}(\theta)$, respectively. Then, the equilibrium welfare, denoted by $W(\theta)$, can be written in the form as

$$W(\theta) := u(\bar{E}(\theta), \bar{c}(\theta)).$$

This welfare function is well-defined. We call a money growth rate θ^{**} *second-best* if it maximizes equilibrium welfare, $W(\theta)$, on $[\theta^f, \infty[$. The existence of second-best rate is guaranteed by the following proposition.

Proposition 3 *A second-best money growth rate θ^{**} exists and is unique. Furthermore, it is characterized by*

$$\theta^{**} = \frac{(1+\rho)\beta + (1-\alpha)(1+\lambda\rho)\gamma}{\lambda(1+\rho)\alpha\gamma},$$

the value of which is positive.

As a corollary of the last proposition, we can obtain a claim on suboptimality of the Friedman rule:

Corollary 1 *In the case of pure monetary policy, the money growth rate corresponding to the Friedman rule, θ^f, is neither first-best nor second-best.*

This is a remarkable difference from the traditional monetary theory, which claims that the optimality of the Friedman rule. The following proposition might help us to understand the suboptimality of the Friedman rule.

Proposition 4 *For the stationary monetary equilibrium (q, E, c, k) given θ^{**}, it holds that $k < k^*$, $E < E^*$, and $c > c^*$.*

Because $k < k^*$, the equilibrium outcome is dynamically efficient. However, as argued above, any stationary monetary equilibrium cannot attain golden rule optimal allocation, provided that $\rho > 0$. As the sources of inefficiency, in short, we can consider the possibilities that (a) agents invest more in money instead of the environmental quality (so-called a free-rider problem) or (b) the consumption externality is extremely high so as to decrease the level of the environmental quality, since $E < E^*$ and $c > c^*$. In any cases, agents might invest too much in money. On the other hand, the lump-sum money transfer decreases the rate of return of money. Therefore, the growth of money decreases the investment in money and remedies the inefficiency caused by the free-rider problem and the consumption externality.

Remark 3 As the tradition from Friedman (1969), it is well-known that the Friedman rule, a policy such that the nominal interest rate will be zero, will be optimal when there is no market friction. In fact, even in the standard monetary OLG model, the optimal money growth rate follows the Friedman rule (see, e.g., Wallace, 1980, Ch.10). Differently from the standard argument in the monetary OLG model, we have shown that the second-best rate of money growth, which maximizes the welfare at stationary monetary equilibrium, exists and is positive.[6] This is a remarkable difference between ours and previous studies.

Remark 4 In the pure-endowment case, i.e., when $\rho = 0$ and $\lambda = 1$, the policymaker can achieve golden rule optimality by controlling the money growth rate only. In this case, a stationary monetary equilibrium (q, E, c, k) given θ is characterized by $k = 0$, $q = c$,

$$\frac{u_2(E,c)}{u_1(E,c)} = (1+\theta)\gamma, \quad \text{and} \quad \frac{\beta+\gamma}{\gamma}c + \frac{\alpha}{\gamma}E = \omega,$$

where it holds that $\theta = i$. Therefore, the policymaker can attain golden rule optimality by choosing

$$\check{\theta} := \frac{\beta + (1-\alpha)\gamma}{\alpha\gamma} > 0$$

as the money growth rate. Actually, at the stationary monetary equilibrium given $\check{\theta}$, it holds that $u_2(E,c)/u_1(E,c) = (1+\check{\theta})\gamma = (\beta+\gamma)/\alpha$. Then, it follows from Proposition 2 that the equilibrium allocation is optimal. One might find that this observation is an analogue of Proposition 3 of Bloise, Currarini, and Kikidis (2002), which explored optimal inflation rates in a pure-endowment OLG model with accumulated public goods, to a production economy.

6.3 Optimal Policies

We now consider the situation that the policymaker can control tax instruments in addition to the money growth rate. Recall that, in such a situation, a stationary monetary equilibrium (q, E, c, k) given θ is characterized by

$$\frac{u_2(E,c)}{u_1(E,c)} = \frac{1+\lambda i}{1-\tau^E}\frac{\gamma}{1+\rho}, \tag{17}$$

$$\frac{\beta+\gamma}{\gamma}c + \frac{\alpha}{\gamma}E = \omega + \rho k, \tag{18}$$

$$q = \lambda c, \text{ and} \tag{19}$$

$$k = [(1-\lambda)c - \tau^o]/(1+\rho), \tag{20}$$

where $i := (1+\rho)(1+\theta) - 1$. Among these equations, we should note that the money growth rate, θ, and the rate of environmental subsidy, τ^E, affect equilibrium outcomes only through the

[6] Optimality of growth in the money stock is recently reported in the OLG model with spatial frictions (Haslag and Martin, 2007), the monetary OLG model with search (Zhu, 2008), and so on. Differently from the previous results, we show the optimality of money growth in the OLG model with the environment.

condition on the marginal rate of substitution, (10). Especially, any combinations of θ and τ, which realize the same value of $(1+\lambda i)\gamma/(1-\tau^E)(1+\rho)$, yield the same equilibrium outcome (q, E, c, k).

Comparing the above equilibrium characterizations with Proposition 2, we can now find an *optimal policy* $(\theta, \tau^y, \tau^o, \tau^E)$, which implements a golden rule allocation as an equilibrium outcome. In order to verify this fact, define the set

$$A := \left\{ (\theta, \tau^E) \in [\theta^f, \infty[\times [0, 1[\;\middle|\; \frac{1+\lambda i(\theta)}{1-\tau^E} = (1+\rho)\frac{\beta+\gamma}{\alpha\gamma} \right\},$$

where $i(\theta) := (1+\rho)(1+\theta) - 1$. Then, for any $(\theta, \tau^E) \in A$, Eq.(17) can be rewritten as

$$\frac{u_2(E, c)}{u_1(E, c)} = \frac{\beta+\gamma}{\alpha}.$$

Therefore, by setting $k = \omega = k^*$, Eqs.(17) and (18) yield that $E = E^*$ and $c = c^*$ given $(\theta, \tau^E) \in A$. Finally, in order to support (E^*, c^*, k^*), especially k^*, as a stationary equilibrium allocation, set $\tau^o = (1-\lambda)c^* - (1+\rho)\omega =: \tau^{o*}$ and $\tau^y = -(1-\tau^E)\tau^o + \tau^E\lambda c^* = (1-\tau^E)[(1+\rho)\omega - c^*] + \lambda c^* =: \tau^{y*}(\tau^E)$. One can easily verify that, for any $(\theta, \tau^E) \in A$, Eqs.(17)-(20) yield $(E, c, k) = (E^g, c^g, k^g)$ given $(\theta, \tau^{y*}(\tau^E), \tau^{o*}, \tau^E)$. Summarizing these arguments, we can obtain the following proposition.

Proposition 5 *For any policy $(\theta, \tau^y, \tau^o, \tau^E)$ satisfying that*

$$\frac{1+\lambda i(\theta)}{1-\tau^E} = (1+\rho)\frac{\beta+\gamma}{\alpha\gamma},$$

$\tau^y = \tau^{y*}(\tau^E)$, and $\tau^o = \tau^{o*}$, the stationary monetary equilibrium given it yields a golden rule optimal allocation.

As shown in the last proposition, there exists a degree of freedom in setting θ (or i) and τ^E. This is because both of θ and τ^E determine only the relative price between the environmental quality and the consumption good as appeared in Eq.(17). Then, as a corollary of Proposition 5, we can now obtain a claim on optimality of the Friedman rule:

Corollary 2 *The money growth rate corresponding to the Friedman rule, θ^f, can implement a golden rule allocation as a stationary monetary equilibrium allocation given $(\theta, \tau^y, \tau^o, \tau^E)$ if and only if $\tau^y = \tau^{y*}(\tau^E)$, $\tau^o = \tau^{o*}$, and*

$$\tau^E = \frac{(1+\rho)\beta + (1-\alpha+\rho)\gamma}{(1+\rho)(1+\beta)} \in]0, 1].$$

As a summary, we can find that there exists a continuum of combination of the money growth rates and tax instruments (especially, subsidies on environmental investment), which implements the golden rule optimal allocation and the Friedman rule is only one special case.[7] Note however that, in order to implement the golden rule optimal allocation, the policymaker cannot determine money growth rates and tax instruments independently.

7 Concluding Remarks

Motivated by recent climate actions of central banks and supervisors in a lot of countries, this study has explored, in an overlapping generations model of the environment and money, a role of monetary policy to resolve conflicts between the environment and economic activities. It has been shown that, when the policymaker controls money growth rates only, the first-best allocation cannot be attained. In such a case, we have shown that a second-best money growth rate, which maximizes equilibrium welfare, is positive and the Friedman rule is neither first-best nor second best. In contrast, when the policymaker can adopt tax instruments in addition to controlling money growth rates, it has been shown that there exists a continuum of combination of money growth rates and tax instruments implementing the first-best allocation as an equilibrium outcome. This result represents that, to obtain the first-best allocation, money growth rates and tax instruments cannot be determined independently. Therefore, it suggests that monetary and fiscal authorities may be required to coordinate with each other in order to resolve environmental issues.

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Appendix: Proofs of Propositions

Proof of Proposition 1. By Eqs.(11) and (13), we can obtain that

$$\left(\frac{1+\lambda\rho}{1+\rho}+\frac{\beta}{\gamma}\right)c+\frac{\alpha}{\gamma}E=\omega-\frac{\rho}{1+\rho}\tau^o.$$

Thus, we can write

$$E=\frac{\gamma}{\alpha}\left[\omega-\frac{\rho}{1+\rho}\tau^o-\left(\frac{1+\lambda\rho}{1+\rho}+\frac{\beta}{\gamma}\right)c\right]=:\phi(c).$$

[7]This observation is an analogue of Proposition 1(ii) of Gahvari (2007), which explores optimal policy instruments in the OLG model with production.

It is immediate to verify that $\phi'(c) < 0$ and $\phi(\bar{c}) = 0$, where

$$\bar{c} := \frac{\omega - \rho\tau^o/(1+\rho)}{(1+\lambda\rho)/(1+\rho) + \beta/\gamma}.$$

Note that $\bar{c} > 0$ because $\omega > \rho\tau^o/(1+\rho)$. Let $V(c) = u_2(\phi(c), c)/u_1(\phi(c), c)$. It follows then that V is continuously differentiable and

$$V'(c) := \frac{(u_{21} \cdot \phi' + u_{22}) \cdot u_1 - u_2 \cdot (u_{11} \cdot \phi' + u_{12})}{(u_1)^2} < 0.$$

Hence, V is strictly monotone decreasing. Furthermore, it follows from the boundary conditions on u that $\lim_{c \downarrow 0} V(c) = \infty$ and $\lim_{c \uparrow \bar{c}} V(c) = 0$. Therefore, there exists a unique number $c \in (0, \bar{c})$ such that

$$\frac{u_2(\phi(c), c)}{u_1(\phi(c), c)} = V(c) = \frac{1+\lambda i}{1-\tau^E}\frac{\gamma}{1+\rho}.$$

Then, $(q, \phi(c), c, k)$ is a unique stationary monetary equilibrium given a stationary policy $(\sigma, \tau^y, \tau^o, \tau^E)$, where q and k are defined as in Eqs.(12) and (13), respectively. Q.E.D.

Proof of Proposition 2. The characterization of golden rule optimal allocation immediately follows from the definition of the golden rule optimality. Thus, we show its existence and uniqueness. Let $\phi_*(c) := (\gamma/\alpha)[(1+\rho)\omega - (\beta+\gamma)c/\gamma]$ and $V_*(c) = u_2(\phi_*(c), c)/u_1(\phi_*(c), c)$. Then, one can find, as in Proposition 1, that $\phi_*'(c) < 0$, $V_*'(c) < 0$, $\lim_{c\downarrow 0} V_*(c) = \infty$, and $\lim_{c\uparrow \bar{c}^*} V_*(c) = 0$, where $\bar{c}^* := (1+\rho)\omega\gamma/(\beta+\gamma)$. Therefore, it follows from the intermediate value that there exists a unique number c^* such that $V_*(c^*) = (\beta+\gamma)/\gamma$ and $(\beta+\gamma)c^*/\gamma + \alpha E^*/\gamma = (1+\rho)\omega$, where $E^* := \phi_*(c^*)$. Q.E.D.

Proof of Proposition 3. Because the nominal interest rate i and the money growth rate θ has a one-to-one relation, we consider the nominal interest rate i instead of θ in this proof. Even in this case, the stationary monetary equilibrium outcome is obviously well-defined and therefore we denote it by $(\hat{q}(i), \hat{E}(i), \hat{c}(i), \hat{k}(i))$. Then, the equilibrium welfare is denoted by $\hat{W}(i) = u(\hat{E}(i), \hat{c}(i))$.

We first claim that $\hat{c}'(i) < 0$ for each $i \geq 0$. In order to verify it, define the function $\Phi : \Re_{++} \times \Re_+ \to \Re$ by, for all $(c, i) \in \Re_{++} \times \Re_+$,

$$\Phi(c, i) := -(1+\lambda i)\frac{\gamma}{1+\rho}u_1(\phi(c), c) + u_2(\phi(c), c),$$

where ϕ is defined as in the proof of Proposition 1. Note that $\hat{E}(i) = \phi(\hat{c}(i))$ by its definition. It follows immediately that $\Phi(\hat{c}(i), i) = 0$ by Eq.(16). It also follows from the implicit function theorem that $\hat{c}'(i) < 0$ because $\Phi_1(\hat{c}(i), i) = -[(1+\lambda i)\gamma/(1+\rho)][u_{11} \cdot \phi' + u_{12}] + [u_{21} \cdot \phi' + u_{22}] < 0$ and $\Phi_2(\hat{c}(i), i) = -[\lambda\gamma/(1+\rho]u_1 < 0$.

By differentiating $\hat{W}$, we can now obtain that

$$\begin{aligned}
\hat{W}'(i) &= u_1(\phi(\hat{c}(i)), \hat{c}(i))\phi'(\hat{c}(i))\hat{c}'(i) + u_2(\phi(\hat{c}(i), \hat{c}(i))\hat{c}'(i) \\
&= \hat{c}'(i)u_1(\phi(\hat{c}(i)), \hat{c}(i)) \left[-\left(\frac{1+\lambda\rho}{1+\rho} + \frac{\beta}{\gamma} \right) \frac{\gamma}{\alpha} + (1+\lambda i)\frac{\gamma}{1+\rho} \right] \\
\left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} & \quad 0 \quad \textit{iff} \quad i \quad \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \quad \frac{1}{\lambda} \frac{(1-\alpha+\lambda\rho)\gamma + (1+\rho)\beta}{\alpha\gamma} =: i^{**},
\end{aligned}$$

where the second equality follows from the fact that $\Phi(\hat{c}(i), i) = 0$. Therefore, i^{**} is the unique nominal interest rate that maximizes $\hat{W}$. The value of θ^{**}presented in the proposition can be calculated by $\theta^{**} = (1+i^{**})/(1+\rho) - 1$. Q.E.D.

Proof of Proposition 4. The fact that $k < k^*$ is already shown. Thus, we should show that $E < E^*$ and $c > c^*$. First, suppose that $E \geq E^*$. Then, it holds that $c < c^*$ because

$$\frac{\beta+\gamma}{\gamma}c + \frac{\alpha}{\gamma}E = \omega + \rho k < \omega = \frac{\beta+\gamma}{\gamma}c^* + \frac{\alpha}{\gamma}E^*$$

Hence, it follows that

$$u_1(E^*, c^*) \geq u_1(E^*, c) \geq u_1(E, c) \quad \text{and} \quad u_2(E^*, c^*) \leq u_2(E, c^*) \leq u_2(E, c),$$

which imply that

$$\frac{u_2(E^*, c^*)}{u_1(E^*, c^*)} \leq \frac{u_2(E, c)}{u_1(E, c)}.$$

However, this contradicts the fact that

$$\frac{u_2(E^*, c^*)}{u_1(E^*, c^*)} = \frac{\beta+\gamma}{\alpha} > \frac{\gamma}{\alpha}\left(\frac{1+\lambda\rho}{1+\rho} + \frac{\beta}{\gamma}\right) = \frac{u_2(E, c)}{u_1(E, c)}.$$

Therefore, $E < E^*$.

The fact that $c > c^*$ follows from an easy calculation that

$$\begin{aligned}
0 &\leq \frac{\alpha}{\gamma}(E^* - E) \\
&= \frac{\beta+\gamma}{\gamma}c - \rho k - \frac{\beta+\gamma}{\gamma}c^* \\
&< \frac{\beta+\gamma}{\gamma}(c - c^*),
\end{aligned}$$

where k is positive as shown before. Q.E.D.

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