TCER Working Paper Series

Optimal redistributive policy under disaster risk: self-protection, social mitigation and social adaptation

Shuichi Tsugawa

March 2024

Working Paper E-203
https://www.tcer.or.jp/wp/pdf/e203.pdf

TOKYO CENTER FOR ECONOMIC RESEARCH
1-7-10-703 Iidabashi, Chiyoda-ku, Tokyo 102-0072, Japan

©2024 by Shuichi Tsugawa.
All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including ©notice, is given to the source.

Abstract

In this paper, we examine the optimal mixed taxation of polluting goods and subsidies for self-protection under nonlinear income tax. The novel contribution of this paper is that we take into account disaster risk, the probability of which is determined by the total amount of polluting goods consumed by all individuals. We derive the properties of optimal allocations in the first-best and second-best scenarios, and the tax wedges. Additionally, we obtain the optimal tax scheme in cases in which the government cannot observe each individual's consumption of polluting goods. The optimal tax rate on polluting goods includes the Pigouvian term and the screening term under asymmetric information, and the optimal subsidy rate on goods for self-protection is discriminatory, which reflects that screening term. Additionally, we consider public expenditure aimed at reducing losses incurred as a result of disasters, in addition to other fiscal policies in an asymmetric information setting, and find that the optimal level is determined by a modified Samuelson rule that includes the screening term between households.

Shuichi Tsugawa
Ryukoku University
Faculty of Economics
67 Tsukamoto-cho, Fukakusa Fushimi-ku,
Kyoyo, 612-8577 Japan
tsugawa.shuichi@office.ryukoku.ac.jp

Optimal redistributive policy under disaster risk: self-protection, social mitigation and social adaptation

Shuichi Tsugawa*

March 26, 2024

Abstract

In this paper, we examine the optimal mixed taxation of polluting goods and subsidies for self-protection under nonlinear income tax. The novel contribution of this paper is that we take into account disaster risk, the probability of which is determined by the total amount of polluting goods consumed by all individuals. We derive the properties of optimal allocations in the first-best and second-best scenarios, and the tax wedges. Additionally, we obtain the optimal tax scheme in cases in which the government cannot observe each individual's consumption of polluting goods. The optimal tax rate on polluting goods includes the Pigouvian term and the screening term under asymmetric information, and the optimal subsidy rate on goods for self-protection is discriminatory, which reflects that screening term. Additionally, we consider public expenditure aimed at reducing losses incurred as a result of disasters, in addition to other fiscal policies in an asymmetric information setting, and find that the optimal level is determined by a modified Samuelson rule that includes the screening term between households.

JEL Classification: D62, H21, Q54

Keywords: Optimal taxation, externalities, self-protection, adaptation, asymmetric information

*Ryukoku University; tsugawa.shuichi@office.ryukoku.ac.jp

1 Introduction

Over the past few decades, numerous papers have been published in public economics and environmental economics journals about optimal mixed taxation under externalities. The studies have examined scenarios in which tax authorities pursue redistributive objectives using both nonlinear labor income taxation and linear commodity taxation under consumption externalities and asymmetric information between the government and consumers. Some of the most relevant contributions have been provided by Cremer et al. (1998) and Micheletto (2008). In these studies, the authors found that optimal commodity tax rates have additive properties, which means that the term ”correcting externalities,” or the Pigouvian term, is included in the tax system proposed by Sandmo (1975).

Additionally, in the field of environmental economics, there has been interest in ”mitigation-adaptation” and ”self-protection” policies in relation to climate change, whereby individuals purchase self-protective insurance; or policymakers either tax them or provide subsidies, or they reduce disaster risk by regulating economic activities that generate negative externalities (i.e., mitigation) or building infrastructure to protect urban areas from climate-related disasters. The global increase in greenhouse gases has increased disaster risk, with global warming already causing increased flooding and wildfires. Firms, households, and governments aim to protect themselves from natural disasters. Thus, as a mitigation strategy, governments encourage economic agents to reduce their consumption of polluting goods while building or improving infrastructure aimed at protecting the community from climate-change-induced natural disasters. In particular, governments play an important role in two schemes. To reduce disaster risks in advance, they levy environmental taxes on polluting goods, and they

I would like to thank Masayoshi Hayashi, Takuya Obara, Hikari Ogawa, Yoshihiro Takamatsu, Jun Yoshida, and all participants in seminars and conferences at Kyoto University, Kyushu University, University of Tokyo, Hokkaido University, the 78th Annual Congress of the Japan Institute of Public Finance, the 2022 International Conference on Public Economic Theory (PET 2022), the 22nd Journees Louis-André Gérard-Varet (LAGV), and the J-TREE Regional Workshop in Sendai. This work was supported by financial aid from Kyushu University Platform of Inter/Transdisciplinary Energy Research, Okinawa University, Ryukoku University, the Tokyo Center for Economic Research, and JSPS KAKENHI Grant Number JP23K12496.

build and maintain infrastructure for the purpose of protecting the community from natural disasters. In this paper, I call these climate-related controls ”social mitigation and adaptation.” In addition to these two forms of policy intervention, governments subsidize spending on self-protection measures.

In the IPCC’s Fifth Assessment Report AR5 (2014), edited by Edenhofer (2015), it was noted that, from the point of view of economics, mitigating the risk of disaster risks was a type of public good, while dealing with the actual damage was a type of private good. For instance, the effects of reducing greenhouse gases involve every economic agent all over the world, reducing disaster risks globally. Conversely, building river levees and maintaining forests mitigates the damage that results from floods and forest fires for local residents. Additionally, social mitigation and adaptation, and subsidies for self-protection are related to fiscal policies. Regarding social mitigation, taxing polluting goods, such as petroleum, raises revenue for the government, whereas the other two environmental policies increase spending. In numerous studies in the field of economics, researchers have examined government strategies aimed at mitigation and adaptation in the context of public good provision and strategic interactions, with some of the earlier studies explaining the fiscal impact of these green policies.[1]

Specifically, we analytically derive optimal commodity taxes on polluting goods and subsidies for self-protection with nonlinear labor income taxes in a two-class economy. Each household has a different wage level and experiences a different degree of damage as a result of disasters, and information asymmetries exist between households and the government with regard to their wage and the extent of the damage. In contrast to Cremer et al. (1998) and Micheletto (2008), we explicitly introduce the probability of a disaster occurring, which is determined by the total amount of polluting goods consumed by individuals.

In this paper, we show that the tax authority should impose taxes on polluting goods in the form of Pigouvian taxes, or, to correct for externalities. Moreover, we consider the second-best setting and show that it should develop an optimal subsidy schedule whereby low-productivity households face positive marginal subsidy rates which result in their being willing to buy more self-protective goods in the second-best setting. Additionally, we show

[1] For instance, Zehaie (2009) analyzed governments’ strategic impact on the timing of adaptation and mitigation measures. Regarding fiscal policy, Barrage (2020b) and Barrage (2020a) used the DICE model to examine the optimal carbon tax policy in a dynamic setting. The model used in this paper differs from that model in that it uses a simple static setting.

that the optimal linear commodity tax rate on polluting goods is composed of a Pigouvian term and a screening term when it can only observe their aggregate demand for polluting goods.

Furthermore, we embed public expenditure for infrastructure aimed at reducing losses from disasters into the above redistributive policy model under asymmetric information. In addition to providing subsidies for self-protection, this expenditure reduces the amount of damage from natural disasters. This public spending is a type of pure public good; thus, the government sets the level in conjunction with the other optimal fiscal policies. Based on this analysis, the optimal level of public expenditure is determined by the modified Samuelson rule. This means that manipulating the level of public expenditure plays a role in screening low-skilled and high-skilled households, thereby enabling a relaxation of the incentive constraint, which is analogous to the mechanics of linear commodity tax under the observation of aggregate demand for polluting goods.

1.1 Related Literature

In previous relevant studies, researchers have focused on optimal taxation with environmental externalities. Sandmo (1975) conducted a seminal study on optimal commodity taxation with externalities that derived the optimum commodity tax rate on polluting goods. This invoked the inverse elasticity rule with a Pigouvian term, which corrects for the externalities. Cremer et al. (1998) and Micheletto (2008) studied the optimal commodity tax rate on polluting goods under nonlinear labor income taxation. Cremer et al. (1998) studied both nonlinear and linear commodity taxation under nonlinear labor income taxation, and showed that commodity taxation only incorporates the Pigouvian term and not the screening term. Micheletto (2008) extended their model in terms of the different externalities that individuals encounter. The novel contribution of this study is the introduction of disaster risk as determined by the total amount of consumption.[2]

Regarding the optimal nonlinear income taxation, Mirrlees (1971), Stern (1982), and

[2]Treich and Yang (2021) studied the level of public safety under linear labor income taxation. This level determines the agents' survival rates; thus, the model is similar to my model. However, the difference is that my survival (or disaster) probability is determined by the total consumption of polluting goods, whereas the survival probability of Treich and Yang (2021) depends on the level of public safety or public goods provided by the government.

Stiglitz (1982) examined the optimal nonlinear labor income taxation by applying the principal-agent model to labor income taxation, and assumed that there was asymmetric information regarding individuals' productivity between the government and those agents. In particular, following Stern (1982) and Stiglitz (1982), we employ nonlinear income taxation where agents are classified into one of two types: high productivity or low productivity. We also consider the optimal taxation on polluting goods and subsidies for self-protection. Therefore, this study is related to previous studies in which researchers examine optimal mixed taxation, such as Atkinson and Stiglitz (1976).

Regarding self-insurance and self-protection, there are numerous previous studies such as Ehrlich and Becker (1972) and Lohse et al. (2012). However, this study is the first to investigate the fiscal policies for self-protection, social mitigation, and social adaptation under nonlinear labor income taxation.[3]

The remainder of this paper is organized as follows. In Section 2, we present the basic model and examine the laissez-faire and the first-best allocation. In Section 3, we examine the optimal tax and subsidy rules in the second-best setting. In Section 4, we extend the model to the case of linear commodity taxation, and in Section 5, we introduce public expenditure for eliminating losses as a result of disasters. In Section 6, I discuss several points as to results obtained in the preceding sections, and we conclude the paper in Section 7.

2 The Model

We consider a two-class economy in which each agent ($i = H, L$) possesses ability or unit wage w_i, where $w_H > w_L > 0$[4]. We assume that the total population in this economy equals one and the fraction of agents with w_H is denoted by $n \in (0, 1)$. The various abilities are private information for agents, who earn labor income through their labor supply l.

In this economy, agents consume polluting goods denoted by x, in addition to numeraire c. Let x_i be the consumption level of agent i and the total amount of consumption $E =$

[3] Golosov et al. (2014) and Barrage (2020b) studied dynamic optimal carbon and other taxation under climate change. The main difference between our model and these two papers is to consider the subsidies for self-protection or not.

[4] In this paper, we employ the optimal nonlinear income tax model under the two-class economy in Stiglitz (1982).

$nx_H + (1-n)x_L$ affects the probability p of disasters occurring; hence, the odds can be written as a function of E: $p(E)$. We assume that for any $E \in \mathbb{R}$, $p(E) \in (0, 1)$, and p is strictly increasing and differentiable. With probability $p(E)$, agents incur cost $\phi(D)$, where D is the degree of damage and ϕ is strictly increasing, strictly convex, and differentiable. To protect their assets from such disasters or mitigate the damage, agents can spend s on self-protection. When an agent i pays s_i, the individual can eliminate cost $\phi(D - s)$.

All individuals differ in terms of the level of damage incurred D_i and productivity w_i. In this model, there are two types of profile regarding ability and the degree of damage: (w_H, D_H) and (w_L, D_L), with $w_H > w_L$. There are two possible cases regarding D: positive correlation $D_H > D_L$ and negative correlation $D_H < D_L$. Agent i's expected utility is given by

$$U(c_i, x_i) - p(E)\phi(D_i - s_i) - h(l_i), \tag{1}$$

where $U(c_i, x_i)$ is the utility on numeraire c_i and polluting goods x_i, and $h(l_i)$ is the disutility of labor supply l_i. We assume that U is strictly concave and twice differentiable, and that h is strictly increasing, strictly convex, and differentiable. Individual i can purchase polluting goods x_i and self-protection s_i at prices q_x and q_s, in the competitive markets respectively. Additionally, agent i selects labor supply l_i that determines pretax income $I_i = w_i l_i$. Thus, this is equivalent to choosing pretax income I_i.

2.1 Individual's optimization

To examine the individual's optimization without taxation, I formulate the problem as follows:

$$\begin{aligned} &\max_{c_i, x_i, s_i, I_i} U(c_i, x_i) - p(E)\phi(D_i - s_i) - h(\frac{I_i}{w_i}) \\ &\text{s.t. } \ c + q_x x_i + q_s s_i \leq I_i. \end{aligned} \tag{2}$$

Let θ be the Lagrangian multiplier. Then, the first-order conditions (FOCs) are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_i} &= U_c(c_i, x_i) - \theta = 0 \\ \frac{\partial \mathcal{L}}{\partial x_i} &= U_x(c_i, x_i) - \theta q_x = 0 \\ \frac{\partial \mathcal{L}}{\partial s_i} &= p(E)\phi'(D_i - s_i) - \theta q_s = 0 \\ \frac{\partial \mathcal{L}}{\partial I_i} &= -\frac{1}{w_i} h'(\frac{I_i}{w_i}) + \theta = 0, \end{aligned} \tag{3}$$

where U_c and U_x are the derivatives of U with respect to c and x, respectively. It is worth noting that individuals cannot manipulate the total amount of polluting goods E because they are atomless [5]. With respect to the consumption of polluting goods x, self-protection s, and pretax income or labor supply I, individual i's marginal rates of substitution for the numeraire are given by

$$MRS^i_{xc} = \frac{U_c(c_i, x_i)}{U_x(c_i, x_i)} = q_x \quad MRS^i_{sc} = \frac{p(E)\phi'(D_i - s_i)}{U_c(c_i, x_i)} = q_s \quad MRS^i_{Ic} = \frac{\frac{1}{w_i} \times h'(\frac{I_i}{w_i})}{U_c(c_i, x_i)} = 1.$$

All marginal rates of substitution equal these prices of goods x and s or 1. Therefore, to achieve a desirable allocation, the government should set the taxation schedule with tax wedges to cater for the optimum conditions.

2.2 First-best allocation

First, I consider the first-best allocation. In this paper, a government maximizes the weighted sum of all individuals' utilities $U(c_H, x_H) + \mu U(c_L, x_L)$, where $\mu (> 0)$ represents the redistribution preference. If $\mu > 1$, the tax authority prefers redistribution from the rich to the poor; otherwise, the preference is to redistribute from the poor to the rich. In the first-best scenario, the allocation $(c_i, x_i, s_i, I_i)_{i=H,L}$ is observable; thus, the tax authority considers the following

[5] On this point, in this paper, we follow Cremer et al. (1998). Conversely, if we assume that the total population is finite and countable, then each agent can affect E through his or her consumption of polluting goods x.

optimization problem:

$$\max_{\{c_i,x_i,s_i,I_i\}_{i=H,L}} U(c_H, x_H) + \mu U(c_L, x_L) - p(E)\{\phi(D_H - s_H) + \mu\phi(D_L - s_L)\} - \{h(\frac{I_H}{w_H}) + \mu h(\frac{I_L}{w_L})\}$$
$$\text{s.t. } n(I_H - c_H - q_x x_H - q_s s_H) + (1-n)(I_L - c_L - q_x x_L - q_s s_L) \geq 0. \tag{4}$$

Let $\mathcal{L}^{FB}$ be the Lagrangian and λ be the multiplier. The FOCs are [6]

$$\begin{aligned}
\frac{\partial \mathcal{L}^{FB}}{\partial c_H} &= U_1^H - \lambda n = 0, \quad \frac{\partial \mathcal{L}^{FB}}{\partial c_L} = \mu U_1^L - \lambda(1-n) = 0 \\
\frac{\partial \mathcal{L}^{FB}}{\partial x_H} &= U_2^H - np'(E)(\phi^H + \mu\phi^L) - \lambda n q_x = 0 \\
\frac{\partial \mathcal{L}^{FB}}{\partial x_L} &= \mu U_2^L - (1-n)p'(E)(\phi^H + \mu\phi^L) - \lambda(1-n)q_x = 0 \\
\frac{\partial \mathcal{L}^{FB}}{\partial s_H} &= p(E)\phi'(D_H - s_H) - \lambda n q_s = 0 \\
\frac{\partial \mathcal{L}^{FB}}{\partial s_L} &= \mu p(E)\phi'(D_L - s_L) - \lambda(1-n)q_s = 0 \\
\frac{\partial \mathcal{L}^{FB}}{\partial I_H} &= -\frac{1}{w_H}h'(\frac{I_H}{w_H}) + \lambda n = 0 \\
\frac{\partial \mathcal{L}^{FB}}{\partial I_L} &= -\frac{\mu}{w_L}h'(\frac{I_L}{w_L}) + \lambda(1-n) = 0.
\end{aligned}$$

Rearranging the FOCs, I obtain the following expressions for each marginal rate of substitution, for all $i = L, H$:

$$\begin{aligned}
MRS^i_{xc} &= q_x + \frac{p'(E)(\phi^H + \mu\phi^L)}{\lambda} \\
MRS^i_{sc} &= q_s \\
MRS^i_{Ic} &= 1.
\end{aligned} \tag{5}$$

As observed in related studies such as Sandmo (1975), the government takes into consideration correcting the externalities associated with the consumption of polluting goods, whereas it does not have to intervene in the self-protection market. Additionally, in terms of labor supply or pretax income for class L, the government achieves the allocation without taking into consideration the redistribution weights.

[6] Because the government can determine the allocation of all individuals' consumption bundles, it can manipulate E.

2.3 Tax wedges of first-best allocation

From the point of view of the government, it can observe each agent i's (pretax) labor income I_i, but not the labor supply. Additionally, it can check the consumption levels of polluting goods x_i and self-protection s_i. Hence, it sets nonlinear differentiable taxation $T(x_i, s_i, I_i)$ depending on the profile.

Under this tax schedule, each individual i has the following budget constraint:

$$c_i + q_x x_i + q_s s_i \leq I_i - T(x_i, s_i, I_i).$$

To achieve the best allocation, the marginal tax rates must satisfy, for all $i = L, H$:

$$\frac{\partial T(x_i, s_i, I_i)}{\partial x} = \frac{p'(E)(\phi^H + \mu\phi^L)}{\lambda}$$
$$\frac{\partial T(x_i, s_i, I_i)}{\partial s} = \frac{\partial T(x_i, s_i, I_i)}{\partial I} = 0.$$

The key point is that the tax authority should set the tax rate for polluting goods x at an appropriate level to correct for the externalities. This includes the marginal probability of the disaster occurring $p'(E)$ multiplied by the social damages $\phi^H + \mu\phi^L$ divided by the Lagrangian multiplier for tax revenue λ. This represents the marginal social cost of consuming polluting goods.

3 Second-best allocation

In this section, we still assume that all allocations are publicly observable, although the government can no longer observe the individuals' characteristics. In this setup, a new self-selection constraint needs to be considered similar to those of Stiglitz (1982) and Stern (1982).[7] This means that taxpayers with (w_H, D_H) have no incentive to mimic those with (w_L, D_L), and vice versa. Formally, this can be written as follows, for all $i, j = L, H$ with $i \neq j$:

$$U(c_i, x_i) - p(E)\phi(D_i - s_i) - h(\frac{I_i}{w_i}) \geq U(c_j, x_j) - p(E)\phi(D_i - s_j) - h(\frac{I_j}{w_i}).$$

Moreover, I assume that $D_H < D_L$, which enables us to ignore mimickers of (w_L, D_L), the self-selection constraint, because of the single-crossing property. In addition to reducing

[7] In some previous studies, such as Mirrlees (1971), this constraint was termed "incentive compatibility."

the technical burden, this assumption seems plausible because high-skilled workers tend to preempt disaster risks. For example, wealthier workers are able to purchase houses in areas that are less prone to damage from events such as floods.[8]

In this paper, we focus on the case in which $D_H < D_L$. For the second-best allocation, we formulate the following problem:

$$\max_{\{c_i,x_i,s_i,I_i\}_{i=H,L}} U(c_H,x_H)+\mu U(c_L,x_L)-p(E)\{\phi(D_H-s_H)+\mu\phi(D_L-s_L)\}-\{h(\frac{I_H}{w_H})+\mu h(\frac{I_L}{w_L})\}$$
$$\text{s.t. } n(I_H-c_H-q_x x_H-q_s s_H)+(1-n)(I_L-c_L-q_x x_L-q_s s_L)\geq 0$$
$$U(c_H,x_H)-p(E)\phi(D_H-s_H)-h(\frac{I_H}{w_H})\geq U(c_L,x_L)-p(E)\phi(D_H-s_L)-h(\frac{I_L}{w_H}). \tag{6}$$

I retain the notation λ as the multiplier of the resource constraint, and let δ represent that of the self-selection constraint. Solving this problem, I obtain the following expressions for each marginal rate of substitution:

$$MRS^L_{xc}=MRS^H_{xc}=q_x+\frac{p'(E)(\phi^H+\mu\phi^L)}{\lambda}$$
$$MRS^H_{sc}=q_s$$
$$MRS^L_{sc}=q_s+\frac{\delta p(E)}{\lambda(1-n)}(\phi'^{HL}-\phi'^{L})$$
$$MRS^H_{Ic}=1 \text{ and } MRS^L_{Ic}<1,$$

where $\phi^k=\phi(D_k-s_k)$ for each $k=H,L$ and $\phi^{HL}=\phi(D_H-s_L)$. Comparing first-best allocation and second-best allocation, I can identify two properties. First, this is the same allocation rule for polluting goods x in the sense that MRS^i_{xc} for any individual i; that is, information asymmetry does not distort the rule. Second, the allocation rule for self-protection is different from that for first-best allocation. In particular, those with (w_L,D_L) have a different allocation rule, where the marginal tax rate on s is lower under the negative correlation $D_L>D_H$. To summarize, we provide the following proposition.

Proposition 1. *Assume that the government can observe each individual's self-protection,*

[8] It is unclear whether there is any evidence that $D_L < D_H$, but it might be interesting to consider that case as a generalization. Additionally, this extension might be an application of the model of Bastani et al. (2020), but this is beyond the scope of this study.

pretax income, and demand for other consumption goods, but cannot observe their characteristics. When $D_H < D_L$, the second-best allocation rules must satisfy the following conditions:

1. *This is the same allocation rule as that for the first-best allocation (FB) for polluting goods x.*

2. *The allocation rule for self-protection differs from FB; in particular, the marginal rate of substitution for agents with (w_L, D_L) is lower than that in FB.*

Tax wedges of first-best allocation

To implement the second best allocation, the tax schedule must satisfy:

$$\frac{\partial T(x_i, s_i, I_i)}{\partial x} = \frac{p'(E)(\phi^H + \mu\phi^L)}{\lambda} \quad \text{for all } i = L, H$$
$$\frac{\partial T(x_H, s_H, I_H)}{\partial s} = \frac{\partial T(x_H, s_H, I_H)}{\partial I} = 0$$
$$\frac{\partial T(x_L, s_L, I_L)}{\partial s} = \frac{\delta p(E)}{\lambda(1-n)}\{\phi'^{HL} - \phi'^{L}\} < 0$$
$$\frac{\partial T(x_L, s_L, I_L)}{\partial I} > 0.$$

4 Imperfect observation of polluting goods

Thus far, we have maintained the assumption that all consumption bundles are publicly observable. However, tax authorities all over the world have difficulty in observing all consumption bundles. For instance, governments find it difficult to monitor each agent's consumption of polluting goods such as fossil fuels. Conversely, there are several cases in which this assumption is not appropriate. For example, self-protection in the form of residential location or investment in infrastructure is publicly observable and cannot be hidden.

Following the above argument, the government cannot tax polluting goods based on individual consumption levels because of the administrative costs involved. Conversely, it can levy taxes on self-protection measures depending on the amounts involved. Thus, we assume that the government can only observe agents' self-protection measures and pretax income.

The government sets nonlinear taxation based on the profile of agent i's pretax income and self-protection $T(s_i, I_i)$, and linear taxation on polluting goods t_x. In this setup, individual

i's optimization problem can be summarized as follows:

$$\max_{c_i,x_i,s_i,I_i} U(c_i,x_i) - p(E)\phi(D_i - s_i) - h(\frac{I_i}{w_i})$$
$$\text{s.t. } c_i + (q_x + t_x)x_i \leq I_i - q_s s_i - T(s_i, I_i) \equiv B_i,$$

where B_i is the after-tax income after subtracting expenditure on self-protection. Because I_i and s_i are publicly observable for each agent i, whose utility is separable, the following indirect utility determined by $q_x + t_X$ and B_i can be deduced:

$$U(c_i,x_i) - p(E)\phi(D_i - s_i) - h(\frac{I_i}{w_i}) = V(q_x + t_x, B_i) - p(E)\phi(D_i - s_i) - h(\frac{I_i}{w_i}),$$

where $V(q_x + t_x, B)$ is the indirect utility function induced from the following utility maximization problem:

$$\max_{c_i,x_i} U(c_i,x_i)$$
$$\text{s.t. } c_i + (q_x + t_x)x_i \leq B_i.$$

Furthermore, let $c(q_x + t_x, B_i)$ and $x(q_x + t_x, B_i)$ be individual i's demand functions for the numeraire and polluting goods, respectively. I formulate this optimization problem as follows:

$$\begin{aligned}\max_{t_x,\{B_i,s_i,I_i\}_{i=H,L}} \; & V(q_x + t_x, B_H) + \mu V(q_x + t_x, B_L) - p(E)\{\phi(D_H - s_H) + \mu\phi(D_L - s_L)\} - \{h(\frac{I_H}{w_H}) + \mu h(\frac{I_L}{w_L})\} \\ \text{s.t. } \; & n(I_H - B_H - q_s s_H + t_x x(q_x + t_x, B_H)) + (1-n)(I_L - B_L - q_s s_L - t_x x(q_x + t_x, B_L)) \geq 0 \\ & V(q_x + t_x, B_H) - p(E)\phi(D_H - s_H) - h(\frac{I_H}{w_H}) \geq V(q_x + t_x, B_L) + p(E)\phi(D_H - s_L) + h(\frac{I_L}{w_H}).\end{aligned} \tag{7}$$

By solving this, I can derive the optimal tax schedule summarized by the following proposition.

Proposition 2. *Assume that the government can observe individuals' self-protection and pre-tax income, in addition to aggregate demand for other consumption goods. Retaining the assumptions made in relation to* ***Proposition 1****, the optimal tax rate on polluting goods t_x^* satisfies*

$$t_x^* = \frac{p'(E)\{\phi^H + \mu\phi^L - \delta(\phi^{HL} - \phi^H)\}}{\lambda}$$

The Lagrangian, FOCs, and calculation process are presented in the Appendix. Regarding the optimal tax rate on polluting goods t_x^*, first, the tax rate t_x^* is divided into two parts: the Pigouvian term $\frac{p'(E)(\phi^H+\mu\phi^L)}{\lambda}$ and screening term $-\frac{\delta p'(E)}{\lambda}(\phi^{HL} - \phi^H)$. The Pigouvian term is the same mechanism as that used in the previous scenario; thus, it corrects for the externalities caused by the consumption of polluting goods x. The tax rate t_x^* has a novel screening term, but this is similar to those that appeared in previous studies on mixed taxation, such as Cremer et al. (2001). If $D_H < D_L$ or $\phi^{HL} - \phi^H < 0$, there is an incentive to mimic the other type. Thus, to relax the constraint, the government increases the tax rate. From another perspective, to reduce the burden of revealing the true type, the government prevents individuals from consuming polluting goods.

5 Public expenditure on infrastructure for adaptation

In addition to individuals' self-protection, public expenditure on building or maintaining infrastructure such as dams can be considered as a form of adaptation. In this section, based on the second-best setup presented in Section 3, we consider the optimal level of public expenditure, in addition to other fiscal policies.

Let G be the amount of public expenditure aimed at reducing losses from natural disasters. In this case, the government can set both the subsidy s_i for each $i = L, H$ and public spending G. I consider the second best setting presented in Section 3. For this setting, agent i's expected utility is given by

$$U(c_i, x_i) - p(E)\phi(D_i - s_i - G) - h(\frac{I_i}{w_i}).$$

The government incurs public expenditure cost $F(G)$ where F is increasing, differentiable, and convex. The fiscal burden is financed by taxes on people's income and their consumption of polluting goods. Moreover, to guarantee the interior solution and self-selection constraint, it is necessary to assume that both D_L and D_H, with $D_L > D_H$, are sufficiently large.

I define this problem as follows:

$$\max_{\{c_i,x_i,s_i,I_i\}_{i=H,L},G} U(c_H,x_H)+\mu U(c_L,x_L)$$
$$-p(E)\{\phi(D_H-s_H-G)+\mu\phi(D_L-s_L-G)\}-\{h(\frac{I_H}{w_H})+\mu h(\frac{I_L}{w_L})\}$$
$$\text{s.t. } n(I_H-c_H-q_xx_H-q_ss_H)+(1-n)(I_L-c_L-q_xx_L-q_ss_L)\geq F(G) \quad (8)$$
$$U(c_H,x_H)-p(E)\phi(D_H-s_H-G)-h(\frac{I_H}{w_H})$$
$$\geq U(c_L,x_L)-p(E)\phi(D_H-s_L-G)-h(\frac{I_L}{w_H}).$$

By solving this problem, I obtain the following result as Proposition 3.

Proposition 3. *Assume that the government can finance public expenditure G to reduce the damage that results from natural disasters. Retaining the assumptions made in relation to **Proposition 1**, the optimal tax schedule $T(x,s,I)$ satisfies the same conditions as those stated in **Proposition 1**. Moreover, the optimal provision rule satisfies*

$$n\times\frac{p(E)\phi'^H}{U_1^H}+(1-n)\times\frac{p(E)\phi'^L}{U_1^L}+\frac{\delta}{\lambda}p(E)(\phi'^L-\phi'^{HL})=F'(G).$$

The Lagrangian, FOCs, and calculation process are presented in the Appendix. Based on the optimal provision rule, the first two terms on the left-hand side are the sum of the marginal rate of substitution between public expenditure and the numeraire among individuals, while the remaining term is a "screening term." Thus, this rule is modified Samuelson rule. To screen agents, the government sets the differential subsidy level and finances investment expenditure for investment to relax the self-selection constraint. In particular, that $\frac{\partial^2\phi}{\partial G^2}>0$ and $D_L>D_H$ imply that $\phi'^L-\phi'^{HL}>0$, so the third term must be strictly positive if the self-selection constraint binds or $\lambda>0$. If individuals are not allowed to buy self-protective goods and this kind of public expenditure is available, the level of public investment is set to meet the Samuelson condition.

6 Discussion

Before providing the conclusion, we discuss the difference between the heterogeneity of disastrous damages and the various probabilities that individuals encounter the disaster, the results of both imperfect observation and public expenditure examined in the preceding two sections, and the linear subsidy on buying self-protective goods.

6.1 Heterogeneous incident rate of disaster

In this paper, we assume that all individuals have the same probability of encountering a disastrous event. By contrast, it is also natural to suppose that each agent has a different incident rate of disaster. For example, those living in designated flood-hazard areas have a higher probability of encountering such an event.

Suppose that this model assumes that the individuals incur the same actual damage but have the different probabilities. In this case, the government cannot screen them by setting different subsidy rules on self-protection. Even though the individuals have different incident rates of disaster, their probabilities must be determined by the total amount of polluting goods consumed, and each individual is atomless, so can not change the aggregate consumption level and the probability by himself/herself. Moreover, when they all share the same damage level during the disaster, they may have incentives to mimic an agent with a different degree of damage. However, according to Proposition 1, $\phi'^{HL} - \phi'^{L}$ must be equal to zero if $D_H = D_L$, which means that the government should not set the discriminatory subsidy rule in the sense of margin.

To summarize, heterogeneity in the individuals' disaster-related damages plays a key role in the results of the preceding three sections. Clearly, it is possible to obtain key insights into the different subsidy rules if both the individuals' actual damages and the probability of a disaster are set differently. In this case, Pigouvian taxes and the modified Samuelson rule should take the different incident rates into consideration.

6.2 Considering both imperfect observation of polluting goods and public expenditure

Thus far, I have not examined both imperfect observations of polluting goods and financing public infrastructure.

In this model, each individual's level of actual damages determines the losses in a disaster, and public expenditure on infrastructure reduces its level, but does not have a distinct effect on the other individuals' consumption of polluting goods. Additionally, we assume that a government can observe each individual's consumption level on self-protection, which leads to the scenario in which the government can set different subsidy rules for them.

Therefore, there is no direct interaction between the linear Pigouvian tax rate and the level of public infrastructure, and to identify each policy role, I embed the imperfect observation of polluting goods and financing public infrastructure independently.

6.3 Linear subsidy on self-protection

Throughout this paper, a government can observe each household's consumption level on self-protection; hence, the government can subsidize their purchase of self-protective goods depending on their behaviors.

As argued in the previous section, this is a valid assumption; however, a tax authority may incur huge costs of observing the amount of each individual's buying, and adopt the proportional subsidy to save the expenses.

In this case, we follow Section 4 and assume that a tax authority sets a proportional tax on polluting goods and a proportional subsidy on self-protection, in addition to a nonlinear income tax schedule. Let t_s be the linear subsidy on goods s, and the individual encounters the following problem:

$$\max_{c_i,x_i,s_i,I_i} U(c_i, x_i) - p(E)\phi(D_i - s_i) - h(\frac{I_i}{w_i})$$
$$\text{s.t.} \;\; c_i + (q_x + t_x)x_i + (q_s - t_s)s_i \leq I_i - T(I_i) \equiv \hat{B}_i.$$

The government can only observe the before-tax income of individual i, I_i; hence, given after-tax income $B_i \equiv I_i - T(I_i)$, he/she chooses the consumption bundle $\{c_i, x_i, s_i\}$;

$$\max_{c_i,x_i,s_i} U(c_i, x_i) - p(E)\phi(D_i - s_i)$$
$$\text{s.t.} \; c_i + (q_x + t_x)x_i + (q_s - t_s)s_i \leq \hat{B}_i.$$

Here, we do not derive the optimal linear subsidy rate t_s, but I conjecture that this also includes the heterogeneity of individuals' actual damages and plays a key role in screening the agents.

7 Conclusion

In this paper, we investigate the optimal taxation of polluting goods and subsidies for self-protection under nonlinear income taxation in the context of disaster risks. The novel contribution of the paper is that we embed these risks based on the total amount of polluting goods consumed by taxpayers and consider two dimensions, earning ability and the amount of damage from disasters, which are considered private information. In this setting, we characterize both the first-best and second-best allocations, and the tax wedges of various tax/subsidy schedules.

In this setup, the first-best allocation rule implies the government should correct for externalities caused by consuming polluting goods, and this can be implemented by a tax schedule incorporating the Pigouvian term, as shown by Sandmo (1975). Additionally, we consider the second-best allocation rule in scenarios in which the government does not know the true characteristics of all individuals. In particular, the marginal subsidy rate for low-skilled and high-actually-damaged agents must be positive to promote their consumption of self-protective goods, This implies that each class encounters different marginal subsidy rates, and it implies that such subsidy schedule plays a role in screening the agents to resolve information asymmetry.

In the second-best scenario, we examine two types of tax/subsidy schedule: perfect and imperfect observation of agents' consumption of polluting goods. In the former case, the Pigouvian term is included in the marginal tax rate on polluting goods, and the screening term is included in the marginal tax/subsidy rate on self-protection. In the latter case, the marginal tax/subsidy rate on self-protection is similar to that in the former case, but it (or the proportional tax rate on polluting goods) includes the screening term in addition to the Pigouvian term.

Moreover, in the second-best scenario, we investigate the optimal level of public expenditure for reducing losses from disasters while retaining the same setup for other (nonlinear) fiscal policies. The results show that the government determines the amount of public spending by modifying the Samuelson rule. The marginal cost of providing public infrastructure is equal to the sum of the marginal rate of substitution between the numeraire and the public good, and the screening term. To acquire accurate information from households regarding productivity and the degree of damage, the government should increase the level of public expenditure to relax the incentive constraint.

Finally, there are several extensions of this work to be pursued in future research. The first involves the generalization of taxpayers' bi-dimensional characteristics. In this case, the focus is restricted to the negative correlation: $w_H > w_L$ and $D_H < D_L$. However, it may be interesting to extend this setup to a multidimensional screening problem. Second, similar to Micheletto (2008), it is possible for me to consider different consumption externalities, that is, each individual might experience different externalities as a result of consuming polluting goods. These extensions are important for both theoretical and practical reasons. Finally, self-protection may not cover all the damage from disasters; thus, there is scope to reconsider the study setup and assumptions. For instance, it may be interesting to extend this model to incomplete markets, in relation to which there have been recent studies have been conducted on the optimal taxation rate, such as those by Gottardi et al. (2015) and Gottardi et al. (2016).

Appendix

FOCs of the second best scenario under perfect observation

The Lagrangian of the second best allocation under perfect observation can be written as follows:

$$\mathcal{L}^{SB} = U(c_H, x_H) + \mu U(c_L, x_L) - p(E)\{\phi(D_H - s_H) + \mu\phi(D_L - s_L)\} - \{h(\frac{I_H}{w_H}) + \mu h(\frac{I_L}{w_L})\}$$
$$+ \lambda\{n(I_H - c_H - q_x x_H - q_s s_H) + (1-n)(I_L - c_L - q_x x_L - q_s s_L)\}$$
$$+ \delta[U(c_H, x_H) - p(E)\phi(D_H - s_H) - h(\frac{I_H}{w_H}) - U(c_L, x_L) + p(E)\phi(D_H - s_L) + h(\frac{I_L}{w_H})].$$

The FOCs are as follows:

$$\frac{\partial \mathcal{L}^{SB}}{\partial c_H} = (1+\delta)U_1^H - \lambda n = 0, \quad \frac{\partial \mathcal{L}^{SB}}{\partial c_L} = (\mu - \delta)U_1^L - \lambda(1-n) = 0$$
$$\frac{\partial \mathcal{L}^{SB}}{\partial x_H} = (1+\delta)U_2^H - np'(E)(\phi^H + \mu\phi^L) - \lambda n q_x = 0$$
$$\frac{\partial \mathcal{L}^{SB}}{\partial x_L} = (\mu - \delta)U_2^L - (1-n)p'(E)(\phi^H + \mu\phi^L) - \lambda(1-n)q_x = 0$$
$$\frac{\partial \mathcal{L}^{SB}}{\partial s_H} = (1+\delta)p(E)\phi'(D_H - s_H) - \lambda n q_s = 0$$
$$\frac{\partial \mathcal{L}^{SB}}{\partial s_L} = \mu p(E)\phi'(D_L - s_L) - \lambda(1-n)q_s - \delta p(E)\phi'(D_H - s_L) = 0$$
$$\frac{\partial \mathcal{L}^{SB}}{\partial I_H} = -\frac{1+\delta}{w_H}h'(\frac{I_H}{w_H}) + \lambda n = 0$$
$$\frac{\partial \mathcal{L}^{SB}}{\partial I_L} = -\frac{\mu}{w_L}h'(\frac{I_L}{w_L}) + \lambda(1-n) + \frac{\delta}{w_H}h'(\frac{I_L}{w_H}) = 0.$$

Applying $(1+\delta)U_1^H = \lambda n$ and $(\mu - \delta)U_1^L = \lambda(1-n)$ and rearranging the above, I obtain the following;

$$(1+\delta)U_2^H - \frac{(1+\delta)U_1^H}{\lambda}p'(E)(\phi^H + \mu\phi^L) - (1+\delta)U_1^H q_x = 0$$

$$\Leftrightarrow MRS_{xc}^H = q_x + \frac{p'(E)(\phi^H + \mu\phi^L)}{\lambda}$$

$$(\mu-\delta)U_2^L - \frac{(\mu-\delta)U_1^L}{\lambda}p'(E)(\phi^H + \mu\phi^L) - (\mu-\delta)U_1^L q_x = 0$$

$$\Leftrightarrow MRS_{xc}^L = q_x + \frac{p'(E)(\phi^H + \mu\phi^L)}{\lambda}$$

$$(1+\delta)p(E)\phi'(D_H - s_H) - (1+\delta)U_1^H q_s = 0$$

$$\Leftrightarrow MRS_{sc}^H = q_s$$

$$\mu p(E)\phi'(D_L - s_L) - (\mu-\delta)U_1^L q_s - \delta p(E)\phi'(D_H - s_L) = 0$$

$$\Leftrightarrow (\mu-\delta)p(E)\phi'(D_L - s_L) - (\mu-\delta)U_1^L q_s + \delta p(E)\{\phi'(D_L - s_L) - \phi'(D_H - s_L)\} = 0$$

$$\Leftrightarrow (\mu-\delta)p(E)\phi'(D_L - s_L) = (\mu-\delta)U_1^L q_s - \delta p(E)\{\phi'(D_L - s_L) - \phi'(D_H - s_L)\}$$

$$\Leftrightarrow MRS_{sc}^L = q_s + \frac{\delta p(E)}{\lambda(1-n)}\{\phi'(D_H - s_L) - \phi'(D_L - s_L)\}$$

FOCs of the second best scenario under imperfect observation

The Lagrangian of the second best allocation under imperfect observation can be written as follows:

$$\mathcal{L}^{IPSB} = V(q_x + t_x, B_H) + \mu V(q_x + t_x, B_L) - p(E)\{\phi(D_H - s_H) + \mu\phi(D_L - s_L)\} - \{h(\frac{I_H}{w_H}) + \mu h(\frac{I_L}{w_L})\}$$
$$+ \lambda\{n(I_H - B_H - q_s s_H + tx(q_x + t_x, B_H)) + (1-n)(I_L - B_L - q_s s_L + tx(q_x + t_x, B_L))\}$$
$$+ \delta[V(q_x + t_x, B_H) - p(E)\phi(D_H - s_H) - h(\frac{I_H}{w_H}) - V(q_x + t_x, B_L) + p(E)\phi(D_H - s_L) + h(\frac{I_L}{w_H})],$$

where $E \equiv nx(q_x + t_x, B_H) + (1-n)x(q_x + t_x, B_L)$.

The FOCs are as follows:

$$
\begin{aligned}
\frac{\partial \mathcal{L}^{IPSB}}{\partial t_x} =& -(1+\delta)\frac{\partial V^H}{\partial B}x^H - (\mu-\delta)\frac{\partial V^L}{\partial B}x^L - \frac{\partial E}{\partial q_x}p'(E)(\phi^H + \mu\phi^L) \\
&+ \lambda[nx^H + (1-n)x^L + t_x n\frac{\partial x^H}{\partial q_x} + t_x(1-n)\frac{\partial x^L}{\partial q_x}] \\
&+ \delta\frac{\partial E}{\partial q_x}p'(E)(\phi^{HL} - \phi^H) = 0 \\
\frac{\partial \mathcal{L}^{IPSB}}{\partial B_H} =&(1+\delta)\frac{\partial V^H}{\partial B} - n\frac{\partial x^H}{\partial B}p'(E)(\phi^H + \mu\phi^L) \\
&- \lambda n(1 - t_x\frac{\partial x^H}{\partial B}) \\
&+ \delta n\frac{\partial x^H}{\partial B}p'(E)(\phi^{HL} - \phi^H) = 0 \\
\frac{\partial \mathcal{L}^{IPSB}}{\partial B_L} =&(\mu-\delta)\frac{\partial V^L}{\partial B} - (1-n)\frac{\partial x^L}{\partial B}p'(E)(\phi^H + \mu\phi^L) \\
&- \lambda(1-n)(1 - t_x\frac{\partial x^L}{\partial B}) \\
&+ \delta(1-n)\frac{\partial x^L}{\partial B}p'(E)(\phi^{HL} - \phi^H) = 0 \\
\frac{\partial \mathcal{L}^{IPSB}}{\partial s_H} =&(1+\delta)p(E)\phi'(D_H - s_H) - \lambda n q_s = 0 \\
\frac{\partial \mathcal{L}^{IPSB}}{\partial s_L} =&p(E)\{\mu\phi'(D_L - s_L) - \delta\phi'(D_H - s_L)\} - \lambda(1-n)q_s = 0 \\
\frac{\partial \mathcal{L}^{IPSB}}{\partial I_H} =& -\frac{1+\delta}{w_H}h'(\frac{I_H}{w_H}) + \lambda n = 0 \\
\frac{\partial \mathcal{L}^{IPSB}}{\partial I_L} =& -\frac{\mu}{w_L}h'(\frac{I_L}{w_L}) + -\frac{\delta}{w_H}h'(\frac{I_L}{w_H}) + \lambda(1-n) = 0.
\end{aligned}
$$

Rearranging the second and third equations and substituting those into the first one, I obtain

$$
\begin{aligned}
&-[n\frac{\partial x^H}{\partial B}p'(E)(\phi^H + \mu\phi^L) + \lambda n(1 - t_x\frac{\partial x^H}{\partial B}) - \delta n\frac{\partial x^H}{\partial B}p'(E)(\phi^{HL} - \phi^H)]x^H \\
&-[(1-n)\frac{\partial x^L}{\partial B}p'(E)(\phi^H + \mu\phi^L) + \lambda(1-n)(1 - t_x\frac{\partial x^L}{\partial B}) - \delta(1-n)\frac{\partial x^L}{\partial B}p'(E)(\phi^{HL} - \phi^H)]x^L \\
&-\frac{\partial E}{\partial q_x}p'(E)(\phi^H + \mu\phi^L) + \lambda[nx^H + (1-n)x^L + t_x n\frac{\partial x^H}{\partial q_x} + t_x(1-n)\frac{\partial x^L}{\partial q_x}] + \delta\frac{\partial E}{\partial q_x}p'(E)(\phi^{HL} - \phi^H) = 0
\end{aligned}
$$

Note that $\frac{\partial E}{\partial q_x} = n\frac{\partial x^H}{\partial q_x} + (1-n)\frac{\partial x^L}{\partial q_x}$, so I get

$$
\begin{aligned}
& - p'(E)(\phi^H + \mu\phi^L)(nx^H\frac{\partial x^H}{\partial B} + (1-n)x^L\frac{\partial x^L}{\partial B} + \frac{\partial E}{\partial q_x}) \\
& + \delta p'(E)(\phi^{HL} - \phi^H)(nx^H\frac{\partial x^H}{\partial B} + (1-n)x^L\frac{\partial x^L}{\partial B} + \frac{\partial E}{\partial q_x}) \\
& + \lambda t_x(nx^H\frac{\partial x^H}{\partial B} + (1-n)x^L\frac{\partial x^L}{\partial B} + \frac{\partial E}{\partial q_x}) = 0
\end{aligned}
$$

Therefore, I derive the optimal tax rate

$$t_x = \frac{p'(E)\{(\phi^H + \mu\phi^L) - \delta(\phi^{HL} - \phi^H)\}}{\lambda} \tag{9}$$

FOCs of the second best scenario under perfect observation with public expenditure

The Lagrangian of the second best allocation under perfect observation with public expenditure can be written as follows

$$
\begin{aligned}
\mathcal{L}^{SBG} &= U(c_H, x_H) + \mu U(c_L, x_L) \\
& - p(E)\{\phi(D_H - s_H - G) + \mu\phi(D_L - s_L - G)\} \\
& - \{h(\frac{I_H}{w_H}) + \mu h(\frac{I_L}{w_L})\} + \lambda\{n(I_H - c_H - q_x x_H - q_s s_H) \\
& + (1-n)(I_L - c_L - q_x x_L - q_s s_L) - F(G)\} \\
& + \delta[U(c_H, x_H) - p(E)\phi(D_H - s_H - G) - h(\frac{I_H}{w_H}) \\
& - U(c_L, x_L) + p(E)\phi(D_H - s_L - G) + h(\frac{I_L}{w_H})]
\end{aligned}
$$

The FOCs are as follows:

$$\frac{\partial \mathcal{L}^{SBG}}{\partial c_H} = (1+\delta)U_1^H - \lambda n = 0, \quad \frac{\partial \mathcal{L}^{SBG}}{\partial c_L} = (\mu-\delta)U_1^L - \lambda(1-n) = 0$$

$$\frac{\partial \mathcal{L}^{SBG}}{\partial x_H} = (1+\delta)U_2^H - np'(E)(\phi^H + \mu\phi^L) - \lambda n q_x = 0$$

$$\frac{\partial \mathcal{L}^{SBG}}{\partial x_L} = (\mu-\delta)U_2^L - (1-n)p'(E)(\phi^H + \mu\phi^L) - \lambda(1-n)q_x = 0$$

$$\frac{\partial \mathcal{L}^{SBG}}{\partial s_H} = (1+\delta)p(E)\phi'(D_H - s_H) - \lambda n q_s = 0$$

$$\frac{\partial \mathcal{L}^{SBG}}{\partial s_L} = \mu p(E)\phi'(D_L - s_L) - \lambda(1-n)q_s - \delta p(E)\phi'(D_H - s_L) = 0$$

$$\frac{\partial \mathcal{L}^{SBG}}{\partial I_H} = -\frac{1+\delta}{w_H}h'(\frac{I_H}{w_H}) + \lambda n = 0$$

$$\frac{\partial \mathcal{L}^{SBG}}{\partial I_L} = -\frac{\mu}{w_L}h'(\frac{I_L}{w_L}) + \lambda(1-n) + \frac{\delta}{w_H}h'(\frac{I_L}{w_H}) = 0.$$

$$\frac{\partial \mathcal{L}^{SBG}}{\partial G} = p(E)\{\phi'(D_H - s_H - G) + \mu\phi'(D_L - s_L - G)\} - \lambda F'(G)$$
$$+ \delta p(E)\{\phi'(D_H - s_H - G) - \phi'(D_H - s_L - G)\} = 0.$$

Applying $(1+\delta)U_1^H = \lambda n$ and $(\mu-\delta)U_1^L = \lambda(1-n)$ and rearranging $\frac{\partial \mathcal{L}^{SBG}}{\partial G} = 0$, I obtain

$$(1+\delta)p(E)\phi'(D_H - s_H - G) + (\mu-\delta)p(E)\phi'(D_L - s_L - G)$$
$$- \lambda F'(G) - \delta p(E)\{\phi'(D_H - s_L - G) - \phi'(D_L - s_L - G)\} = 0.$$
$$\Rightarrow \lambda n \frac{p(E)\phi'(D_H - s_H - G)}{U_1^H} + \lambda(1-n)\frac{p(E)\phi'(D_L - s_L - G)}{U_1^L}$$
$$+ \delta p(E)\{\phi'(D_L - s_L - G) - \phi'(D_H - s_L - G)\} = \lambda F'(G)$$
$$\Rightarrow n \times \frac{p(E)\phi'^H}{U_1^H} + (1-n) \times \frac{p(E)\phi'^L}{U_1^L} + \frac{\delta}{\lambda}p(E)(\phi'^L - \phi'^{HL}) = F'(G)$$

References

Atkinson, A.B., Stiglitz, J.E., 1976. The design of tax structure: direct versus indirect taxation. Journal of public Economics 6, 55–75.

Barrage, L., 2020a. The fiscal costs of climate change, in: AEA Papers and Proceedings, pp. 107–12.

Barrage, L., 2020b. Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy. The Review of Economic Studies 87, 1–39.

Bastani, S., Blomquist, S., Micheletto, L., 2020. Pareto efficient income taxation without single-crossing. Social Choice and Welfare 55, 547–594.

Cremer, H., Gahvari, F., Ladoux, N., 1998. Externalities and optimal taxation. Journal of Public Economics 70, 343–364.

Cremer, H., Pestieau, P., Rochet, J.C., 2001. Direct versus indirect taxation: the design of the tax structure revisited. International Economic Review 42, 781–800.

Edenhofer, O., 2015. Climate change 2014: mitigation of climate change. volume 3. Cambridge University Press.

Ehrlich, I., Becker, G.S., 1972. Market insurance, self-insurance, and self-protection. Journal of political Economy 80, 623–648.

Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. Econometrica 82, 41–88.

Gottardi, P., Kajii, A., Nakajima, T., 2015. Optimal taxation and debt with uninsurable risks to human capital accumulation. American Economic Review 105, 3443–70.

Gottardi, P., Kajii, A., Nakajima, T., 2016. Constrained inefficiency and optimal taxation with uninsurable risks. Journal of Public Economic Theory 18, 1–28.

Lohse, T., Robledo, J.R., Schmidt, U., 2012. Self-insurance and self-protection as public goods. Journal of Risk and Insurance 79, 57–76.

Micheletto, L., 2008. Redistribution and optimal mixed taxation in the presence of consumption externalities. Journal of Public Economics 92, 2262–2274.

Mirrlees, J.A., 1971. An Exploration in the Theory of Optimal Taxation. Review of Economic Studies 38, 175–208.

Sandmo, A., 1975. Optimal taxation in the presence of externalities. The Swedish Journal of Economics , 86–98.

Stern, N., 1982. Optimum taxation with errors in administration. Journal of Public Economics 17, 181–211.

Stiglitz, J.E., 1982. Self-selection and Pareto efficient taxation. Journal of Public Economics 17, 213–240. doi:10.1016/0047-2727(82)90020-2.

Treich, N., Yang, Y., 2021. Public safety under imperfect taxation. Journal of Environmental Economics and Management 106, 102421.

Zehaie, F., 2009. The timing and strategic role of self-protection. Environmental and Resource Economics 44, 337–350.