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FORMAL AND INTUITIVE RULES OF ORDINAL PROBABILITY IN CASE OF POSITIVE AND NEGATIVE EVENTS

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Abstract

The aim of this paper is to establish whether the rules of probability calculus are fulfilled in the intuitive evaluation of ordinal probability. The rules of probability calculus are based on the rules of propositional calculus, so we also study whether these rules are satisfied in intuitive reasoning. Two experiments were conducted. They are compared to explain the degree of conformity of mathematical rules of probability calculus with the intuitive, subjective probability in case of ordinal probability. In ordinal probability the probabilities are not known. Every person knows only how to order probabilities . We study ordinal probability for negative and positive events. Results of both experiments are similar. There is no conformity of mathematical rules of probability calculus with the intuitive rules of ordinal probability , especially in a case of rule of probability of a sum of events. It is important that respondents distinguish the sum and intersection of events in case of negative events but a lot of them do not do it in case of positive events.

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ABSTRACT

The aim of this paper is to establish whether the rules of probability calculus are fulfilled in the intuitive evaluation of ordinal probability. The rules of probability calculus are based on the rules of propositional calculus, so we also study whether these rules are satisfied in intuitive reasoning. Two experiments were conducted. They are compared to explain the degree of conformity of mathematical rules of probability calculus with the intuitive, subjective probability in case of ordinal probability. In ordinal probability the probabilities are not known. Every person knows only how to order probabilities . We study ordinal probability for negative and positive events. Results of both experiments are similar. There is no conformity of mathematical rules of probability calculus with the intuitive rules of ordinal probability , especially in a case of rule of probability of a sum of events. It is important that respondents distinguish the sum and intersection of events in case of negative events but a lot of them do not do it in case of positive events .

1. INTRODUCTION

The aim of this paper is to establish whether the rules of probability calculus are fulfilled in the intuitive evaluation of ordinal probability. The rules of probability calculus are based on the rules of propositional calculus, so we also study whether these rules are satisfied in the intuitive reasoning. Two experiments were conducted. They are compared to explain the degree of conformity of mathematical rules of probability calculus with the intuitive, subjective reasoning in case of ordinal probability. The probabilities are not known in case of ordinal probability. Every person can only compare probabilities saying which one is higher or lower than other . So, each person knows only how to order probabilities . People very often deal with such situations in real life for example when they compare financial investments. . We consider a case when they use their subjective probability. We study ordinal probability for negative and positive events. As negative we treat uncertain events which can be a threat for someone. In our research we use diseases as negative events. Positive events improve someone's situation. We treat some uncertain events such as being paid well or finding a good spouse as positive events.

It is known that the rules of propositional calculus are not always fulfilled in case of intuitive reasoning. Implication is studied in the famous Wason selection problem (Wason, 1966, Johnsson-Laird 1999) . Kahneman and Tversky (1982) studied a situation called the Linda problem, where people thought that it was more probable that a girl called Linda worked in a bank and was a feminist than that she worked in a bank. So, they though that intersection of two events concerning Linda was more probable than one of these events. This result is also known as a conjunction fallacy and was studied by many authors in many situations.These results are also presented in some mathematical logic handbooks (see Chiswell, Hodges, 2007). In this paper we try experimentally to verify whether the rules of probability calculus are fulfilled in a case of ordinal probability . Because the rules of probability calculus are based on the rules of propositional calculus we also verify whether the rules of propositional calculus are fulfilled in case of ordinal probability.

In our research we deal with intersection and sum of events. In our experiments 6 groups of students of good Polish economic universities were examined. A lot of respondents put a sum of events on the probability scale incorrectly in relation to these events. Moreover, a significant part of respondents order a sum and intersection of events incorrectly in case of positive events.

The paper is constructed as follows. The experiments are described in Section 2. The results are presented in Section 3. Conclusions and plans of future research are formulated in Section 4. The questionnaires are included in the Appendix.

2.EXPERIMENTS

In this section we shall describe questionnaire experiments concerning ordinal probability[1]. The respondents ordered given events depending on probability. They filled a special questionnaire (see Appendix). They constructed a subjective scale of probability by putting the most probable event on the top of the scale, then less and less probable events and ending with the least probable event . Then, they put complementary events, an intersection and a sum of events on the same scale.

Two experiments were conducted on 6 groups of students. We deal with 2 groups of students in case of negative, uncertain events and 4 in case of positive uncertain events . The names of the groups depend on the students major. In case of two groups with the same major their names are numbered. The groups were chosen in such a way that they were disjoint.

We consider serious diseases as negative events. The choice of the diseases was based on a special survey conducted among students . The survey revealed which diseases the students were afraid of. Analogically, in case of positive events the students were asked which positive uncertain events were important for them. The most popular were put in the questionnaire. In case of negative events medical definitions of diseases were not used and it was not expected that respondents would use them. In both cases the students knew neither a strict definition nor the exact probability of the events. They used only their subjective relation „lower-higher probability". The aim of the experiment was to compare the subjective rules of ordinal probability with formal rules. Respondents were not informed that there were proper answers and that their results would be better if they were closer to these answers. Respondents were not paid in order not to cause a situation where some answers were better than others. All answers were good for this experiment because all can be compared with the answers resulting from formal rules.

The experiments were conducted in the following way. At first, the case of negative events was analyzed in "Quantitative Methods II" group (Questionnaire I, Autumn 2006), then after a couple of months in "Psychology II" group (Questionnaire II, Spring 2007). Later, the

[1] In Polish version a term "ordinal risk" was used interchangeably with the term „ordinal probability. It was caused by the fact that the word "risk" in Polish can also mean an undertaking with uncertain result. So, term "risk" may refer to both negative and positive events. In English version more suitable term "ordinal probability" is used but the term "ordinal risk" was left in questionnaires to make them more similar to Polish original.

positive events were analyzed in 4 groups “Sociology”, “Management”, “Psychology I, “Quantitative Methods I” (Questionnaire III, Spring 2008) almost at the same time.
The groups differed in mathematical background. “Quantitative Methods II” group was the best from the mathematical point of view, “Quantitative Methods I” group was the second best. Then “Management”, “Psychology II” and “Psychology I” and “Sociology” at the end.

2.1. NEGATIVE EVENTS

The first experiment was conducted on two groups of students. The questionnaire concerned a risk of going down with selected diseases. Two versions of the questionnaire were used (see Appendix, Questionnaire I, Questionnaire II). The form of both versions is the same. The versions differ only in one disease. “Broken limb” replaced “leukemia”. This change was suggested during the discussion of the results of the first experiment in which Questionnaire I with “leukemia” was used. Disputants suggested to include a disease which occurs often among young people.

(i) Group “Quantitative Methods II”.

The experiment was conducted in Autumn 2006 on a group of 119 undergraduate students of Warsaw School of Economics. They were students of the first year majoring in quantitative methods. School leaving exam in mathematics on extended level is needed in the recruitment process at Warsaw School of Economics. The experiment was conducted during an optional lecture on logic. The students knew propositional calculus and quantifiers. They also had lectures on mathematics. Questionnaire I was used. The experiment was described in Polish in my paper (Sosnowska,2008a). Warsaw School of Economics is the best Polish economic university.

(ii) Group “Psychology II”.

The experiment was conducted in Spring 2007 on a group of 30 undergraduate students of Leon Koźmiński Academy. They were students of the first year majoring in management psychology. School leaving exam on mathematics is not needed in the recruitment process at this university. The experiment was conducted during an obligatory lecture on logic. The students knew propositional calculus and quantifiers. They also had lectures on mathematics. Questionnaire I was used. The experiment was described in Polish in my paper (Sosnowska,2008b). Leon Koźmiński Academy is the best Polish non-state economic university.

2.2. POSITIVE EVENTS. EXPERIMENT III.

I presented my results concerning intuitive and formal rules of ordinal probability over negative events at the conference on economic psychology in spring 2008 in Wrocław , Poland. The results concerned a case of negative events. During the discussion a question whether similar results could be obtained in case of positive events arose. The second experiment was conducted to answer this question. Four groups of students were examined . The number of groups resulted from their small size and intention to examine students who studied neither mathematics nor logic during the time of the experiment. Questionnaire III was used. The questionnaire was prepared in the same way as the previous questionnaires. The diseases were replaced by selected positive, uncertain events.

(i) Group “Sociology”

The group consisted of 8 undergraduate students of Leon Koźmiński Academy. They were first year students and majored in sociology. The experiment was conducted during the obligatory lecture on logic. They knew propositional calculus and quantifiers. They did not attend lectures on mathematics.

(ii) Group “Management”

The group consisted of 9 undergraduate students of Leon Koźmiński Academy. They were first year students majoring in management. The experiment was conducted during an optional lecture on logic. They knew propositional calculus and quantifiers. They also attended lectures on mathematics.

(iii) Group “Psychology II”

The group consisted of 38 undergraduate students of Leon Koźmiński Academy. They were second year students majoring in management psychology. The experiment was conducted during an obligatory lecture on statistics. They had obligatory lectures on logic and mathematics during the previous year..

(iv) Group “Quantitative Methods I”

The group consisted of 27 graduate students of Warsaw School of Economics majoring in quantitative methods. The experiment was conducted during an optional lecture on mathematical economics. They attended a lecture on logic and many lectures on mathematics. It was the only group with a very good mathematical background.

3. INTUITIVE AND FORMAL RULES

In this section we present and analyze the results of experiments.

(i) Connections between probability of an intersection and probability of a sum of events

The analysis of the results of the first experiment shows that a significant part of respondents did not put a sum of events on the scale correctly (see Table 3, detailed description below). However, most of them put a sum and an intersection of events in the correct order - a sum of events as more probable than an intersection of events. We tried to justify the hypothesis that most of the respondents put a sum and an intersection of events in the correct way in case of positive events. The results are presented in Table 1. The correct answers are in the column 3, denoted by "$\wedge \leq \vee$", incorrect – in column 4, denoted "$\vee < \wedge$". The hypothesis was not confirmed in the case of groups "Sociology" and "Psychology I" with the worst mathematical background. Roughly speaking , they had trouble differentiating between a sum and an intersection of events . In other words they had trouble with difference between conjunction and disjunction of sentences.

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group	number of respondents	$\wedge \leq \vee$	$\vee < \wedge$
1.sociology	8	4 (50%)	4 (50%)
2. management	9	6 (66%)	3 (33%)
3.psychology I	38	12 (31%)	26 (68%)
4. quantitative methods I	27	23 (85%)	4 (15%)

Table1. Connection between the probability of an intersection and the probability of a sum of events in case of positive events (a share result in brackets).

(ii) Comparing the probabilities of an intersection and a sum of positive events with the probability of these events

We check whether the respondents put an intersection and a sum of events correctly in relation to these events . The results concerning positive events are presented in Table 2. In column 3, denoted by $\wedge \leq ..., ...; \vee \leq ..., ...$, a situation where respondents put an intersection and a sum of events as less probable than these events (correct in case of sum) is presented. In column 4, denoted by $... < \wedge < ...; ... < \vee < ...$, a situation where respondents put an intersection and a sum of events between these events (incorrect in both cases) is presented. In column 5, denoted by $..., ... \leq \wedge; ..., ... \leq \vee$, a situation where respondents put an intersection and a sum of events as more probable than these events (correct in case of

sum) is presented. Only a few respondents put probability of a sum or an intersection of events as equal to probability of one of the events. Less than 50% of the respondents with worse mathematical background ("Sociology", "Management", "Psychology I") put a sum and an intersection of events incorrectly in relation to these events. Incorrect answers can be also found in the group with very good mathematical background ("Quantitative Methods I"). Incorrect answers may be explained by the Linda problem (Kahneman, Tversky,1982; Chiswell, Hodges, 2007) in case of an intersection of events. The results concerning a sum of events may suggest that the concept of a sum of events (that is also disjunction of sentences) is not intuitive . It may be confirmed by the fact that some of my colleagues (mathematicians specialized in the probability theory) filled in the questionnaires putting a sum of events incorrectly on the probability scale.

group-positive	number of respondents	$\wedge \leq$ ..., ...	... $< \wedge <$ ...	..., ... $\leq \wedge$
1. sociology	8	3 (37%)	4 (50%)	1 (12%)
2. management	9	1 (11%)	6 (66%)	2 (22%)
3.psychology I	38	2 (5%)	20 (53%)	16 (42%)
4. quantitative methods I	27	13 (48%)	10 (37%)	4 (15%)
group - positive	number of respondents	$\vee \leq$ ..., ...	...$< \vee <$ ...	..., ... $\leq \vee$
1. sociology	8	3 (37%)	5 (62%)	0
2. management	9	4 (44%)	3 (33%)	2 (22%)
3. psychology I	38	12 (31%)	18 (47%)	8 (21%)
4. quantitative methods I	27	2 (7%)	10 (37%)	15 (55%)

Table 2. Comparing the probabilities of an intersection and a sum of positive events with the probability of these events. Share result in brackets.

(iii) Comparing the probabilities of an intersection and a sum of negative events with the probability of these events

Results of analogical analysis for negative events are presented in Table 3. Let us note that contrary to the case of positive events most of the respondents put an intersection of events correctly, while results concerning a sum of events are similar to the case of positive events. Most of the respondents put a sum of events incorrectly. Lower correctness in case of positive events may be explained by the fact that people treat negative events more seriously because they feel threatened. Wrong prediction of probability in case of positive events may only result in wasting a chance. It is not a real waste and it does not decrease the actual situation of the respondents. Another explanation is the fact that when considering positive events the respondents felt more optimistic and they did not attach importance to logic. Conversely, considering negative events they felt depressed (see Table 4 – in more than 50% of answers the probability of being well paid is higher than the probability of not being well paid ; similarly in more than of 50% answers the probability of going down with depression is higher than the probability of not going down with it).

group-negative	number of respondents	$\wedge \leq ..., ...$	$... < \wedge < ...$	$..., ... \leq \wedge$
1. quantitative methods II	119	100 (84%)	15 (13%)	4 (3%)
2. psychology II	28	25 (89%)	1 (4%)	2 (7%)
group – negative	number of respondents	$\vee \leq ..., ...$	$...< \vee < ...$	$..., ... \leq \vee$
1. quantitative methods II	119	41 (35%)	35 (30%)	41 (35%)
2. psychology II	26	15 (57%)	7 (27%)	4 (15%)

Table 3. Comparing the probabilities of an intersection and a sum of negative events with the probability of these events. Share result in brackets.

(iv) Order of events and complementary events

Connections between an event and a complementary event are presented in Table 4. Positive events are described by groups 1-4. Most of the respondents were optimistic and thought that

it was more probable that they would be well paid than that they would not be paid well. The results concerning winning on a lottery (column 4) show that the respondents were rational and did not predict an event with a very low probability. Three last columns concern negative events. The percentage of predictions of diseases is higher than the incidence rate. This result may be explained as an example of overestimation of rare events (Berstein, 1996). Another explanation is depressive atmosphere caused by dealing with possibility of going down with serious diseases (see the end of part (iii) in this section).

group	number of respondents	$\neg Z \leq Z$ positive	$\neg W \leq W$ positive	$\neg D \leq D$ negative	$\neg B \leq B$ - negative	$\neg ZK \leq ZK$ negative
1.sociology	8	4 (50%)	2 (25%)	*	*	*
2.management	9	7 (77%)	2 (22%)	*	*	*
3.psychology I	38	33 (87%)	5 (13%)	*	*	*
4. quantitative methods I	27	26 (96%)	1 (3%)	*	*	*
5.quantitative methods II	119	*	*	75 (64%)	23 (19%)	*
6.psychology II	30	*	*	22 (73%)	*	25 (85%)

Table 4. Order of event and complementary event. Share results in brackets. Notation : Z – I shall be well paid, W – I shall win 100.000 zlotys on a lottery, D – I shall go down with depression, B – I shall go down with leukemia, ZK – I shall break a limb.

4. CONCLUSION AND FUTURE RESEARCH

We can formulate the following conclusion: the intuitive, subjective rules of probability calculus in cases of ordinal probability differ from the formal rules. The respondents without special mathematical background may have trouble with differentiating between an intersection and a sum of events. Most respondents, even with good knowledge of

mathematics put a sum of events in relation to these events incorrectly. Rare events are overestimated.

Charnes, Karni and Lewin (2008) conducted an experiment where the respondents could cooperate comparing probabilities of events and their intersection. Cooperation improves correctness of results. I am planning to repeat the experiments described in this paper for case of possible cooperation.

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APPENDIX

QUESTIONAIRE I

You are taking part in a research on ordinal risk. The research consists of two parts.

PART I

Order the following diseases depending on risk of the disease in lines with numbers 1 to 6. You start with the disease with the greatest risk, then with less and less risk and end with disease with the least risk.

DISEASES

AIDS, leukemia, depression, bird flu, cancer, schizophrenia

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1.....

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2.....

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3.....

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4.....

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5.....

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6.....

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PART II

Put the following events depending on risk of such situation in lines without numbers.

a) I shall not have leukemia
b) I shall not have depression
c) I shall have leukemia and depression
d) I shall have AIDS or cancer

QUESTIONAIRE II

You are taking part in a research on ordinal risk. The research consists of two parts.

PART I

Order the following diseases depending on risk of the disease in line with numbers 1 to 6.. You start with the disease with the greatest risk, then with less and less risk and end with disease with the least risk.

DISEASES

AIDS, broken limb, depression, bird flu, cancer, schizophrenia

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1.....

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2.....

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3.....

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4.....

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5.....

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6.....

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PART II

Put the following events depending on risk of such situation in lines without numbers.

a)I shall not break a limb

b)I shall not have depression

c) I shall have AIDS and cancer

d) I shall have AIDS or cancer

QUESTIONAIRE III

ORDINAL RISK – POSITIVE EVENTS

major....., university .....

You are taking part in an experiment on an ordinal risk. The experiment consists on two parts.

PART I.

Positive, uncertain events are listed below. Put them in lines with numbers to order them from the most probable (row 1) to the least probable (row 6). We consider the following events.

Husband/wife – I shall find a good spouse

Travels – I shall travel a lot

Car – I shall have a good car

Higher education – I shall graduate with good results

Win – I shall win 100 000 zlotys on a lottery

Earnings – I shall be paid well

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1.....

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2.....

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3.....

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4.....

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5.....

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6.....

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PART II

Put the following events in lines without numbers

(a)I shall not win 100 000 zlotys on a lottery

(b) I shall not be paid well

(c) I shall have a good car and I shall graduate with good results

(d) I shall have a good car or I shall graduate with good results