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Climate change, financial intermediation, and monetary policy

Eisei Ohtaki

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1-7-10-703 Iidabashi, Chiyoda-ku, Tokyo 102-0072, Japan

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Abstract

Motivated by recent climate actions of central banks and supervisors, this study aims to explore implications of climate change in an economy with financial intermediaries. For this aim, this study develops an overlapping generations model of the environment and financial intermediation. In that model, reactions of financial intermediaries, the monetary steady state, and optimal monetary policy against climate change are studied. Especially, it is demonstrated that the level of the optimal money growth rate depends on how “green” agents are.

Eisei Ohtaki
TCER
and
Kanagawa University
Department of Economics
Rokkakubashi 3-27-1, Kanagawa-ku, Yokohama-shi, Kanagawa 221-8686, Japan
ohtaki@kanagawa-u.ac.jp

Climate change, financial intermediation, and monetary policy

Eisei Ohtaki[†]

[†] Faculty of Economics, Kanagawa university, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama-shi, Kanagawa 221-8686, Japan
email address: ohtaki@kanagawa-u.ac.jp

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Abstract: Motivated by recent climate actions of central banks and supervisors, this study aims to explore implications of climate change in an economy with financial intermediaries. For this aim, this study develops an overlapping generations model of the environment and financial intermediation. In that model, reactions of financial intermediaries, the monetary steady state, and optimal monetary policy against climate change are studied. Especially, it is demonstrated that the level of the optimal money growth rate depends on how "green" agents are.

Keywords: Environment; Climate change; Financial intermediation; Monetary policy; Golden rule optimality; Overlapping generations model.

JEL Classification Number: E40; E50; Q50; M14.

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1 Introduction

Since the establishing of the Network of Central Banks and Supervisors for “green”ing the Financial System (NGFS) at the Paris “One Planet Summit” in December 2017, central banks and supervisors in a lot of countries have directed significant attention towards developing an understanding of the implications of climate change for the economy and monetary policies. In fact, NGFS (2020) reported that all central banks (26 central banks representing 51 countries), which participated its survey, consider climate change to be a challenge its impact on central banks’ operational frameworks. Now, central banks and supervisors in a lot of countries have directed significant attention towards developing an understanding of the implications of climate change for the financial sector and financial stability. As mentioned by Brunetti et al. (2021), however, ‘analysis and research is at an early stage.’ Therefore, it is worthwhile to develop an economic model to argue on effects of climate change on financial intermediation and monetary policies.

To study relationships among climate change, financial intermediation, and the monetary policy, this study considers the economic structure as described in Figure 1. There are three types of economic agents, that is, agents (consumers), financial intermediaries, and the central bank. Agents gain utility from consumption and the quality of the environment, but face the possibilities of liquidity shortage. Financial intermediaries care agents’ welfare and offer agents liquidity insurance, which involves the environmental maintenance. The central bank then conducts the monetary policy to maximize the economic welfare. Climate changes in this study are captured as changes in the speed of reversion of the environment, the degree of negative externalities by consumptions, and the degree of the environmental maintenance.

More formally, this study develops an overlapping generations (OLG) model with the environment and financial intermediation because the environment, including climate change, affects not only a certain generation but also many generations in a long period. The model is constructed by embedding the environment described in John and Pecchenino (1994) in the OLG model with the random relocation mechanism considered in Haslag and Martin (2007). Similar to John and Pecchenino, the environmental quality is modeled by a long-lasting accumulable public good. Similar to Haslag and Martin, money has a transactions role due to the limited communication across separate locations, and financial intermediaries can offer agents liquidity insurance using money. This study adopts the golden rule optimality as an efficiency criterion of stationary allocations and explores the optimal monetary policy.

This study examines how financial intermediaries and markets react against marginal changes in climate parameters (Propositions 1 and 3). It is conjectured that climate change that spoils the environmental quality could increase financial instability (Remarks 3 and 5). Furthermore, it is observed that the model exhibits the conflicts between market activities and the environmental quality, that is, an equilibrium with valued money cannot achieve a golden rule optimal allocation (Corollary 1). Therefore, there is a room for the central bank to explore some optimal policies. This study then examine the level of the optimal money growth rate, that maximizes the equilibrium welfare, and how the level react against marginal changes in climate parameters (Propositions 5 and 6). It is shown that the Friedman rule (Friedman, 1969) is suboptimal. It is also demonstrated that whether the level of the optimal money growth rate becomes greater than one depends on how “green” agents are.

As mentioned in the next section, previous studies explored mainly optimal ‘tax’ policies (John et al., 1995; Ono, 1996; Ono and Maeda, 2002; Jouvet, Michael, and Vidal, 2000; Jouvet, Pestieau, and Ponthiere, 2010; Dao and Dávila, 2014) or market creations (Jouvet, Michael, and Rotillon, 2005) to remedy environmental externalities. Therefore, a role of monetary policies in environmental issues is often overlooked in the literature. This study contributes to the literature by developing the OLG model of the environment and financial intermediation. This study also contributes to the literature by exploring a role of monetary policies in the environmental issues and by characterizing the optimal monetary policy.

The remainder of this paper is organized as follows: Section 2 presents some of the relevant results from the existing literature. Section 3 provides the model in this study. In Section 4, we examines the behavior of the financial intermediary against climate change. Section 5 shows existence and uniqueness of monetary steady state and provides comparative statics on the monetary steady state. Section 6 studies monetary policies. In Subsection 6.1, we define and characterize golden rule optimality. In Subsection

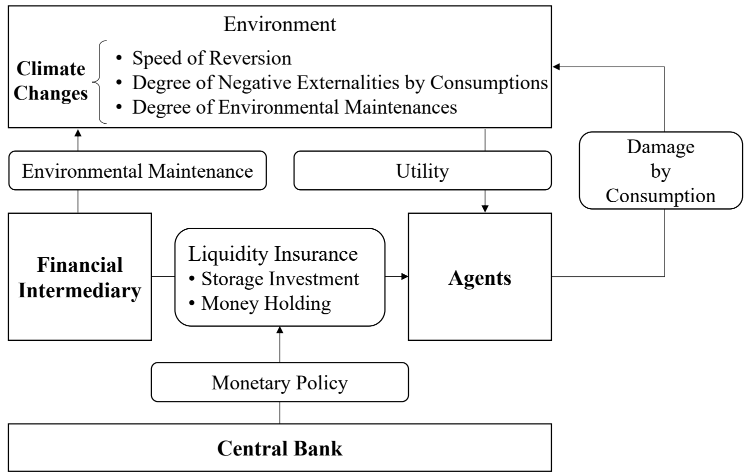

Figure 1: The Structure of the Economy

6.2, we argue the equilibrium efficiency when the policymaker controls money growth rates only. In Subsection 6.3, we explore optimal policies. Section 7 concludes this study with some remarks. Proofs of main propositions are given in the Appendix.

2 Related Literature

There is the ever-growing literature on studies that tried to resolve conflicts between economic activities and the environment that includes climate change. This section presents some of the relevant results from the existing literature.

2.1 On OLG models of the Environment

The environment affects not only a certain generation but also many generations in a long period. To incorporate the effects of the environmental quality on many generations, it is reasonable to adopt the OLG framework. John and Pecchenino (1994) first develops the OLG model of the environment by embedding the environment quality, which is modeled by a long-lasting accumulable public good, in a variant of the OLG model of capital accumulation (Diamond, 1965). They then studied environmental sustainability and found that a competitive equilibrium cannot attain Pareto efficiency.

After John and Pecchenino (1994), the literature paid attention mainly to the role of tax instruments. For example, John et al. (1995), Ono (1996), Ono and Maeda (2002) studied about optimal tax-subsidy systems in the framework of John and Pecchenino (1994). Jouvet, Michael, and Vidal (2000) showed that, in the presence of altruism, proportional taxes should be used in order to neutralize the external effects. Ono (2007) studied about the effect of the environmental tax reform on economic growth and welfare in the OLG model with endogenous growth, unemployment, and pollution. Jouvet, Pestieau, and Ponthiere (2010) considered an OLG model with endogenous longevity and showed that the decentralization of the social optimum requires a tax not only on capital income but also on health expenditures. Dao and Dávila (2014) assumed that both consumption and production generates pollution and studied about optimal tax and transfer policies. Fodha, Seegmuller, and Yamagami (2018) studied about the environmental tax reform in an OLG model with pollution and the government's debt constraint.

There are also other branches of the literature started with John and Pecchenino (1994). Jouvet, Michael, and Rotillon (2005) introduced a market for permits to a variant of John and Pecchenino (1994) and showed that all permits should be auctioned. Prieur (2009) showed that the emergence of the environmental Kuznets curve is no longer the rule when the assimilation capacity of nature is limited and vanishes beyond a critical level of pollution. Bosi and Desmarchelier (2013) considered an OLG model with endogenous fertility and pollution externalities and showed that a raise in the cost of

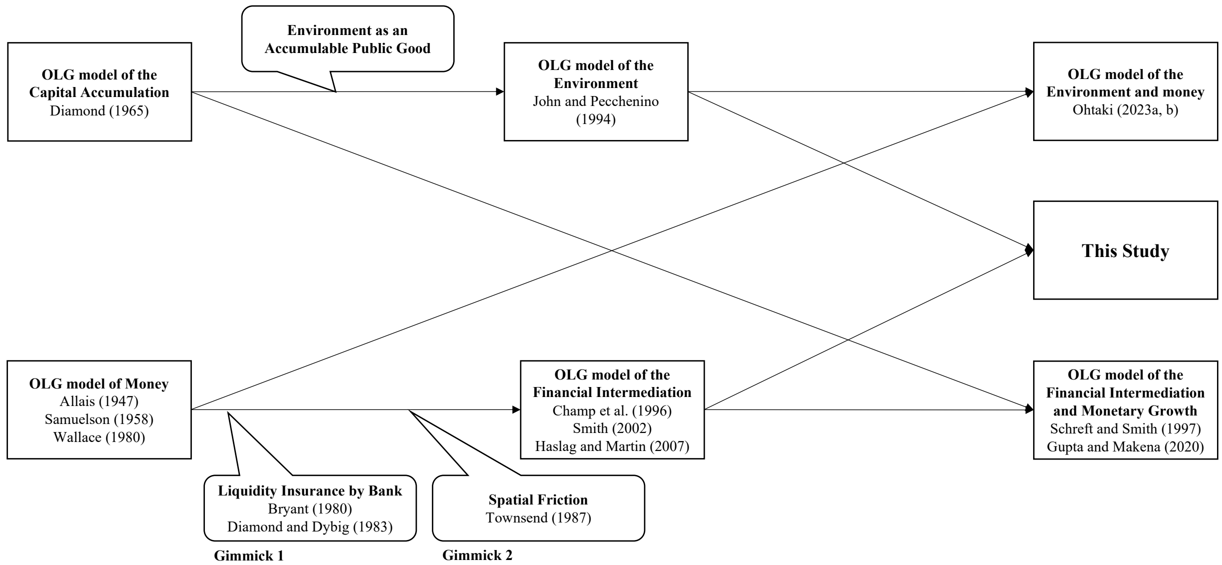

Figure 2: The Literature Tree

rearing children increases (decreases) consumption and decreases (increases) pollution under dominant income (substitution) effects. Ponthiere (2016) characterized the optimal level of pollution when pollution deteriorates survival conditions. Constant and Davin (2019) considered an open OLG economy and examined how the underlying costs can spread from a vulnerable to a non-vulnerable country through international trade. Cisco and Gatto (2021) provided several calibration results for a variant of John and Pecchenino (1994). All of these studies, however, paid no attention to the relation between the environment and monetary policies.[1] Recent climate actions of central banks (see also Dennis (2022)), of course, are not captured at all in these studies.

In contrast, in a pure-endowment monetary OLG model with the environmental quality modeled by John and Pecchenino (1994), Ohtaki (2023a) showed the saddle-point property of the monetary steady state and characterized the optimal money growth rate, which is greater than one. Furthermore, Ohtaki (2023b) characterized the set of optimal policy mix in a monetary OLG model with the environmental quality and a simple intertemporal production technology. However, these two studies paid no attention to financial intermediation. This study belongs to the literature on OLG models of the environment because the modeling of the environmental quality is owed John and Pecchenino (1994). Furthermore, this study contributes to this literature by exploring implications of climate change on monetary policies in an OLG model with financial intermediation.

2.2 On OLG models of financial intermediation

Following Bryant (1980) and Diamond and Dybvig (1983), a body of literature that investigates the consequences of liquidity insurance with financial intermediaries has been persistently expanding. An important stream in such literature was provided by Champ, Smith, and Williamson (1996) and Smith (2002), who incorporated banks as described by Diamond and Dybvig (1983) into a monetary OLG model with spatial separation and showed the suboptimality of the Friedman rule (Friedman, 1969).[2] In their model, there are two islands between which there is no communication (spatial friction).[3] Furthermore, liquidity shocks are modeled by random relocation of agents. Then, a bank that is established by a

[1] Bloise, Currarini, and Kikidis (2002) explored optimal inflation rates in a pure-endowment OLG model with accumulated public goods, whereas they paid no attention to environmental problems.

[2] The OLG model of money was established by Allais (1947) and Samuelson (1958) and sophisticated by Wallace (1980).

[3] The first emphasizing spatial friction is Townsend (1987).

coalition of agents provides deposit contracts, which play the role of liquidity insurance, to agents and relocated agents will withdraw their deposits before they move to the other island.

In this body of the literature, it has been mainly concerned with whether the Friedman rule is optimal. Antinolfi and Keister (2006), Bhattacharya, Haslag, and Russell (2005), Bhattacharya, Haslag, and Martin (2006), Haslag and Martin (2007), Bhattacharya, Haslag, and Martin (2009), and Matsuoka (2011) are such examples. As mentioned in Matsuoka (2011), the constant money supply becomes optimal in a basic OLG model with random relocation and competitive financial intermediaries.

There also exist other branches of the literature. Schreft and Smith (1997) and Gupta and Makena (2020) constructed a monetary growth model with random relocation mechanism. Schreft and Smith (2002) studied the consequences for a central bank of a declining stock of government debt. Bhattacharya and Singh (2008) showed that, money growth targeting realizes higher welfare under nominal shocks and inflation targeting realizes higher welfare under small nominal shocks. Matsuoka (2012) explored the implications of a lender of last resort. Ohtaki (2014) considered more-than-two locations to introduce asymmetric liquidity shocks and showed that there is no room for monetary policy to improve social welfare when the number of locations is extremely large. However, the behavior of financial intermediaries and the central bank against climate change has not been studied yet.

This study also lies on this stream of the literature by using random relocation mechanism to describe valued money and financial intermediation. This study contributes to this body of the literature by exploring implications of climate change on the behavior of financial intermediaries, the monetary steady state, and the optimal monetary policy.

2.3 Recent Quantitative Studies

As mentioned in NGFS (2018), it is broadly agreed that climate change will impact economic conditions through two main channels, *physical risks* and *transition risks*. Physical risks are associated with climate change itself. They are also categorized as *chronic risks* such as increasing temperatures and *acute risks* such as storms, floods or droughts. Transition risks result from changes in climate policy, technology, and consumer and market sentiment during the adjustment to a lower-carbon economy. It is reasonable to construct the economic model of policy design against these risks. This study lies in this line by capturing these risks in an abstracted form. For example, the environment as the long-lasting good models chronic risks and changes in parameters of the environment quality might capture acute risks. The fact that agents gain from the environment in this study might also models transition risks.

More quantitative studies including climate risks as above has been also provided. Dafermos et al. (2018) calibrated a *stock-flow-fund ecological macroeconomic model* using global data and simulations are conducted for the period 2016-2120. Their analysis, however, had no micro-foundation. In contrast, McKibbin et al. (2020) surveyed quantitative studies on macroeconomic models of climate policies and summarized the nature of the macroeconomic modeling framework that is needed to better quantitative analyse climate and monetary policy interactions. Chen et al. (2021) builded an environmental dynamic stochastic general equilibrium (E-DSGE) model and studied the policy mix of monetary and climate policies. Annicchiarico and Dio (2017), Economides and Xepapadeas (2018), and Kantur and Ozkan (2022) provided quantitative studies in new Keynesian frameworks.[4] It is not so easy to directly compare results of these quantitative studies with ours based on a pure theoretical model. However, there are two major differences between these studies and this study. One is that these quantitative studies have not yet considered financial intermediation. The other is whether a damage function as in the dynamic integrated climate-economy (DICE) model is adopted. Adopting a damage function is convenient for quantitative analysis but neglects the effects of environmental maintenances on the damage, which is considered in this study.

Both Diluiso et al. (2021) and Ferrari and Landi (2023) studies implications of a "Green Quantitative Easing" (Green QE) in New Keynesian frameworks. Their model includes banks based on Gertler and

[4]Kantur and Ozkan (2022) introduced *model uncertainty*, which is one of modelings of *Knightian uncertainty* in a new Keynesian model. There is the ever-growing literature of applications of Knightian uncertainty. Even in the OLG model, one can find Fukuda (2008), Wigniolle (2014), Ohtaki and Ozaki (2015), Ohtaki (2016), Ohtaki (2021b), and Ohtaki (2023c) as examples of applications of Knightian uncertainty.

Karadi (2011), which assumed an ad-hoc borrowing constraint. In contrast, financial intermediaries in our model arise with a role to provide liquidity insurances as described by Bryant (1980) and Diamond and Dybvig (1983).

3 The Model

This section develops an OLG model of the environment and financial intermediation.

3.1 Ingredients

Time runs discretely from $t = 1$ to infinity. The economy consists of two spatially separated but symmetric locations. There are two types of goods in each period. One is a perishable good, called the *consumption good*. The other is an accumulable public good, called the *environmental quality*.

The economy has two types of intertemporal production technologies. One is the *storage technology*. Each units of the consumption good put into storage in period t yields $R > 1$ units of the consumption good in period $t+1$, where R is a known constant. The other is the *environmental maintenance* (or the environmental improvement). The environmental quality, E_t, is an accumarable public good that evolves according to

$$(1) \quad E_{t+1} = (1-\theta)E_t - \gamma C_t + \zeta z_t,$$

where $\theta \in [0, 1]$ measures the speed of reversion of the environmental quality, $\gamma > 0$ measures the degree of the negative externality of the total consumption C_t of the old in period t,[5] and ζ measures the degree of the environmental maintenance, z_t, in period t.[6] Note that an increase in θ makes the speed of reversion of the environment quality slower.

Each location is populated by a continuum of agents of unit mass. Agents live for two periods. Agents born in period t gain utility $u(E_{t+1}) + v(c_{t+1})$ from pairs $(E_{t+1}, c_{t+1}) \in \Re^2_+$, where E_{t+1} and c_{t+1} are the level of the environmental quality and the quantity of the consumption good enjoyed in old age, respectively. Assume that u and v are twice continuously differentiable and that, on the interiors of their domains, $u'(E) > 0$, $u''(E) < 0$, $v'(c) > 0$, $v''(c) < 0$. Also assume that $\lim_{E\downarrow 0} u'(E) = \infty$ and $\lim_{c\downarrow 0} v'(c) = \infty$. Agents born in period t receive ω units of the consumption good when young and nothing when old.

Agents born in period t are ex-ante identical but, at the end of period t, learn their types, α or β. Type α agents in one location move to the other location in the beginning of the second period of their lives, whereas type β agents stay the same location. Let π_α and π_β be the probability that an agent will be type α and β, respectively. It is assumed that π_α, $\pi_\beta \geq 0$ and $\pi_\alpha + \pi_\beta = 1$. It is also assumed that a low of large numbers holds so that π_α is also the measure of agents that are relocated.

3.2 Money and the Central Bank

To creates a transactions role for money as in Townsend (1987) and Champ, Smith, and Williamson (1996), this study emphasizes the limited communication across separate locations. In particular, agents can trade and communicate only with other agents at the same location. Therefore, type α agents must hold money as this is the only way for them to acquire goods in the new location.

The central bank can affect the money supply in the economy through lump-sum money transfers. Let M_t be the period t money stock. M_t is assume to grow at a constant gross rate σ such that $M_{t+1} = \sigma M_t$. To guarantee the nonnegativity of the net nominal interest rate, it is assumed that $\sigma \geq R^{-1}$.[7] The period t lump-sum subsidies/taxes τ_t are imposed on young agents according to

$$\tau_t = p_t(M_t - M_{t-1}) = p_t M_t \left(1 - \frac{1}{\sigma}\right),$$

[5] C_t will be equal to $c^\alpha_t \pi_\alpha + c^\beta_t \pi_\beta$ at an equilibrium, where α, β, c^α, c^β, π_α, and π_β are introduced later.

[6] Distinguish locations by indexing $\ell = A, B$. One might consider that the law of motion of the environmental quality should be replaced with the form that $E_{t+1} = (1-\theta)E_t - \gamma\bar{C}_t + \zeta Z_t$, where $\bar{C}_t = \sum_{\ell=A,B}(c^{\ell,\alpha}_t \pi_\alpha + c^{\ell,\beta}_t \pi_\beta)$ and $Z_t = z^A_t + z^B_t$. Due to the symmetry of the two locations in the current settings, however, such replacement have little effect on main results.

[7] To be more precise, we assume that $\sigma > R^{-1}$ and that the condition $\sigma = R^{-1}$ represents the limiting case as $\sigma \to R^{-1}$. See also Haslag and Martin (2007, p.1744).

where p_t is the real price of money in period t.

Remark 1 One can find that the present model has intersections with several previous studies. When $\pi_\beta = 1$ (no relocation) and $M_t \equiv 0$ (no money), the present model becomes a variant of John and Pecchenino (1994), whereas John and Pecchenino considered a neoclassical production function, not a storage technology, as an intertemporal production technology. When $\pi_\beta = 1$ (no relocation), $\theta = \zeta = 1$, $\gamma = 0$ (no environment), the present model becomes a variant of Wallace (1980) with the reinterpretation that z_t is the consumption when young.[8] When $\pi_\beta = 1$ (complete relocation), $R = 0$ (no storage technology), the present model agrees with Ohtaki (2023a).[9] When $\theta = 1$, $\gamma = \zeta = 0$ (no environment and no consumption when young), the present model agrees with Haslag and Martin (2007).[10]

4 Behavior of Financial Intermediaries

The previous section build up the fundamental framework of the model. This section introduces financial intermediaries to the framework and examines the effects of climate change on the behavior of financial intermediaries.

4.1 Decision of Financial Intermediary

Agents born in period t at some location deposit their entire after-tax endowments, $\omega + \tau_t$, with a financial intermediary at the same location, who behaves competitively and takes as given real money prices p_t and p_{t+1} in periods t and $t+1$, respectively. The financial intermediary divides its deposits among money acquired from old agents, m_t, investment in a storage technology, s_t, and investment in the environmental maintenance, z_t. This can be represented by the *balance sheet constraint*

(2) $\quad p_t m_t + s_t + z_t \leq \omega + \tau_t.$

The financial intermediary must have sufficient liquidity to meet the needs of type α agents. This is captured by the *liquidity constraint*

(3) $\quad c^\alpha_{t+1}\pi_\alpha \leq p_{t+1}m_t,$

where c^α_{t+1} is the consumption of each of type α agents. A similar condition for type β agents, who consume all the proceeds from the storage technology, is given by

(4) $\quad c^\beta_{t+1}\pi_\beta \leq Rs_t,$

where c^β_{t+1} is the consumption of each of type β agents.

Remark 2 The environmental maintenance, z_t, by the financial intermediary might be interpreted as the investment in firms enthusiastic on environmental activities.[11] There is also another interpretation. Combining the constraint (3) and the constraint (4), we can obtain the participation constraint for the financial intermediary,

$$(p_{t+1}m_t + Rs_t) - (c^\alpha_{t+1}\pi_\alpha + c^\beta_{t+1}\pi_\beta) \geq 0,$$

where the left-hand and the right-hand sides of the inequality are the profit and the reservation profit of the financial intermediary, respectively. Because the environmental maintenance modeled in this study does not yield the profit of the financial intermediary, it can be interpreted as one of nonprofitable activities based on *corporate social responsibility* (CSR).

[8] In such a Wallacian OLG model, a monetary steady state exists if and only if $\sigma^{-1} \geq R$, which is the case implicitly excluded in this study. See also the footnote 7.

[9] The assumption of the complete relocation, $\pi_\beta = 1$, is imposed to guarantee that agents invest their entire after-tax endowments in money.

[10] When $\theta = 1$, $\gamma = 0$ (no environment) but $\zeta = 1$, the present model becomes an extension of Haslag and Martin (2007) by introducing the consumption when young, z_t.

[11] The environmental maintenance can also be interpreted as an 'abstracted' "green" finance.

The financial intermediary is then assumed to maximize the welfare function

$$u(E_{t+1}) + u(c^{\alpha}_{t+1})\pi_{\alpha} + u(c^{\beta}_{t+1})\pi_{\beta}$$

subject to constraints (2), (1), (3), and (4) given E_t, C_t, p_t and p_{t+1}.[12]

Because of monotonicity and concavity of u and v, the pair of amounts of money and the storage investment, that solves the optimization problem of the financial intermediary, satisfies (2), (1), (3), and (4) with equality and is a solution of the system of equations

$$\frac{u'(E_{t+1})}{v'(c^{\alpha}_{t+1})} = \frac{p_{t+1}}{\zeta p_t} \tag{5}$$

and

$$\frac{u'(E_{t+1})}{v'(c^{\beta}_{t+1})} = \frac{R}{\zeta}. \tag{6}$$

As usual, these equations can be interpreted as equalities between marginal rates of substitution and relative prices. These equations also indicate that the behavior of the financial intermediary follows two motives. One of two motives is to smooth consumptions between the environmental quality and the consumption good. The other is to realize risk sharing between type α and type β agents. The latter motive follows from the fact that the combination of the above two equations implies that $\frac{p_{t+1}}{p_t} v'(c^{\alpha}_{t+1}) = Rv'(c^{\beta}_{t+1})$.

4.2 Climate Change and Behavior of Financial Intermediary

This subsection addresses the issue how the financial intermediary should behave against climate changes, which are captured by marginal changes in parameters θ, γ, ζ in this study.

A marginal increment in either θ or γ decreases directly the environmental quality through (1), so that the financial intermediary will need to decrease the amounts of money and the storage investment to increase the environmental maintenance. In contrast, effects of a marginal increment in the degree of the environmental maintenance, ζ, on the decision of the financial intermediary is not so straightforward. This is because a marginal increment in ζ affects the decision of the financial intermediary not only through (1) but also through right-hand sides of equations (5) and (6).[13] However, the effects of a marginal increment in ζ are expected to be determined under some sufficient conditions. As shown in the following proposition, these intuitions are true.

Proposition 1 *Let $m_t = m_t(\theta, \gamma, \zeta)$, $s_t = s_t(\theta, \gamma, \zeta)$, and $z_t = z_t(\theta, \gamma, \zeta)$ solve the optimization problem of the financial intermediary given E_t, C_t, p_t, and p_{t+1}. Then, it follows that*

$$\frac{\partial m_t}{\partial \theta} < 0, \quad \frac{\partial s_t}{\partial \theta} < 0, \quad \frac{\partial z_t}{\partial \theta} > 0,$$
$$\frac{\partial m_t}{\partial \gamma} < 0, \quad \frac{\partial s_t}{\partial \gamma} < 0, \quad \frac{\partial z_t}{\partial \gamma} > 0.$$

Furthermore, it holds that

$$\frac{\partial m_t}{\partial \zeta} < 0, \quad \frac{\partial s_t}{\partial \zeta} < 0, \quad \frac{\partial z_t}{\partial \zeta} > 0$$

if $(1-\theta)E_t - \gamma C_t > 0$ and $u'(E) \geq -Eu''(E)$ on the domain of u. In contrast,

$$\frac{\partial m_t}{\partial \zeta} > 0, \quad \frac{\partial s_t}{\partial \zeta} > 0, \quad \frac{\partial z_t}{\partial \zeta} < 0$$

if $(1-\theta)E_t - \gamma C_t < 0$ and $u'(E) \leq -Eu''(E)$ on the domain of u.

Table 1 summarizes the effects of marginal increments in climate parameters on the behavior of the financial intermediary described in Proposition 1. As shown in Proposition 1, a marginal increment in each of θ and γ decreases the money holding and the storage investment but increases the environmental maintenance. In contrast, the effect of a marginal increment in ζ on the behavior of the financial intermediary depends on the sign of $(1-\theta)E_t - \gamma C_t$ and the sign of $u'(E) + Eu''(E)$.[14]

Remark 3 In the present model, $p_t m_t/(\omega + \tau_t)$ represents a *reserve-to-deposit ratio*.[15] Generally, monetary authorities use the reserve-to-deposit ratio ratio to protect banks from a sudden decline in liquidity, which can result in a financial crisis. Following Proposition 1, however, we can observe that the financial intermediary decreases its reserve-to-deposit ratio against each of marginal increments in climate parameters θ and γ. Therefore, it is valid to consider that such changes in θ and θ increase financial instability. Similarly, a marginal increments in ζ will also increase financial instability, provided that $(1-\theta)E_t > \gamma C_t$ and $u'(E) \geq -Eu''(E)$ on the domain of u. The effect of a marginal increments in ζ, however, should be examined at an equilibrium.

This study also provides an example with specified utility index functions.

Example 1 Specify the utility index functions u and v as $u(E) = \varepsilon \ln E$ and $v(c) = (1-\varepsilon)\ln c$, respectively. Here, $\varepsilon \in (0,1)$ is the weight on the utility index for the environmental quality and describes how "green" agents are. Then, the optimal choice for the financial intermediary is given by (m_t, s_t, z_t), where

$$\begin{aligned} m_t &= \frac{1}{p_t}\left[(\omega+\tau_t) + \frac{1}{\zeta}[(1-\theta)E_t - \gamma C_t]\right]\pi_\alpha(1-\varepsilon), \\ s_t &= \left[(\omega+\tau_t) + \frac{1}{\zeta}[(1-\theta)E_t - \gamma C_t]\right]\pi_\beta(1-\varepsilon), \\ z_t &= (\omega+\tau_t)\varepsilon - \frac{1}{\zeta}[(1-\theta)E_t - \gamma C_t](1-\varepsilon). \end{aligned}$$

One can easily verify that Proposition 1 holds and, especially, the effect of a marginal increment in ζ on z_t depends only on the sign of $(1-\theta)E_t - \gamma C_t$ because u is a logarithmic function and satisfies that $u'(E) = -Eu''(E)$ for each $E > 0$.

We close this section with a remark on climate actions of financial intermediaries.

Remark 4 We can also consider strategic conflicts between financial intermediaries of two locations by assuming, as mentioned in the footnote 6, that the law of motion of the environmental quality depends on the environmental maintenances of two locations, not of a single location. In such a framework, as the case of the standard public good, one of financial intermediaries decreases the environmental maintenance

Table 1: Marginal Increments in Climate Parameters and Behavior of the Financial Intermediary

	m_t	s_t	z_t	Additional Conditions
θ	$-$	$-$	$+$	
γ	$-$	$-$	$+$	
ζ	$-$	$-$	$+$	$(1-\theta)E_t > \gamma C_t$ and $u'(E) \geq -Eu''(E)$
	$+$	$+$	$-$	$(1-\theta)E_t < \gamma C_t$ and $u'(E) \leq -Eu''(E)$

[12] This optimization problem can be easily rewritten as a similar optimization problem to determin deposit contracts as in Haslag and Martin (2007).

[13] In other words, a change in ζ distorts the relative prices.

[14] Although there is no risk in this study, the condition that $u'(E) + Eu''(E) \geq 0$ can be interpreted as the index of the relative risk aversion being less than or equal to one. In contrast, the condition that $u'(E) + Eu''(E) \leq 0$ can be interpreted as the index of the relative risk aversion being greater than or equal to one.

[15] Note that, if agents' deposit is modeled as $\omega + \tau_t - z_t$, then the reserve-to-deposit ratio is defined by $p_t m_t/(\omega + \tau_t - z_t)$.

against a marginal increment in the environmental maintenance by the other. Due to the symmetry of the two locations in the current settings, however, both financial intermediaries choose the same amount of the environmental maintenance at an equilibrium.[16]

5 Monetary Steady State

The previous section introduced financial intermediaries and studied their behavior. This section defines a monetary steady state, and examines basic properties of monetary steady states.

5.1 Definition, Existence, and Uniqueness of Monetary Steady State

Given the behavior of the financial intermediary, a *monetary equilibrium* occurs when the money market is cleared, i.e., when $m_t = M_t$ in each period t. Following the behavior of the financial intermediary and the fact that $C_t = p_t M_{t-1} + Rs_{t-1}$ at an equilibrium, a monetary equilibrium can be constructed by a pair of $\{\rho_t\}$ and $\{s_t\}$ satisfying that

$$\sigma\rho_t\zeta u'((1-\theta)E_t + \zeta\omega - (\gamma+\zeta)\rho_t - (\gamma R s_{t-1} + \zeta s_t)) = \rho_{t+1} v'\left(\frac{\rho_{t+1}}{\pi_\alpha}\right)$$

and

$$\zeta u'((1-\theta)E_t + \zeta\omega - (\gamma+\zeta)\rho_t - (\gamma R s_{t-1} + \zeta s_t)) = R v'\left(\frac{R s_t}{\pi_\beta}\right),$$

where $\rho_t = p_t M_t/\sigma$ represents an *inflation-adjusted* real money balance and E_{t+1} follows the equation (1) with $C_t = p_t M_{t-1} + Rs_{t-1}$.[17]

A monetary steady state can be defined by a monetary equilibrium, at which $E_{t+1} = E_t$, $\rho_{t+1} = \rho_t$, and $s_t = s_{t-1}$ hold for each t. More formally, this study gives the next definition.

Definition 1 A pair $(\rho, s) \in \mathbb{R}^2_{++}$ is a *monetary steady state* if it satisfies that

$$\sigma\zeta u'\left(\frac{1}{\theta}[\zeta\omega - (\gamma+\zeta)\rho - (\gamma R + \zeta)s]\right) = v'\left(\frac{\rho}{\pi_\alpha}\right) \tag{7}$$

and

$$\zeta u'\left(\frac{1}{\theta}[\zeta\omega - (\gamma+\zeta)\rho - (\gamma R + \zeta)s]\right) = R v'\left(\frac{Rs}{\pi_\beta}\right). \tag{8}$$

We are now ready to demonstrate the existence and uniqueness of monetary steady state.

Proposition 2 *Given a vector of parameters $\boldsymbol{a} = (\theta, \gamma, \zeta, \sigma)$ with $\theta \in (0,1)$, $\gamma > 0$, $\zeta > 0$, and $\sigma > R^{-1}$, a monetary steady state, denoted by $(\rho, s) = (\phi(\boldsymbol{a}), \psi(\boldsymbol{a})) \in \mathbb{R}^2_{++}$, exists uniquely.*

Because a monetary steady state is unique for each $\boldsymbol{a} = (\theta, \gamma, \zeta, \sigma)$, we can explore effects of changes in these parameters.

[16] This is because both financial intermediaries must face the same amount of the environmental quality at an equilibrium.

[17] In this model, an inflation rate is given by $p_t/p_{t+1} = \sigma q_t/q_{t+1}$, where $q_s = p_s M_s$ is the real money balance in period $s = t,\ t+1$ and reduces to σ at a monetary steady state.

5.2 Comparative Statics on Monetary Steady State

This subsection examines effects of marginal changes in parameters θ, γ, ζ, and σ on the monetary steady state.

It would be natural to think that the effects of marginal increments in climate parameters, θ, γ, and ζ on the monetary steady state are taken over from those of individual optimization behavior of financial intermediaries (see Proposition 1). In contrast, an increase in the money growth rate σ will decrease inflation-adjusted real money balance and increase the storage investment and the environmental maintenance because it increases the opportunity cost of money holding.[18] To be more precise, we can obtain the following proposition.

Proposition 3 *Let $\chi(\boldsymbol{a}) = \omega - \phi(\boldsymbol{a}) - \psi(\boldsymbol{a})$, which is the amount of the environmental maintenance at the monetary steady state. Then, the following four statements hold.*

(a) $\dfrac{\partial \phi}{\partial \theta} < 0, \quad \dfrac{\partial \psi}{\partial \theta} < 0, \quad \dfrac{\partial \chi}{\partial \theta} > 0.$

(b) $\dfrac{\partial \phi}{\partial \gamma} < 0, \quad \dfrac{\partial \psi}{\partial \gamma} < 0, \quad \dfrac{\partial \chi}{\partial \gamma} > 0.$

(c) $\dfrac{\partial \phi}{\partial \zeta} > 0, \quad \dfrac{\partial \psi}{\partial \zeta} > 0, \quad \dfrac{\partial \chi}{\partial \zeta} < 0$ *if $u'(E) \leq -Eu''(E)$ on its domain.*

(d) $\dfrac{\partial \phi}{\partial \sigma} < 0, \quad \dfrac{\partial \psi}{\partial \sigma} > 0, \quad \dfrac{\partial \chi}{\partial \sigma} > 0,$ *and* $\dfrac{\partial \psi}{\partial \sigma} \bigg/ \dfrac{\partial \phi}{\partial \sigma} \in (-1, 0).$

Table 2 summarizes the effects of marginal increments in climate parameters, θ, γ, ζ, and in the gross rate of money growth, σ, on the behavior of the financial intermediary described in Proposition 3. Fundamentally, these effects are similar to those on the financial intermediary. As shown in Proposition 3, a marginal increment in each of θ and γ decreases the inflation-adjusted real money balance and the storage investment but increases the environmental maintenance. In contrast, under the additional condition that $u'(E) + Eu''(E) \leq 0$, a marginal increment in ζ increases the inflation-adjusted real money balance and the storage investment but decreases the environmental maintenance. Furthermore, a marginal increment in σ decreases the inflation-adjusted real money balance but increases the storage investment and the environmental maintenance.

Remark 5 Recall that $p_t m_t/(\omega + \tau_t)$ represents a reserve-to-deposit ratio. One can verify that, at the monetary steady state $(\rho, s) = (\phi(\boldsymbol{a}), \psi(\boldsymbol{a}))$, the reserve-to-deposit rate reduces to $D \equiv \sigma\phi(\boldsymbol{a})/[\omega + (\sigma - 1)\phi(\boldsymbol{a})]$. It follows then that $\partial D/\partial \theta < 0$, $\partial D/\partial \gamma < 0$, and, if $u'(E) \leq -Eu''(E)$ on its domain, $\partial D/\partial \zeta > 0$. Therefore, we can consider that marginal increments in θ and γ increase financial *in*stability and that, with a certain condition, a marginal increment in ζ increase financial stability.

This study also provides an example with specified utility index functions.

Table 2: Comparative Statics on Monetary Steady State

	ρ	s	z	Additional Conditions
θ	$-$	$-$	$+$	
γ	$-$	$-$	$+$	
ζ	$+$	$+$	$-$	$u'(E) \leq -Eu''(E)$
σ	$-$	$+$	$+$	

[18] The opportunity cost of holding money can be measured by the gross nominal interest rate $I_t = Rp_t/p_{t+1}$, which reduces to $I = \sigma R$ at the monetary steady state.

Example 2 Specify the utility index functions u and v as in Example 1, respectively. Let $\boldsymbol{a} = (\theta, \gamma, \zeta, \sigma)$. Then, the monetary steady state $(\rho, s) = (\phi(\boldsymbol{a}), \psi(\boldsymbol{a}))$ and the corresponding amount of the environmental maintenance, $z = \chi(\boldsymbol{a})$, are given as follows.

$$\begin{aligned}
\phi(\boldsymbol{a}) &= \frac{\zeta \pi_\alpha (1-\varepsilon)\omega}{\sigma[\theta\zeta\varepsilon + (\gamma R + \zeta)\pi_\beta(1-\varepsilon)] + (\gamma+\zeta)\pi_\alpha(1-\varepsilon)}, \\
\psi(\boldsymbol{a}) &= \frac{\sigma \zeta \pi_\beta (1-\varepsilon)\omega}{\sigma[\theta\zeta\varepsilon + (\gamma R + \zeta)\pi_\beta(1-\varepsilon)] + (\gamma+\zeta)\pi_\alpha(1-\varepsilon)}, \\
\chi(\boldsymbol{a}) &= \frac{[\sigma[\theta\zeta\varepsilon + \gamma R \pi_\beta(1-\varepsilon)] + \gamma\pi_\alpha(1-\varepsilon)]\,\omega}{\sigma[\theta\zeta\varepsilon + (\gamma R + \zeta)\pi_\beta(1-\varepsilon)] + (\gamma+\zeta)\pi_\alpha(1-\varepsilon)}.
\end{aligned}$$

One can easily verify that Proposition 3 holds.

6 Optimal Monetary Policy

We have studies market activities in previous sections. We now turn to study the optimal monetary policy. Subsection 6.1 defines and characterizes golden rule optimality. Suboptimality of the monetary steady state is also argued. Subsection 6.2 then studies the second-best monetary policy.

6.1 Golden Rule Optimality

Given s_0 and E_1, a *feasible allocation* of this economy is a triplet $(E_t, c_t, s_{t-1})_{t\geq 1}$ of the environmental qualities E_t, the consumptions $c_t = (c_t^\alpha, c_t^\beta)$, and the storage investments s_{t-1} with $s_{t-1} \in [0, \omega]$ such that there exists a nonnegative sequence of environmental investments, $\{z_t\}_{t\geq 1}$, satisfying the law of motion of environmental quality (1) and the equation that, for each $t \geq 1$,

$$(9) \quad c_t^\alpha \pi_\alpha + c_t^\beta \pi_\beta + s_t - s_{t-1} + z_t = (R-1)s_{t-1} + \omega,$$

which means that the sum of consumption, storage investment, and environmental investment must be equal to the total output in each period. Summing up equations (1) and (9), we can obtain a resource constraint

$$(10) \quad \frac{\gamma+\zeta}{\zeta}[c_t^\alpha \pi_\alpha + c_t^\beta \pi_\beta] + [s_t - s_{t-1}] + \frac{1}{\zeta}[E_{t+1} - (1-\theta)E_t] = (R-1)s_{t-1} + \omega,$$

for each $t \geq 1$. Note that $(E_t, c_t, s_{t-1})_{t\geq 1}$ is a feasible allocation if and only if the equation (10) and the inequalities, $[(R-1)s_{t-1} + \omega] - [c_t^\alpha \pi_\alpha + c_t^\beta \pi_\beta] - [s_t - s_{t-1}] \geq 0$ for each date $t \geq 1$, hold.[19] A feasible allocation is *stationary* if it is independent of periods t. It is often identified with a quadruple of nonnegative numbers, $(E, c^\alpha, c^\beta, s)$, such that

$$(11) \quad \frac{\gamma+\zeta}{\zeta}[c^\alpha \pi_\alpha + c^\beta \pi_\beta] + \frac{\theta}{\zeta}E = (R-1)s + \omega,$$

provided that $s \in [0, \omega]$ and $(R-1)s + \omega - [c^\alpha \pi_\alpha + c^\beta \pi_\beta] \geq 0$.

As a criterion of optimality, this study adopts golden rule optimality, which is Pareto optimality defined over stationary feasible allocations.

Definition 2 A stationary feasible allocation $(E, c^\alpha, c^\beta, s)$ is a *golden rule optimal allocation* if it satisfies that $u(E) + v(c^\alpha)\pi_\alpha + v(c^\beta)\pi_\beta \geq u(\tilde{E}) + v(\tilde{c}^\alpha)\pi_\alpha + v(\tilde{c}^\beta)\pi_\beta$ for any feasible stationary allocation $(\tilde{E}, \tilde{c}^\alpha, \tilde{c}^\beta, \tilde{s})$.

Golden rule optimality can be characterized as follows.

Proposition 4 *An interior stationary feasible allocation $(E, c^\alpha, c^\beta, s)$ is golden rule optimal if and only if the following two conditions hold.*

[19] The last inequality ensures the nonnegativity of the environment maintenance, z_t.

(A) $s = \omega$.

(B) $\dfrac{u'(E)}{v'(c^\alpha)} = \dfrac{u'(E)}{v'(c^\beta)} = \dfrac{\theta}{\gamma+\zeta}$.

The condition (A) of Proposition 4 represents a requirement on productive efficiency. To be more precise, it requires the total endowment when young, ω, is entirely invested in the storage technology. This is because the net return of the storage technology, $R-1$, is positive constant in the current setting.

Similar to John and Pecchenino (1994), the environment qualities in our model play a role as the public good. As shown in the equation (11) that the per-unit cost, measured by the (total) consumption good, of producing the environmental quality is equal to $(\gamma+\zeta)/\theta$. Therefore, the condition (B) of Proposition 4, which means that the marginal rates of substitution are equal to the marginal cost of producing the environmental good, can be interpreted as an intertemporal variation of the Samuelson condition for the optimal provision of the public good (Samuelson, 1954). The condition (B) of Proposition 4 also implies that $v'(c^\alpha) = v'(c^\beta)$, which represents the optimal risk-sharing condition between type α and type β agents and implies that $c^\alpha = c^\beta$.

As a corollary of Proposition 4, we can observe suboptimality of the monetary steady state.

Corollary 1 *For any σ, the monetary steady state $(\rho, s) = (\phi(\theta,\gamma,\zeta,\sigma), \psi(\theta,\gamma,\zeta,\sigma))$ is suboptimal.*

There are two reasons for this suboptimality. First, at the monetary steady state, not all initial endowments are available for the storage investment. This fact represents a productive inefficiency, which is often observed in OLG frameworks. Second, at the monetary steady state, the marginal rate of substitution between the environmental quality and the consumption for type β agent is fixed to be equal to R/ζ (see the equation (8)). This fact also implies a lack of optimal risk-sharing between type α and type β agents. In summary, at the monetary steady state, we can observe the trade-off among productive efficiency, efficient provision of the environmental quality (the environmental efficiency), and risk sharing.

6.2 Optimum Quantity of Money

We now turn to study "optimal" monetary policy. The *best* choice of money growth rate for the central bank is such that the monetary steady state given the rate generates a golden rule optimal allocation. However, as shown in the previous section, the monetary steady state never generates golden rule optimal. The central bank therefore should find a second-best choice. As the objective function of the central bank, the *equilibrium welfare* is defined by

$$W(\sigma) = u\left(\frac{1}{\theta}[\zeta\omega - (\gamma+\zeta)\phi(\boldsymbol{a}) - (\gamma R+\zeta)\psi(\boldsymbol{a})]\right) + v\left(\frac{\phi(\boldsymbol{a})}{\pi_\alpha}\right)\pi_\alpha + v\left(\frac{R\psi(\boldsymbol{a})}{\pi_\beta}\right)\pi_\beta,$$

where $\boldsymbol{a} = (\theta,\gamma,\zeta,\sigma)$. A money growth rate σ^{**} is *second-best* if it maximizes $W(\sigma)$.

To describe the result on the second-best money growth rate, several notations are introduced. Let

$$\Sigma(\sigma) = \sigma - \frac{\gamma R + (1-\theta)\zeta}{\theta\zeta} \frac{\frac{\partial\psi}{\partial\sigma}(\theta,\gamma,\zeta,\sigma)}{\frac{\partial\phi}{\partial\sigma}(\theta,\gamma,\zeta,\sigma)},$$

which satisfies that $\Sigma(\sigma) > \sigma$ for each σ because $(\partial\psi/\partial\sigma)/(\partial\phi/\partial\sigma) < 0$ (see Proposition 3). Also let

$$\bar{\sigma} = \frac{\gamma+\zeta}{\theta\zeta},$$

which satisfies that $\bar{\sigma} > 1$ because $\gamma + \zeta - \theta\zeta = \gamma + (1-\theta)\zeta > 0$. We can then obtain the following proposition.

Proposition 5 *A second-best money growth rate σ^{**} exists uniquely in the open interval $(R^{-1}, \bar{\sigma})$ and is characterized by the solution of the equation that $\Sigma(\sigma^{**}) = \bar{\sigma}$ if Σ is strictly monotone increasing on its domain. Furthermore, it holds that*

$$(12)\quad \sigma^{**} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 1 \quad \text{if and only if} \quad \Sigma(1) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \bar{\sigma}.$$

Because of nonlinearity in Σ, the existence and the uniqueness of the second-best money growth rate are not trivial. Proposition 5 then shows that the strict monotonicity of Σ is sufficient for the existence and the uniqueness of the second-best money growth rate. The proposition also says that the second-best money growth rate can be greater than or less than one, which depends on the relationship between $\Sigma(1)$ and $\bar{\sigma}$. This study provides a sufficient condition for the strict monotonicity of Σ.

Corollary 2 *Let*

$$\xi(\sigma) = \frac{u''\left(\dfrac{1}{\theta}[\zeta\omega - \phi(\boldsymbol{a}) - \psi(\boldsymbol{a})]\right)}{v''\left(\dfrac{R\psi(\boldsymbol{a})}{\pi_\beta}\right)}.$$

If $\xi'(\sigma) > 0$ on its domain, then $\Sigma'(\sigma) > 0$ on its domain, so that there exists a second-best money growth rate σ^{**} as in Proposition 5.

This study also provides two remarks on Proposition 5. The first remark is on the suboptimality of the Friedman rule (Friedman, 1969).

Remark 6 Although the second-best money growth rate can be less than one, the Friedman rule is neither best nor second-best.[20] This can be verify the fact that $\sigma^{**} > R^{-1}$. The suboptimality of the Friedman rule is due to the trade-off among productive efficiency, efficient provision of the environmental quality, and risk sharing. If the central bank follows the Friedman rule, type α and type β agents can consume exactly the same quantities and are fully insured. At the same time, however, productive efficiency and the environmental efficiency are sacrificed. The second-best money growth rate balances this trade-off.

The second remark is on relations with previous studies.

Remark 7 One can find that the last proposition includes results of several previous studies as special cases. When $\pi_\alpha = 1$ and $R = 0$ (no storage technology), the present model degenerates into Ohtaki (2023a) and the second-best money growth rate is given by $\sigma^{**} = \bar{\sigma}$ because $\Sigma(\sigma) = \sigma$ for any σ. When $\theta = \zeta = 1$ and $\gamma = 0$ (no environment), the present model degenerates into a variant of Haslag and Martin (2007) (see also Remark 1 and the footnote 10) and the second-best money growth rate is given by $\sigma^{**} = 1$ because $\Sigma(\sigma) = \sigma$ for any σ and $\bar{\sigma} = 1$.

As mentioned as above, it is not so trivial in general whether sufficient conditions for the existence and the uniqueness of the second-best money growth rate hold. However, we can obtain a closed solution when utility index functions belong to the class of logarithmic functions.

Example 3 Specify the utility index functions u and v as in Example 1, respectively. Under such a specification,

$$\frac{\dfrac{\partial\psi}{\partial\sigma}(\theta,\gamma,\zeta,\sigma)}{\dfrac{\partial\phi}{\partial\sigma}(\theta,\gamma,\zeta,\sigma)} = -\frac{(\gamma+\zeta)\pi_\beta(1-\varepsilon)}{\theta\zeta\varepsilon + (\gamma R + \zeta)\pi_\beta(1-\varepsilon)} \in (-1, 0),$$

[20] The *Friedman rule* is a policy for the net nominal interest rate to be zero. In the current setting, the money growth rate corresponding to the Friedman rule is $\sigma^f = R^{-1}$. See also the footnote 18 for the nominal interest rate.

which is independent of the money growth rate σ. This implies that Σ is actually strictly monotone increasing. Then, the unique second-best money growth rate is given by

$$(13)\quad \sigma^{**} = \frac{(\gamma+\zeta)[\varepsilon+\pi_\beta(1-\varepsilon)]}{\theta\zeta\varepsilon+(\gamma R+\zeta)\pi_\beta(1-\varepsilon)},$$

which belongs to $(R^{-1}, \bar{\sigma})$.[21] This σ^{**} is positive, but may or may not be greater than one, depending on the parameters. For example, $\sigma^{**} \to \bar{\sigma} > 1$ as $\varepsilon \to 1$ and $\sigma^{**} \to (\gamma+\zeta)/(\gamma R+\zeta) \in (R^{-1}, 1)$ as $\varepsilon \to 0$.[22] Focusing on the size of ε, it holds that

$$\sigma^{**} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 1 \quad \text{if and only if} \quad \varepsilon \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \bar{\varepsilon}_0 \equiv \frac{(R-1)\gamma\pi_\beta}{\gamma+(R-1)\gamma\pi_\beta+(1-\theta)\zeta}.$$

Therefore, we can consider that the level of the second money growth rate depends on the level of ε, which is the parameter that describes how "green" agents are.

We close this section by examining the reactions of the second-beset money growth rate against marginal increments in climate parameters when utility index functions are specified as in Example 1.

Proposition 6 *Specify utility index functions u and v as in Example 1, respectively. Then, the following three statements hold.*

(a) $\dfrac{\partial\sigma^{**}}{\partial\theta} < 0.$

(b) $\dfrac{\partial\sigma^{**}}{\partial\gamma} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \textit{if and only if} \quad \varepsilon \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \bar{\varepsilon}_1 \equiv \dfrac{(R-1)\pi_\beta}{1+(R-1)\pi_\beta}.$

(c) $\dfrac{\partial\sigma^{**}}{\partial\zeta} > 0.$

Table 3: Marginal Increments in Climate Parameters and the Second-best Money Growth Rate

	σ^{**}	Additional Conditions
θ	$-$	
γ	$+$	$\varepsilon > \bar{\varepsilon}_1$
	0	$\varepsilon = \bar{\varepsilon}_1$
	$-$	$\varepsilon < \bar{\varepsilon}_1$
ζ	$+$	

[21] Acturally,

$$\begin{aligned} \sigma^{**} - \frac{1}{R} &= \frac{R[\gamma\varepsilon+\zeta\pi_\beta(1-\varepsilon)]+(R-\theta)\zeta\varepsilon+(R-1)\zeta\pi_\beta(1-\varepsilon)}{R[\theta\zeta\varepsilon+(\gamma R+\zeta)\pi_\beta(1-\varepsilon)]} > 0 \\ \bar{\sigma} - \sigma^{**} &= \frac{(\gamma+\zeta)\pi_\beta(1-\varepsilon)[\gamma R+\zeta[1-\theta\pi_\beta(1-\varepsilon)]]}{\theta\zeta[\theta\zeta\varepsilon+(\gamma R+\zeta)\pi_\beta(1-\varepsilon)]} > 0. \end{aligned}$$

Note that the number R^{-1} is the money growth rate corresponding to the Friedman rule and the number $\bar{\sigma}$ is the second-best money growth rate in Ohtaki (2023a), which considers the case without random relocation.

[22] The latter fact, $\sigma^{**} \to (\gamma+\zeta)/(\gamma R+\zeta) \in (R^{-1}, 1)$ as $\varepsilon \to 0$, indicates discontinuity between the present model and Haslag and Martin (2007). When $\varepsilon = 0$, the present model should degenerate into Haslag and Martin (2007), in which the second-best money growth must be one, whereas it is less than one in the present model. This discontinuity might come from consumption externalities. In fact, when γ converges to 0 from above, $(\gamma+\zeta)/(\gamma R+\zeta) \to 1$.

Table 3 summarizes the effects of marginal increments in climate parameters, θ, γ, and ζ on the level of the second-best money growth rate described in Proposition 6. As shown in Proposition 6, the level of the second-best money growth rate decreases against a marginal increment in θ and increases against a marginal increment in ζ. In contrast, an effect of a marginal increment in γ on the level of the second-best money growth rate depends on the size of ε, which is the parameter describing how "green" agents are. Against a marginal increment in γ, the level of the second-best money growth rate increases if $\varepsilon > \bar{\varepsilon}_1$ and decreases if $\varepsilon < \bar{\varepsilon}_1$.[23]

7 Concluding Remarks

Motivated by recent climate actions of central banks and supervisors in a lot of countries, this study has developed an OLG model of the environment and financial intermediation. In the present model, how financial intermediaries and markets react against climate changes has been studied. It has been also shown the existence of conflicts between the environment and economic activities, so that the monetary policies to resolve the conflicts are studied. It has been shown that the Friedman rule is suboptimal as in the previous studies but, differently from the previous studies, the level of the optimal money growth rate is not necessarily equal to one and depends on how "green" agents are. It has been also studied that how the optimal monetary policy react against climate changes.

We close this study with two remarks. First, we should remark that this study considered a storage technology as an intertemporal production technology. To obtain a more tractable macroeconomic model, a storage technology should be replaced with a more general neoclassical production function. Second, we should remark that the "optimal" monetary policy in this study is the "second-best." Recently, Krogstrup and Oman (2019) surveyed the rapidly growing literature on the role of macroeconomic and financial policy tools in issues about climate changes. They concluded that 'fiscal tools are first in line and central, but can and may need to be complemented by financial and monetary policy instruments.' As they mentioned, however, the literature on the most effective policy mix of these tools is scarce. The optimal policy mix to obtain the optimal allocation should be explored. The answers to these remarks are left for the future researches.

Conflict of Interest Statement

The author declares that there is no conflict of interest.

Appendix: Proofs of Propositions

Proof of Proposition 1. The system of equations (5) and (6) can be rewritten as the system of equations that $g_1(m_t, s_t, \theta, \gamma, \zeta) = 0$ and $g_2(m_t, s_t, \theta, \gamma, \zeta) = 0$, where

$$g_1(m_t, s_t, \theta, \gamma, \zeta) = -\zeta p_t u'\left((1-\theta)E_t - \gamma C_t + \zeta(\omega + \tau_t - p_t m_t - s_t)\right) + p_{t+1} v'\left(\frac{p_{t+1} m_t}{\pi_\alpha}\right)$$

and

$$g_2(m_t, s_t, \theta, \gamma, \zeta) = -\zeta u'\left((1-\theta)E_t - \gamma C_t + \zeta(\omega + \tau_t - p_t m_t - s_t)\right) + R v'\left(\frac{R s_t}{\pi_\beta}\right).$$

Because

$$(14) \quad G \;=\; \det \begin{bmatrix} \dfrac{\partial g_1}{\partial m_t} & \dfrac{\partial g_1}{\partial s_t} \\ \dfrac{\partial g_2}{\partial m_t} & \dfrac{\partial g_2}{\partial s_t} \end{bmatrix} = \frac{(\zeta p_{t+1})^2}{\pi_\alpha} u'' v''_\alpha + \frac{(\zeta R p_t)^2}{\pi_\beta} u'' v''_\beta + \frac{(R p_{t+1})^2}{\pi_\alpha \pi_\beta} v''_\alpha v''_\beta > 0,$$

[23]Note that

$$\bar{\varepsilon}_1 - \bar{\varepsilon}_0 = \frac{(R-1)(1-\theta)\zeta\pi_\beta}{[1 + (R-1)\pi_\beta][\gamma + (R-1)\gamma\pi_\beta + (1-\theta)\zeta]} > 0.$$

it follows from the implicit function theorem that

$$\begin{aligned}
\begin{bmatrix} \dfrac{\partial m_t}{\partial \theta} & \dfrac{\partial m_t}{\partial \gamma} & \dfrac{\partial m_t}{\partial \zeta} \\ \dfrac{\partial s_t}{\partial \theta} & \dfrac{\partial s_t}{\partial \gamma} & \dfrac{\partial s_t}{\partial \zeta} \end{bmatrix} &= -\begin{bmatrix} \dfrac{\partial g_1}{\partial m_t} & \dfrac{\partial g_1}{\partial s_t} \\ \dfrac{\partial g_2}{\partial m_t} & \dfrac{\partial g_2}{\partial s_t} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{\partial g_1}{\partial \theta} & \dfrac{\partial g_1}{\partial \gamma} & \dfrac{\partial g_1}{\partial \zeta} \\ \dfrac{\partial g_2}{\partial \theta} & \dfrac{\partial g_2}{\partial \gamma} & \dfrac{\partial g_2}{\partial \zeta} \end{bmatrix} \\
&= -\frac{1}{G}\begin{bmatrix} \dfrac{\partial g_2}{\partial s_t} & -\dfrac{\partial g_1}{\partial s_t} \\ -\dfrac{\partial g_2}{\partial m_t} & \dfrac{\partial g_1}{\partial m_t} \end{bmatrix} \begin{bmatrix} \dfrac{\partial g_1}{\partial \theta} & \dfrac{\partial g_1}{\partial \gamma} & \dfrac{\partial g_1}{\partial \zeta} \\ \dfrac{\partial g_2}{\partial \theta} & \dfrac{\partial g_2}{\partial \gamma} & \dfrac{\partial g_2}{\partial \zeta} \end{bmatrix}
\end{aligned}$$

for $\theta \in (0,1)$, $\gamma > 0$, and $\zeta > 0$, where, in equation (14),

$$\begin{aligned}
u'' &= u''\left((1-\theta)E_t - \gamma C_t + \zeta(\omega + \tau_t - p_t m_t - s_t)\right), \\
v''_\alpha &= v''_\alpha\left(\frac{p_{t+1} m_t}{\pi_\alpha}\right), \\
v''_\beta &= \left(\frac{R s_t}{\pi_\beta}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\begin{bmatrix} \dfrac{\partial m_t}{\partial \theta} \\ \dfrac{\partial s_t}{\partial \theta} \end{bmatrix} &= -\frac{1}{G}\begin{bmatrix} \dfrac{\partial g_2}{\partial s_t}\dfrac{\partial g_1}{\partial \theta} - \dfrac{\partial g_1}{\partial s_t}\dfrac{\partial g_2}{\partial \theta} \\ -\dfrac{\partial g_2}{\partial m_t}\dfrac{\partial g_1}{\partial \theta} + \dfrac{\partial g_1}{\partial m_t}\dfrac{\partial g_2}{\partial \theta} \end{bmatrix} = -\frac{1}{G}\begin{bmatrix} \dfrac{R^2}{\pi_\beta}\zeta p_t E_t u'' v''_\beta \\ \dfrac{p_{t+1}^2}{\pi_\alpha}\zeta E_t u'' v''_\alpha \end{bmatrix} \in \mathbb{R}^2_{--}, \\
\begin{bmatrix} \dfrac{\partial m_t}{\partial \gamma} \\ \dfrac{\partial s_t}{\partial \gamma} \end{bmatrix} &= -\frac{1}{G}\begin{bmatrix} \dfrac{\partial g_2}{\partial s_t}\dfrac{\partial g_1}{\partial \gamma} - \dfrac{\partial g_1}{\partial s_t}\dfrac{\partial g_2}{\partial \gamma} \\ -\dfrac{\partial g_2}{\partial m_t}\dfrac{\partial g_1}{\partial \gamma} + \dfrac{\partial g_1}{\partial m_t}\dfrac{\partial g_2}{\partial \gamma} \end{bmatrix} = -\frac{1}{G}\begin{bmatrix} \dfrac{R^2}{\pi_\beta}\zeta p_t C_t u'' v''_\beta \\ \dfrac{p_{t+1}^2}{\pi_\alpha}\zeta C_t u'' v''_\alpha \end{bmatrix} \in \mathbb{R}^2_{--}.
\end{aligned}$$

Now, assume further that $(1-\theta)E_t > \gamma C_t$ and $u'(E) + Eu''(E) \geq 0$ for each $E > 0$. Then, it follows that

$$\begin{aligned}
& [u'(E_{t+1}) + \zeta(\omega + \tau_t - p_t m_t - s_t)u''(E_{t+1})] - [u'(E_{t+1}) + E_{t+1}u''(E_{t+1})] \\
= \; & -[(1-\theta)E_t - \gamma C_t]u''(E_{t+1}) > 0,
\end{aligned}$$

where E_{t+1} is as in equation (1). This implies that

$$u'(E_{t+1}) + \zeta(\omega + \tau_t - p_t m_t - s_t)u''(E_{t+1}) > u'(E_{t+1}) + E_{t+1}u''(E_{t+1}) \geq 0.$$

We can therefore obtain that

$$\begin{aligned}
\begin{bmatrix} \dfrac{\partial m_t}{\partial \zeta} \\ \dfrac{\partial s_t}{\partial \zeta} \end{bmatrix} &= -\frac{1}{G}\begin{bmatrix} \dfrac{\partial g_2}{\partial s_t}\dfrac{\partial g_1}{\partial \zeta} - \dfrac{\partial g_1}{\partial s_t}\dfrac{\partial g_2}{\partial \zeta} \\ -\dfrac{\partial g_2}{\partial m_t}\dfrac{\partial g_1}{\partial \zeta} + \dfrac{\partial g_1}{\partial m_t}\dfrac{\partial g_2}{\partial \zeta} \end{bmatrix} \\
&= -\frac{1}{G}\begin{bmatrix} -\dfrac{p_t R^2}{\pi_\beta}[u'(E_{t+1}) + \zeta(\omega + \tau_t - p_t m_t - s_t)u''(E_{t+1})]v''_\beta \\ -\dfrac{p_{t+1}^2}{\pi_\alpha}[u'(E_{t+1}) + \zeta(\omega + \tau_t - p_t m_t - s_t)u''(E_{t+1})]v''_\alpha \end{bmatrix} \in \mathbb{R}^2_{--}.
\end{aligned}$$

A similar argument guarantees that $\begin{bmatrix} \frac{\partial m_t}{\partial \zeta} & \frac{\partial s_t}{\partial \zeta} \end{bmatrix}' \in \mathbb{R}^2_{++}$ if $(1-\theta)E_t < \gamma C_t$ and $u'(E) + Eu''(E) \leq 0$ for each $E > 0$.[24]

[24] For each vector $\boldsymbol{v}$, $\boldsymbol{v}'$ is the transpose of $\boldsymbol{v}$.

Finally, because the effects of θ, γ, ζ on z_t are determined through the equation that $z_t = (\omega + \tau_t) - p_t m_t - s_t$, the effects on z_t have signs opposite to those on m_t and s_t. This completes the proof of Proposition 1. Q.E.D.

Proof of Proposition 2. The proof strategy is similar to Koda (1984). Let $\boldsymbol{a} = (\theta, \gamma, \zeta, \sigma)$. The system of equations (7) and (8) can be then rewritten as the system of equations that $f_1(\rho, s, \boldsymbol{a}) = 0$ and $f_2(\rho, s, \boldsymbol{a}) = 0$, where

$$f_1(\rho, s, \boldsymbol{a}) = -\sigma\zeta u'\left(\frac{1}{\theta}[\zeta\omega - (\gamma+\zeta)\rho - (\gamma R + \zeta)s]\right) + v'\left(\frac{\rho}{\pi_\alpha}\right)$$

and

$$f_2(\rho, s, \boldsymbol{a}) = -\zeta u'\left(\frac{1}{\theta}[\zeta\omega - (\gamma+\zeta)\rho - (\gamma R + \zeta)s]\right) + Rv'\left(\frac{Rs}{\pi_\beta}\right).$$

Because f_2 is continuous in s and satisfies that $\lim_{s\downarrow 0} f_2(\rho, s, \boldsymbol{a}) > 0$ and $\lim_{s\uparrow \bar{s}} f_2(\rho, s, \boldsymbol{a}) < 0$ for $\rho \in (0, \zeta\omega/(\gamma+\zeta))$, it follows from the intermediate value theorem that there is some $\tilde{s}(\rho, \boldsymbol{a}) \in (0, \bar{s})$ such that $f_2(\rho, \tilde{s}(\rho, \boldsymbol{a}), \boldsymbol{a}) = 0$, where $\bar{s} = [\zeta\omega - (\gamma+\zeta)\rho]/(\gamma R + \zeta)$. Note that such a $\tilde{s}(\rho, \boldsymbol{a})$ is unique because f_2 is strictly monotone decreasing in s.[25] Furthermore, by the implicit function theorem, $\tilde{s}$ is continuous.[26] Note that $\tilde{s}(\rho, \boldsymbol{a}) \downarrow 0$ as $\rho \uparrow \bar{\rho} \equiv \zeta\omega/(\gamma+\zeta)$. Also note that $\partial\tilde{s}/\partial\sigma = 0$ because f_2 is independent of σ.

Now let $\Phi(\rho, \boldsymbol{a}) = f_1(\rho, \tilde{s}(\rho, \boldsymbol{a}), \boldsymbol{a})$. Because Φ is continuous in ρ and satisfies that $\lim_{\rho\downarrow 0} \Phi(\rho, \boldsymbol{a}) > 0$ and $\lim_{\rho\uparrow\bar{\rho}} \Phi(\rho, \boldsymbol{a}) < 0$, it follows from the intermediate value theorem that there is some $\phi(\boldsymbol{a}) \in (0, \bar{\rho})$ such that $\Phi(\phi(\boldsymbol{a}), \boldsymbol{a}) = 0$. Note that, by the implicit function theorem that $\phi(\boldsymbol{a})$ is continuous. Also note that the uniqueness of $\phi(\boldsymbol{a})$ follows from the fact that Φ is strictly monotone decreasing in ρ.[27]

Define $\psi(\boldsymbol{a}) = \tilde{s}(\phi(\boldsymbol{a}), \boldsymbol{a})$. Then, $(\rho, s) = (\phi(\boldsymbol{a}), \psi(\boldsymbol{a}))$ establishes Proposition 2. Q.E.D.

Proof of Proposition 3. Let $\boldsymbol{a} = (\theta, \gamma, \zeta, \sigma)$. The system of equations (7) and (8) can be rewritten as the system of equations that $f_1(\rho, s, \theta, \gamma, \zeta, \sigma) = 0$ and $f_2(\rho, s, \theta, \gamma, \zeta, \sigma) = 0$, where f_1 and f_2 are defined as

[25] In fact, it holds that

$$\frac{\partial f_2}{\partial s} = \frac{\zeta(\gamma R+\zeta)}{\theta}u''\left(\frac{1}{\theta}[\zeta\omega - (\gamma+\zeta)\rho - (\gamma R+\zeta)\tilde{s}(\rho, \boldsymbol{a})]\right) + \frac{R^2}{\pi_\beta}v''\left(\frac{Rs}{\pi_\beta}\right) < 0.$$

[26] We can also obtain that

$$\frac{\partial\tilde{s}}{\partial\rho}(\rho, \boldsymbol{a}) = -\frac{\dfrac{\partial f_2}{\partial\rho}(\rho, \tilde{s}(\rho, \boldsymbol{a}), \boldsymbol{a})}{\dfrac{\partial f_2}{\partial\rho}(\rho, \tilde{s}(\rho, \boldsymbol{a}), \boldsymbol{a})} = -\frac{\dfrac{\gamma+\zeta}{\theta}\zeta u''(E)}{\dfrac{\gamma R+\zeta}{\theta}\zeta u''(E) + \dfrac{R^2}{\pi_\beta}v''(c^\beta)} \in (-1, 0),$$

where

$$E = \frac{1}{\theta}[\zeta\omega - (\gamma+\zeta)\rho - (\gamma R+\zeta)\tilde{s}(\rho, \boldsymbol{a})] \quad \text{and} \quad c^\beta = \frac{R\tilde{s}(\rho, \boldsymbol{a})}{\pi_\beta}.$$

[27] In fact, it holds that

$$\begin{aligned}\frac{\partial\Phi}{\partial\rho}(\rho, \boldsymbol{a}) &= \frac{\sigma\zeta}{\theta}\left[(\gamma+\zeta) + (\gamma R+\zeta)\frac{\partial\tilde{s}}{\partial\rho}(\rho, \boldsymbol{a})\right]u''(E) + \frac{v''(c^\alpha)}{\pi_\alpha} \\ &= \frac{\sigma\zeta}{\theta}\frac{(\gamma+\zeta)\dfrac{R^2}{\pi_\beta}v''(c^\beta)}{\dfrac{\gamma R+\zeta}{\theta}\zeta u''(E) + \dfrac{R^2}{\pi_\beta}v''(c^\beta)}u''(E) + \frac{v''(c^\alpha)}{\pi_\alpha} < 0,\end{aligned}$$

where E and c^β are as in the footnote 26 and $c^\alpha = \rho/\pi_\alpha$.

in the proof of Proposition (2). From Proposition 2, it holds that $f_1(\phi(\boldsymbol{a}), \psi(\boldsymbol{a}), \boldsymbol{a}) = f_2(\phi(\boldsymbol{a}), \psi(\boldsymbol{a}), \boldsymbol{a}) = 0$ for $\boldsymbol{a} = (\theta, \gamma, \zeta, \sigma)$. Because

$$(15) \quad F = \det \begin{bmatrix} \dfrac{\partial f_1}{\partial \rho} & \dfrac{\partial f_1}{\partial s} \\ \dfrac{\partial f_2}{\partial \rho} & \dfrac{\partial f_2}{\partial s} \end{bmatrix} = \frac{\zeta(\gamma R + \zeta)}{\theta \pi_\alpha} u'' v''_\alpha + \frac{\sigma \zeta (\gamma + \zeta) R^2}{\theta \pi_\beta} u'' v''_\beta + \frac{R^2}{\pi_\alpha \pi_\beta} v''_\alpha v''_\beta > 0,$$

it follows from the implicit function theorem that, at $(\rho, s, \boldsymbol{a}) = (\phi(\boldsymbol{a}), \psi(\boldsymbol{a}), \theta, \gamma, \zeta, \sigma)$,

$$\begin{aligned} \begin{bmatrix} \dfrac{\partial \phi}{\partial \theta} & \dfrac{\partial \phi}{\partial \gamma} & \dfrac{\partial \phi}{\partial \zeta} & \dfrac{\partial \phi}{\partial \sigma} \\ \dfrac{\partial \psi}{\partial \theta} & \dfrac{\partial \psi}{\partial \gamma} & \dfrac{\partial \psi}{\partial \zeta} & \dfrac{\partial \psi}{\partial \sigma} \end{bmatrix} &= - \begin{bmatrix} \dfrac{\partial f_1}{\partial \rho} & \dfrac{\partial f_1}{\partial s} \\ \dfrac{\partial f_2}{\partial \rho} & \dfrac{\partial f_2}{\partial s} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{\partial f_1}{\partial \theta} & \dfrac{\partial f_1}{\partial \gamma} & \dfrac{\partial f_1}{\partial \zeta} & \dfrac{\partial f_1}{\partial \sigma} \\ \dfrac{\partial f_2}{\partial \theta} & \dfrac{\partial f_2}{\partial \gamma} & \dfrac{\partial f_2}{\partial \zeta} & \dfrac{\partial f_2}{\partial \sigma} \end{bmatrix} \\ &= -\frac{1}{F} \begin{bmatrix} \dfrac{\partial f_2}{\partial s} & -\dfrac{\partial f_1}{\partial s} \\ -\dfrac{\partial f_2}{\partial \rho} & \dfrac{\partial f_1}{\partial \rho} \end{bmatrix} \begin{bmatrix} \dfrac{\partial f_1}{\partial \theta} & \dfrac{\partial f_1}{\partial \gamma} & \dfrac{\partial f_1}{\partial \zeta} & \dfrac{\partial f_1}{\partial \sigma} \\ \dfrac{\partial f_2}{\partial \theta} & \dfrac{\partial f_2}{\partial \gamma} & \dfrac{\partial f_2}{\partial \zeta} & \dfrac{\partial f_2}{\partial \sigma} \end{bmatrix} \end{aligned}$$

for $\theta \in (0, 1)$, $\gamma > 0$, $\zeta > 0$, and $\sigma > R^{-1}$, where, in equation (15),

$$\begin{aligned} u'' &= u''(\bar{E}), \\ v''_\alpha &= v''_\alpha \left(\frac{\phi(\boldsymbol{a})}{\pi_\alpha} \right), \\ v''_\beta &= \left(\frac{R\psi(\boldsymbol{a})}{\pi_\beta} \right), \\ \bar{E} &= \frac{1}{\theta} [\zeta \omega - (\gamma + \zeta) \phi(\boldsymbol{a}) - (\gamma R + \zeta) \psi(\boldsymbol{a})]. \end{aligned}$$

Note that $\partial f_2 / \partial \sigma = 0$. Therefore,

$$\begin{aligned} \begin{bmatrix} \dfrac{\partial \phi}{\partial \theta} \\ \dfrac{\partial \psi}{\partial \theta} \end{bmatrix} &= -\frac{1}{F} \begin{bmatrix} \dfrac{\partial f_2}{\partial s} \dfrac{\partial f_1}{\partial \theta} - \dfrac{\partial f_1}{\partial s} \dfrac{\partial f_2}{\partial \theta} \\ -\dfrac{\partial f_2}{\partial \rho} \dfrac{\partial f_1}{\partial \theta} + \dfrac{\partial f_1}{\partial \rho} \dfrac{\partial f_2}{\partial \theta} \end{bmatrix} = -\frac{1}{F} \begin{bmatrix} \dfrac{\sigma \zeta R^2}{\theta \pi_\beta} \bar{E} u'' v''_\beta \\ \dfrac{\zeta}{\theta \pi_\alpha} \bar{E} u'' v''_\alpha \end{bmatrix} \in \mathbb{R}^2_{--}, \\ \begin{bmatrix} \dfrac{\partial \phi}{\partial \gamma} \\ \dfrac{\partial \psi}{\partial \gamma} \end{bmatrix} &= -\frac{1}{F} \begin{bmatrix} \dfrac{\partial f_2}{\partial s} \dfrac{\partial f_1}{\partial \gamma} - \dfrac{\partial f_1}{\partial s} \dfrac{\partial f_2}{\partial \gamma} \\ -\dfrac{\partial f_2}{\partial \rho} \dfrac{\partial f_1}{\partial \gamma} + \dfrac{\partial f_1}{\partial \rho} \dfrac{\partial f_2}{\partial \gamma} \end{bmatrix} = -\frac{1}{F} \begin{bmatrix} \dfrac{\sigma \zeta R^2}{\theta \pi_\beta} (\phi(\boldsymbol{a}) + R\psi(\boldsymbol{a})) u'' v''_\beta \\ \dfrac{\zeta}{\theta \pi_\alpha} (\phi(\boldsymbol{a}) + R\psi(\boldsymbol{a})) u'' v''_\alpha \end{bmatrix} \in \mathbb{R}^2_{--}, \\ \begin{bmatrix} \dfrac{\partial \phi}{\partial \sigma} \\ \dfrac{\partial \psi}{\partial \sigma} \end{bmatrix} &= -\frac{1}{F} \begin{bmatrix} \dfrac{\partial f_2}{\partial s} \dfrac{\partial f_1}{\partial \sigma} \\ -\dfrac{\partial f_2}{\partial \rho} \dfrac{\partial f_1}{\partial \sigma} \end{bmatrix} = -\frac{1}{F} \begin{bmatrix} -\zeta u'(\bar{E}) \left(\dfrac{\zeta(\gamma R + \zeta)}{\theta} u'' + \dfrac{R^2 v''_\beta}{\pi_\beta} \right) \\ \zeta u'(\bar{E}) \dfrac{\zeta(\gamma + \zeta)}{\theta} u'' \end{bmatrix} \in \mathbb{R}_{--} \times \mathbb{R}_{++}. \end{aligned}$$

It is easy to verify that

$$\frac{\dfrac{\partial \psi}{\partial \sigma}}{\dfrac{\partial \phi}{\partial \sigma}} = -\frac{\dfrac{\zeta(\gamma + \zeta)}{\theta} u''}{\dfrac{\zeta(\gamma R + \zeta)}{\theta} u'' + \dfrac{R^2}{\pi_\beta} v''_\beta} \in (-1, 0).$$

Now, assume further that $u'(E) + E u''(E) \leq 0$ for each $E > 0$. Then, it follows that

$$u'(\bar{E}) + \frac{1}{\theta} (\zeta \omega - \zeta \phi(\boldsymbol{a}) - \zeta \psi(\boldsymbol{a})) u''(\bar{E}) < u'(\bar{E}) + \bar{E} u''(\bar{E}) \leq 0.$$

We can therefore obtain that

$$\begin{bmatrix} \dfrac{\partial \phi}{\partial \zeta} \\ \dfrac{\partial \psi}{\partial \zeta} \end{bmatrix} = -\frac{1}{F}\begin{bmatrix} \dfrac{\partial f_2}{\partial s}\dfrac{\partial f_1}{\partial \zeta} - \dfrac{\partial f_1}{\partial s}\dfrac{\partial f_2}{\partial \zeta} \\ -\dfrac{\partial f_2}{\partial \rho}\dfrac{\partial f_1}{\partial \zeta} + \dfrac{\partial f_1}{\partial \rho}\dfrac{\partial f_2}{\partial \zeta} \end{bmatrix}$$

$$= -\frac{1}{F}\begin{bmatrix} -\dfrac{\sigma R^2}{\pi_\beta}\left[u'(\bar{E}) + \dfrac{\zeta}{\theta}(\omega - \phi(\boldsymbol{a}) - \rho(\boldsymbol{a}))u''(\bar{E})\right] v''_\beta \\ -\dfrac{1}{\pi_\alpha}\left[u'(\bar{E}) + \dfrac{\zeta}{\theta}(\omega - \phi(\boldsymbol{a}) - \rho(\boldsymbol{a}))u''(\bar{E})\right] v''_\alpha \end{bmatrix} \in \mathbb{R}^2_{++}.$$

Finally, the effects of parameters on z_t are determined through the equation that $z = \omega - \phi - \psi$. The effects of θ, γ, and ζ on z therefore have signs opposite to those on ϕ and ψ. In contrast, it holds that

$$\frac{\partial z}{\partial \sigma} = -\frac{\partial \phi}{\partial \sigma} - \frac{\partial \psi}{\partial \sigma} = -\frac{1}{F}\zeta u'(\bar{E})\left[\frac{\gamma\zeta}{\theta}(R-1)u'' + \frac{R^2}{\pi_\beta}v''_\beta\right] > 0.$$

This completes the proof of Proposition 3. Q.E.D.

Proof of Proposition 4. An interior golden rule optimal allocation is an interior stationary feasible allocation $(E, c^\alpha, c^\beta, s)$ maximizing $u(E) + v(c^\alpha)\pi_\alpha + v(c^\beta)\pi_\beta$. Under the current assumptions on utility index functions u and v, an interior golden rule optimal allocation $(E, c^\alpha, c^\beta, s)$ is without doubt characterized by the existence of some Lagrange mutiplier $\lambda \in \mathbb{R}$ such that

$$\begin{aligned} 0 &= u'(E) - \lambda\frac{\theta}{\zeta}, \\ 0 &= v'(c^\alpha)\pi_\alpha - \lambda\frac{\gamma+\zeta}{\zeta}\pi_\alpha, \\ 0 &= v'(c^\beta)\pi_\beta - \lambda\frac{\gamma+\zeta}{\zeta}\pi_\beta, \end{aligned}$$

and $s = \omega$, which establishes Proposition 4. Q.E.D.

To prove Proposition 5, we prepare a lemma.

Lemma 1 *There exists some $\sigma^{**} \in (R^{-1}, \bar{\sigma})$ such that $\Sigma(\sigma^{**}) = \bar{\sigma}$. Furthermore, such a σ^{**} is unique if Σ is strictly monotone increasing on its domain.*

Proof of Lemma 1. The continuity of Σ and the fact that $\Sigma(\bar{\sigma}) > \bar{\sigma}$ are easy to verify, so that we should verify that $\Sigma(R^{-1}) < \bar{\sigma}$. It holds that

$$\begin{aligned} &\bar{\sigma} - \Sigma(R^{-1}) \\ =\; & \frac{1}{\theta\zeta\dfrac{\partial\phi}{\partial\sigma}}\left[(\gamma + \zeta - R^{-1}\theta\zeta)\frac{\partial\phi}{\partial\sigma} + [\gamma R + (1-\theta)\zeta]\frac{\partial\psi}{\partial\sigma}\right] \\ =\; & -\frac{\zeta u'(\bar{E})}{F\theta\zeta\dfrac{\partial\phi}{\partial\sigma}}\left[(-\gamma - \zeta + R^{-1}\theta\zeta)\left[\frac{\zeta(\gamma+\zeta)u''(\bar{E})}{\theta} + \frac{R^2 v''(c^\beta)}{\pi_\beta}\right] + [\gamma R + (1-\theta)\zeta]\frac{\zeta(\gamma+\zeta)u''(\bar{E})}{\theta}\right] \\ =\; & -\frac{\zeta u'(\bar{E})}{F\theta\zeta\dfrac{\partial\phi}{\partial\sigma}}\left[\zeta^3(R^{-1}-1)u''(\bar{E}) + \frac{R^2 v''(c^\beta)}{\pi_\beta}\left[-\gamma - (1 - R^{-1}\theta)\zeta\right]\right] > 0, \end{aligned}$$

where F is as defined as in the proof of Proposition 3 (see also the equation (15)). Because Σ is a continuous function such that $\Sigma(R^{-1}) < \bar{\sigma} < \Sigma(\bar{\sigma})$, it follows from the intermediate value theorem that there is some $\sigma^{**} \in (R^{-1}, \bar{\sigma})$ such that $\Sigma(\sigma^{**}) = \bar{\sigma}$. Finally, it is straightforward to verify that σ^{**} is unique if Σ is strictly monotone increasing on its domain. Q.E.D.

Proof of Proposition 5. Suppose that Σ is strictly monotone increasing on its domain. As shown in Lemma 1, there is a unique σ^{**} such that $\Sigma(\sigma^{**}) = \bar{\sigma}$. Because Σ is strictly monotone increasing in σ and $\partial\phi/\partial\sigma$ is negative-valued (see Proposition 3 and Table 2), it holds that

$$\begin{aligned} W'(\sigma) &= -\left(\frac{\gamma+\zeta}{\theta}\frac{\partial\phi}{\partial\sigma} + \frac{\gamma R+\zeta}{\theta}\frac{\partial\psi}{\partial\sigma}\right)u'(\bar{E}) + \frac{\partial\phi}{\partial\sigma}v'\left(\frac{\phi(\boldsymbol{a})}{\pi_\alpha}\right) + \frac{\partial\phi}{\partial\sigma}Rv'\left(\frac{R\psi(\boldsymbol{a})}{\pi_\beta}\right) \\ &= \zeta u'(\bar{E})\frac{\partial\phi}{\partial\sigma}\left(\sigma - \frac{\gamma R+(1-\theta)\zeta}{\theta\zeta}\frac{\frac{\partial\psi}{\partial\sigma}}{\frac{\partial\phi}{\partial\sigma}} - \frac{\gamma+\zeta}{\theta\zeta}\right) \\ &= \zeta u'(\bar{E})\frac{\partial\phi}{\partial\sigma}(\Sigma(\sigma)-\bar{\sigma})\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\}0 \quad \text{if and only if} \quad \sigma\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}\sigma^{**}, \end{aligned}$$

where the last equality follows from equations (7) and (8). Therefore, σ^{**} maximizes W.

Finally, the equation (12) follows from strict monotonicity of Σ. This completes the proof of Proposition 5. Q.E.D.

Proof of Corollary 2. First, it holds that

$$\begin{aligned} \frac{\partial}{\partial\sigma}\left(\frac{\frac{\partial\psi}{\partial\sigma}(\boldsymbol{a})}{\frac{\partial\phi}{\partial\sigma}(\boldsymbol{a})}\right) &= -\frac{\theta\zeta(\gamma+\zeta)R^2}{\pi_\beta[\zeta(\gamma R+\zeta)u''(\bar{E})+\theta R^2v''(c^\beta)]^2}\left[u'''(\bar{E})v''(c^\beta)\frac{\partial\bar{E}}{\partial\sigma} - u''(\bar{E})v'''(c^\beta)\frac{\partial c^\beta}{\partial\sigma}\right] \\ &= -\frac{\theta\zeta(\gamma+\zeta)R^2}{\pi_\beta[\zeta(\gamma R+\zeta)u''(\bar{E})+\theta R^2v''(c^\beta)]^2}\{v'''(c^\beta)\}^2\frac{\partial}{\partial\sigma}\xi'(\sigma) < 0, \end{aligned}$$

where $\bar{E}$ is as defined as in the proof of Proposition 3 and $c^\beta = R\psi(\boldsymbol{a})/\pi_\beta$. Therefore, for each σ,

$$\Sigma'(\sigma) = 1 + \frac{\gamma R+(1-\theta)\zeta}{\theta\zeta}\frac{\theta\zeta(\gamma+\zeta)R^2}{\pi_\beta[\zeta(\gamma R+\zeta)u''(\bar{E})+\theta R^2v''(c^\beta)]^2}\{v'''(c^\beta)\}^2\frac{\partial}{\partial\sigma}\xi'(\sigma) > 0.$$

Then, the existence and the uniqueness of the second-best money growth rate follows from Proposition 5. Q.E.D.

Proof of Proposition 6. It follows from the equation (13) that

$$\begin{aligned} \frac{\partial\sigma^{**}}{\partial\theta} &= -\frac{\zeta\varepsilon(\gamma+\zeta)[\varepsilon+\pi_\beta(1-\varepsilon)]}{[\theta\zeta\varepsilon+(\gamma R+\zeta)\pi_\beta(1-\varepsilon)]^2} < 0, \\ \frac{\partial\sigma^{**}}{\partial\zeta} &= \frac{(1-\theta)\varepsilon}{[\theta\zeta\varepsilon+(\gamma R+\zeta)\pi_\beta(1-\varepsilon)]^2} > 0, \end{aligned}$$

and

$$\frac{\partial\sigma^{**}}{\partial\gamma} = \frac{[1+(R-1)\pi_\beta\varepsilon-(R-1)\pi_\beta]}{[\theta\zeta\varepsilon+(\gamma R+\zeta)\pi_\beta(1-\varepsilon)]^2}\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\}0 \quad \Leftrightarrow \quad \varepsilon\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\}\bar{\varepsilon}_1 \equiv \frac{(R-1)\pi_\beta}{1+(R-1)\pi_\beta}.$$

This completes the proof of Proposition 6. Q.E.D.

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