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Costly Advertising and Information Congestion: Insights from Pigou’s Successors

Ryoji Jinushi

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TOKYO CENTER FOR ECONOMIC RESEARCH
1-7-10-703 Iidabashi, Chiyoda-ku, Tokyo 102-0072, Japan

Abstract

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Ryoji Jinushi
Seikei University
Faculty of Economics
3-3-1 Kichijoji-Kitamachi, Musashino-shi,
Tokyo 180-8633, Japan
ryoji-jinushi@econ.seikei.ac.jp

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Abstract

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*Seikei University. 3 Chome-3-1 Kichijoji Kitamachi, Musashino, Tokyo, Japan 180-8633 (e-mail:ryoji-jinushi@econ.seikei.ac.jp). Acknowledgements: Discussions with Richard Arnott and Takamasa Iryo have contributed to this work. I gratefully appreciate their support. I am grateful as well for the very useful comments of Minoru Osawa, Daisuke Fukuda, Guillaume Rocheteau, Siyang Xiong, Victor Ortego-Marti, Yang Xie, Simon Anderson, Ichiro Obara, Barton Lipman, Tomoya Tajika, George Mailath and Nozomu Muto. For helpful comments, I thank many seminar speakers at UCR who discussed this project with me and the following seminar audiences: the 20th International Industrial Organization Conference 2021, the 48th Annual Conference of European Association for Research in Industrial Economics 2021, the 16th Economics Graduate Students' Conference 2021, the Royal Economic Society Symposium of Junior Researchers 2022, the Summer Workshop on Economic Theory 2022 at Otaru, the Midwest Economic Theory Conference 2024 at Indianapolis, the SING 19, the 2024 Conference on Mechanism and Institution Design, the 7th world congress of the Game Theory Society, the 24th Decentralization Conference in Japan, the 2024 October Workshop on Search and Platform, and microeconomics seminars at Keio University, Kyoto University and Musashi University. This work was supported by JSPS KAKENHI Grant Number JP23K18789 and the Center for Asian and Pacific Studies at Seikei University.

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1 Introduction

Due to the limited capacity of consumers to process information, advertisers have to compete across industries to gain consumer attention. This creates what is called *information congestion*[1]. Information congestion produces a certain loss of social welfare. For example, the advertiser and consumer both lose out when a useful message is missed because of too many other messages[2]. In a primitive advertising structure where consumer information is not available, advertisers cannot avoid sending unsolicited messages to masses of consumers, including non-potential customers. Spam mail is a typical example of such unsolicited messages[3]. It is a large part of all email traffic, approximately 55% (Symantec 2019) to 85% (Cisco 2019), and impairs the efficiency of our economy because of the negligible societal benefit (Rao and Reiley 2012). In a primitive structure in which many advertisers send unsolicited messages, information congestion occurs.

In this study, we theoretically analyze an implementation problem with both hidden actions and information and identify the primitive structure's limitations. This can be used as a benchmark to assess the efficiency of advanced modern advertising structures. In particular, we answer the following question: How can we achieve an efficient allocation of limited consumer attention across many advertisers and industries, when advertisers send messages regardless of consumer interests? Throughout this study, we support the claim that traditional interventions[4] cannot always solve information congestion, and costly interventions are required to guarantee efficient allocation.

In our information congestion model, a continuum of message-senders (i.e., advertisers) sends a discrete number of messages to obtain receivers' limited attention capacity, as in the study by Anderson and de Palma (2013). When receivers deal with a message at least once, the sender obtains benefits which depend on the sender's exogenous type. The unit cost of sending a message is exogenous and

[1] Early literature, such as Hirshleifer (1973), points to the externality among firms wanting to get consumer attention as a congestion effect.

[2] This is often called *information overload*. Malhotra (1984) discusses earlier contributions regarding information overload in consumer decision making, using TV advertising as an example. Eppler and Mengis (2004) review the early literature about information overload across disciplines.

[3] As Anderson and de Palma (2009) indicate, conventional advertising (such as billboards, radio/television and telemarketing) has a similar structure, where advertisers create and deliver messages regardless of consumer interest or attention.

[4] Increasing costs of actions with negative externalities is an example of traditional intervention.

identical. When the total number of messages is larger than the capacity, receivers allocate their attention to each message at random. Therefore, each message has a negative externality for the other active senders; and there are sometimes multiple equilibria, including Pareto-inefficient one, for a given unit cost.

To assess whether the system converges to an efficient allocation after a policy intervention, we apply the *population game* framework (Sandholm 2010) along with evolutionary dynamics; in particular, we apply best response dynamics (Gilboa and Matsui 1991) to information congestion (Anderson and de Palma 2013). Our following discussion reflects the difficulty in controlling this primitive structure where senders send messages regardless of receivers' prior interests or actions.

Although a typical solution for congestion problems is to increase the cost of the action associated with the negative externality, an additional cost per message can have unintended consequences. This additional cost sometimes forces small-benefit-type senders to withdraw from the competition. Consequently, the probability of receiving the same information multiple times increases if large-benefit-type senders continue to send multiple messages. Therefore, an additional cost sometimes hampers the information flow "efficiency"[5]. In addition, once the advertising competition intensifies, sometimes, any amount of additional cost cannot make the system converge to an efficient equilibrium (see subection 5.3). This negative result provides a new insight, unlike the discussion in previous studies[6], which supports placing an additional cost on sending messages when information congestion occurs. In other words, traditional static pricing, as discussed in the previous literature, sometimes does not work well.

Since the traditional one-shot intervention does not necessarily work well, we consider an idealized setting in which a policymaker can put in place a new type of policy: two-step cost change. We can always achieve a Pareto efficient equilibrium with almost minimum social loss by following two steps: (1) raising the senders' cost per message until sending no messages becomes the dominant strategy for all senders (and waiting for the senders' adjustment); (2) gradually reducing the senders' cost until congestion appears (or all senders send a message). Although this policy works regardless of the initial condition and distribution of the senders' benefit, it requires a complicated process that may take time and financial resources. This policy is desirable in certain situations, but its requirements limit its feasibility. A simpler policy may exist, but as far as we can deduce, in the primitive structure, costly policies are required to guarantee the best allocation of receivers' (consumer) attention.

The two-step cost change satisfies the following typical requirements of *evolutionary implemen-*

[5]This study focuses on the benefit of information transmission and ignores the benefit of repetition for senders (and receivers).

[6]See Van Zandt (2004), and Anderson and de Palma (2009, 2012, 2013).

tation for situations where players' actions are anonymous (Sandholm 2002). First, the two-step cost change adjusts the cost of actions regardless of the players' type. It does not require policymakers to monitor private information and actions. Additionally, global convergence is guaranteed, at least under the best response dynamics.

However, the logic behind the two-step cost change is distinct from that of a traditional evolutionary implementation. In the traditional evolutionary implementation, thanks to exogenous information, before implementation begins, a policymaker can fully understand the externality at each aggregate state[7], namely, the number of players who select each action. Thus, in the beginning, the policymaker can create a complete path-independent policy for each aggregate state.

Our approach does not require this complete knowledge. In the two-step cost change, senders' aggregate reactions gradually reveal the distribution of the benefit. This is because senders will continue to send messages only if the cost of sending the message is less than the exogenous benefit from the received message. Eventually, policymakers will understand the externality among players and will decide the timing of each step in the policy. Although the two-step cost change depends on the unique characteristics of information congestion, the underlying logic is more general; evolutionary implementation, in its process, can reveal extensive information about externalities, which policymakers can use to design a policy that achieves an efficient state[8]. We contribute to the field of evolutionary implementation by highlighting this novel advantage.

We contribute to the field of environmental economics because the two-step cost change is a special case of the charges and standards approach (Baumol and Oates 1971, 1988). Briefly, this approach sets environmental standards and tries various tax patterns until the standards are satisfied. Baumol and Oates mention its two weak points, but these points are not valid in our context. First, Baumol and Oates concern multiple equilibria. Because their models are static, their discussion cannot guarantee global convergence, but we do by evolutionary dynamics. Second, their approach does not guarantee the first-best outcome. However, Baumol and Oates point out that their approach would be valid for many problems. Information congestion may be one of the best applications for it because Pareto efficiency is achieved when attention is fully utilized in equilibrium.

[7]In the framework of the population game, we call a strategy distribution a (population) state.

[8]Callander (2011) also discusses a similar situation in which the outcomes of each policy are uncertain, the set of feasible policies are uncountable, and policymakers gradually learn the outcomes through trial and error. However, uncertainty follows a purely exogenous random motion (Brownian motion). In our discussion, policymakers gradually learn the externality in the game by observing endogenous reactions to the policy.

Callander et al. (2022, 2023) analyze the dynamic interaction between market competition and policymakers' interventions. In their settings, the externality in the game is known to the policy makers, and this is different from our setting. In addition, players understand each other in the market competition, and thus equilibria based on the strategic reasoning are discussed. We analyze problems where players may not know each other well.

Table 1: Comparison of Pigouvian Approach and Its Successors

	Pigouvian Approach	**Strategy-Proof Mechanism Design**	**Traditional EI**	**EI with Experiments**
Externality	Known	Unknown	Known	Unknown
Monitoring	No	Who does What	How many players select each action	How many players select each action
Counts	Once	Once	Infinite	Finite
Multi Eq	No	Yes	Yes	Yes

Both the evolutionary implementation and the charges and standards approach are successors of the Pigouvian approach. Table 1 compares the Pigouvian approach, strategy-proof mechanism design, traditional evolutionary implementation (EI), and our approach (which we call EI with experiments). The Pigouvian approach puts additional cost or subsidy on actions with externality such that private marginal profit equals the social one at efficient equilibrium. Thus, the policymaker needs to forecast the equilibrium and the externality. To avoid this difficulty, mechanism design collects private information from players. However, it requires the centralized system monitoring individuals. Traditional EI eases the requirement, but it requires complete knowledge about externalities. Our approach, EI with experiments, does not require individual monitoring and the complete knowledge. In addition, EI with experiments requires only a finite number of cost changes (interventions for mechanism design). Except for Pigouvian approach, all of them can gurantee Pareto efficiency after interventions, even if original games have multiple equilibria.

We utilize evolutionary game theory to compare policies in the setting with multiple equilibria. Dijkstra and De Vries (2006) also utilize evolutionary game theory to compare policies with multiple equilibria. They analyze location choices of firms and consumers, and point out that the Pigouvian approach can fail to guarantee global convergence to efficient resource allocation. Our discussion is similar to their discussion, but we discuss a situation where policymakers do not know externality among players at the beginning.

In this study, we utilize a regular Taylor evolutionarily stable state as a solution concept. This concept is stronger than the Nash equilibrium and has asymptotically local stability under many popular dynamics, including the best response and impartial pairwise comparison dynamics (Sandholm, 2010). The field of information congestion focuses on competition across industries, and thus, many advertisers play the game. As it is difficult for all advertisers always to select the optimal choice that keeps equilibrium, we focus on locally stable equilibria.

The rest of the study is organized as follows. In Section 2, we briefly review the associated literature and our contributions to the field of advertising theory. Section 3 formulates information congestion in the framework of the population game. Section 4 defines the criterion of local stability, regular Taylor evolutionarily stable state, and presents a sufficient condition for the existence of (Pareto) suboptimal equilibrium with local stability. This existence demonstrates the structural inefficiency and the importance of interventions. In Section 5, we discuss a sufficient condition under which any static pricing on unit message cannot lead the system to the efficient equilibrium. The case demonstrates that naïve interventions do not always work. In addition, we propose and analyze the dynamic policy achieving a Pareto efficient equilibrium with stability. Section 6 summarizes our results and implications.

2 Literature

In this section, we situate this paper within the economic analysis of advertising and related fields. Further, we explain our contribution to the field of information congestion. The difference between information congestion and rational inattention is also explained.

In economic theory, advertising informs consumers *about the availability of new products* at a minimum (Renault 2015) and complements consumer search[9]. Despite information from other consumers, consumers sometimes fail to find their best choice without the aid of advertising[10]. This makes advertising an important element in the study of market competition.

The economic analysis of advertising usually focuses on effects of advertising on market competition and social welfare. In contrast, our study focuses on the process of advertising and its efficiency as a means of information transmission. Our study contributes to the field by supplying a useful tool for extending the mainstream discussions on the economic analysis of advertising. Bagwell (2007) offers an overview of advertising research in industrial organization. Bagwell proposes three advertising categories, namely, the persuasive, the informative, and the complementary perspectives of advertising. The differences in the categories are derived from the impact of advertising on consumer preferences for the advertised goods[11], and consequently, on social welfare. Most advertising literature analyzes the impact of advertising on market competition and can be classified using Bagwell's three categories.

[9]For example, see Stigler (1961) and Butters (1977). Renault (2015) reviews the literature relating to advertising and consumer search.

[10]For example, see Ali (2018) and Niehaus (2011).

[11]For example, if an advertisement just persuades people to buy meaningless items, then the consumer surplus would eventually be 0 or negative in the long term.

However, our study does not fall within these categories as it does not assume receivers' preferences. Therefore, we can put additional constraints on receivers' attention in those mainstream models without conflicting assumptions. For example, Van Zandt (2004) inspired Anderson and de Palma (2012), who analyzed the impact of consumers' limited attention on market competition.

Our discussion complements previous literature, focusing on advertising and consumers' limited attention, by analyzing an implementation problem with information congestion. Papers in the field put weight on interesting equilibrium with free entry[12], static pricing/policies[13] and impacts of advanced structures[14]. However, as we see in subection 5.3, in our setting, any static pricing for sending a message cannot guarantee an efficient allocation after the policy. To understand the performance of the primitive structure, we discuss implementation through dynamic interventions.

The field of rational inattention (RI) also examines cognitive limitations but focuses on different research targets. For example, Sims (2003) analyzes a decision maker who decides how much information to collect. There are two types of shocks: temporal measurement errors and long-term fundamental shocks. The decision maker can only partially distinguish between these two shocks because of capacity limitations on the accuracy of information. Therefore, the decision maker gradually adjusts the understanding of economic fundamentals, instead of accepting the latest information. There are two differences between information congestion and RI. First, in RI, decision makers are active and rational. They have a basic understanding of the economy and the distribution of errors. They completely control the messages they receive. In information congestion, receivers may not understand anything about the economy. Second, RI discusses long-term optimization. Information congestion examines the more myopic players. This difference is rooted in their respective research targets. RI aims to analyze serious problems for decision makers. Information congestion explains the situation in which the expected benefits for each player may be low.

3 Formulation and Equilibrium

This section introduces the information congestion model. In 3.1, we introduce the optimization problems of heterogeneous senders. In 3.2, the optimization problems are applied to the population game framework. In 3.3, we define the Nash equilibrium.

[12]See Falkinger (2007, 2008) and Hefti (2018).

[13]See Van Zandt (2004) and Anderson and de Palma (2009, 2012, 2013).

[14]Hefti and Liu (2020) and Anderson and Peitz (2023)

3.1 Formulation (Optimization Problem of Senders)

Information congestion models include message-senders and message-receivers. In this study, we consider a situation in which a continuum of senders cannot distinguish among a continuum of receivers and focus on a mass of homogeneous receivers. The receivers' volume is normalized to one. We consider a situation in which each receiver cannot adjust the capacity for processing messages. For example, if many flyers/spam mails are present in a mailbox, each receiver may unintentionally recognize the top one. The capacity for receivers to deal with messages is $\phi > 0$. If the total number of messages by senders, denoted by $N \in \mathbb{R}_+$, is more than the capacity (ϕ), messages are randomly dealt with by receivers. We apply this constant-capacity assumption to simplify the receivers' side and focus on senders' behaviors[15]. The presented basic model with discrete players is essentially identical to the basic model with two senders in Anderson and de Palma (2013).

First, we explain the basic model with discrete numbers of heterogeneous senders before introducing the population game with a continuous mass of players. Each sender i gains benefit π^i when all receivers receive at least one message from the sender. For example, if half of the receivers receive a message from sender i, the sender's benefit is $\pi^i/2$. We are interested in information transmission, and if a receiver deals with two or more messages from the same sender, it cannot convey any additional information, making it redundant for senders and receivers in our model[16]. The cost of sending each message is $\gamma > 0$. Senders decide their number of messages l by considering the tradeoff between the probability of getting attention and the cost of sending a message. Each sender i can select only discrete numbers $l \in \{0, 1, 2, ..., l_{max}\}$ as the number of messages, where l_{max} denotes the exogenous maximum number of messages for senders.

Suppose that there exist senders including a sender i and aggregate Q number of messages by

[15] If consumers has already minimized their exposure to unsolicited messages, ϕ can be interpreted as the minimum uncontrollable exposure. The constant-capacity assumption is utilized in Van Zandt (2004), Anderson and de Palma (2012, 2013), and Hefti and Liu (2020) perhaps because they are modestly consistent with some empirical literature in advertising research. For example, in Riebe and Dawes (2006), respondents recall 1.1 ads per three ads in low-clutter groups and 1.4 ads per nine ads in high-clutter groups. Hammer, Riebe, and Kennedy (2009) analyze four types of advertising data and conclude that the probability of recall for each is low/high in high/low-clutter situations. As long as an alternative function of ϕ/N is differentiable and decreasing in N when $N > \phi$, we conjecture that our main results hold because the utility of each sender depends on ϕ/N and does not directly depend on ϕ.

[16] As Renault (2015) states, the minimum consensus in the economic analysis of advertising is that advertising communicates to consumers the availability of new products. We focus on this aspect of advertising.

others. The optimization problem for sender i is

$$\max_{l^i \in \{0,1,2,\ldots,l_{max}\}} U^i = \begin{cases} \pi^i(1-(1-\frac{\phi}{(Q+l^i)})^{l^i}) - \gamma l^i & \text{if } \phi < Q + l^i \\ \pi^i \mathbb{1}(l^i > 0) - \gamma l^i & \text{if } \phi \geqq Q + l^i \end{cases} \tag{1}$$

where $\mathbb{1}$ is the indicator function. When $\phi < Q + l^i$, by sending l messages, sender i gets the expected benefit of the message being received ($\pi^i(1-(1-\phi/(Q+l^i))^{l^i})$) and pays the cost of sending l messages (γl). When $\phi \geqq Q + l^i$, by sending at least one message ($l > 0$), sender i gets π^i and pays the cost of sending l messages (γl).

$(1-(1-\phi/(Q+l^i))^{l^i})$ in (1) is the probability that at least one message from sender i is received by each receiver. When we apply the population game framework, we assume that each of senders is small, so that they do not consider the impact of their message on the total number of messages, like $(1-(1-\phi/(Q))^{l^i})$ in this example. We consider advertising competition among many firms across industries, and this assumption would be reasonable. Anderson and de Palma (2013) explain that this assumption is similar to a standard monopolistic competition assumption where firms do not count the impact of their actions on other firms.

As Anderson and de Palma (2013) point out that, when Q is large enough, the breakthrough probability $(1-(1-\phi/(Q+l^i))^{l^i})$ is decreasing in l^i; this characteristic is consistent with the shape of advertising response functions typically confirmed in empirical advertising research. For example, Taylor, Kennedy and Sharp (2009) confirm the decreasing marginal sales of advertising by analyzing data from four non-durable-goods categories [17]. Vakratsas et al. (2004) show the concave advertising response function with the probabilistic thresholds of advertising expenditure for attracting additional market share in durable goods markets. Our discussion supplies a theoretical explanation of these results.

3.2 Formulation (Population Game Framework)

$R = \{1, 2, ..., m\}$ is an exogenous benefit set for heterogeneous senders. Each sender belongs to a single type $r \in R$, and senders with the same type r get the same benefit $\pi(r) > \gamma$ if their message is dealt with. The minimum benefit difference among senders is $MD = \min_{r,r' \in R: r \neq r'} |\pi(r) - \pi(r')|$. We assume $\pi(r) \neq \pi(r')$ if $r, r' \in R$ and $r \neq r'$, and thus $MD > 0$. The senders' maximum benefit is $\pi_{max} = \max_{r \in R} \pi(r)$. The minimum benefit is $\pi_{min} = \min_{r \in R} \pi(r)$. The population of type $r \in R$

[17] Taylor, Kennedy and Sharp (2009) report an exception, but the conclusion about the decreasing marginal sales of advertising typically holds.

is denoted by $d(r) > 0$. We assume that there exists a minimum basic unit $\omega > 0$, and for any $r \in R$ there exists $j \in \mathbb{N} = \{1, 2, ...\}$ s.t. $d(r) = j\omega$[18]. The total population of senders is $D = \sum_{r \in R} d(r)$. x^r_l denotes the population of type r who selects strategy l. The set of available numbers of sending messages is denoted by $L = \{0, 1, 2, ..., l_{max}\}$. For simplicity, we assume $D \geqq \phi$[19].

The total number of pure strategies for all types is $n = \sum_{r \in R} l_{max} + 1$. The set of all possible states (strategy distributions) for type r is $X^r = \{x^r \in \mathbb{R}^{l_{max}+1}_+ : \sum_{l \in L} x^r_l = d(r)\}$. The set of all possible states in the system (state space) is $X = \prod_{r \in R} X^r = \{\boldsymbol{x} = (x^0, ..., x^r) \in \mathbb{R}^n_+ : x^r \in X^r\}$. TX is the tangent space of X s.t. $TX = \prod_{r \in R} TX^r$ where $TX^r = \{z^r \in \mathbb{R}^{l_{max}+1} : \sum_{l \in L} z^r_l = 0\}$. The total number of messages by all senders[20] is $N(\boldsymbol{x}) = \sum_{r \in R} \sum_{l \in L} l x^r_l$.

The population game framework matches with information congestion. In population games, players sporadically and myopically[21] select their strategies. Information congestion occurs in advertising competition across industries, and each sender may not understand the other players well. Thus, myopic decision making based on realized payoffs is reasonable for analyzing such a situation with a large number of players.

3.3 Definition of Equilibrium

Before introducing a Nash equilibrium in this game, we formally define a (Lipschitz continuous) payoff function and the average payoff for type r at x.

$$U(r, l, \boldsymbol{x}) = \pi(r)\mathbb{1}(l > 0)\left(1 - \mathbb{1}(\phi \leqq N(\boldsymbol{x}))(1 - \phi/N(\boldsymbol{x}))^l\right) - \gamma l \tag{2}$$

Equation (2) shows that, the utility depends on the benefits, the costs, and the probability that at least one message is dealt with by receivers. The average payoff is denoted by the following:

$$\bar{U}(r, \boldsymbol{x}) = \frac{1}{d(r)} \sum_{l \in L} x^r_l U(r, l, \boldsymbol{x}). \tag{3}$$

By using these definitions,

[18] We use this assumption when we decide the amount of (finite) cost changes (later defined by γ'_z) in policies.

[19] We discuss more general situations in our previous version Jinushi (2023).

[20] N plays the role of Q in the previous discrete example, but because each sender is small, senders ignore the impact of their actions on N.

[21] Here, "sporadically" suggests that some portion of players can simultaneously change their strategies. "Myopically" suggests that all players make their decision by assuming that other players retain their strategies.

Definition 1. *State $\boldsymbol{x} \in X$ is a Nash equilibrium in this information congestion game if and only if*

$$\begin{aligned} U(r,l,\boldsymbol{x}) &\leqq \bar{U}(r,\boldsymbol{x}) \\ x_l^r(\bar{U}(r,\boldsymbol{x}) - U(r,l,\boldsymbol{x})) &= 0 \qquad\qquad \forall r \in R \text{ and } \forall l \in L \\ x_l^r &\geqq 0 \end{aligned} \tag{4}$$

(4) indicates that, in equilibrium, all players select their best strategies to maximize their utility and each population cannot be negative. Since U is continuous in X, from Theorem 2.1.1. in Sandholm (2010), this *population game admits at least one Nash equilibrium.*

4 Stability of Equilibria

We use the Regular Taylor Evolutionarily Stable State (RTESS) as a criterion of stability. Whether an equilibrium is an RTESS only depends on the payoff function of its neighborhood. Therefore, this concept enables us to discuss the stability of equilibria without assuming a specific dynamic.

From Sandholm (2010), the definition of an RTESS is as follows:

Definition 2. *State $\boldsymbol{x} \in X$ is a Regular Taylor ESS if the utility function U is Lipschitz continuous in X and the following two conditions are satisfied.*

$$\text{For any } r \in R \text{ and } i, j \in L,\ U(r,i,\boldsymbol{x}) = \bar{U}(r,\boldsymbol{x}) > U(r,j,\boldsymbol{x}) \text{ if } x_i^r > 0 \text{ and } x_j^r = 0. \tag{5}$$

$$\text{For any } \boldsymbol{y} \in X - \{\boldsymbol{x}\},\ (\boldsymbol{y}-\boldsymbol{x})' DU(\boldsymbol{x})(\boldsymbol{y}-\boldsymbol{x}) < 0 \qquad \text{if } (\boldsymbol{y}-\boldsymbol{x})'U(\boldsymbol{x}) = 0. \tag{6}$$

Here, $(\boldsymbol{y}-\boldsymbol{x})'$ is the transpose of the matrix $\boldsymbol{y}-\boldsymbol{x}$. The Jacobian matrix DU is the derivative of the linear map such that $U(\boldsymbol{y}) = U(\boldsymbol{x}) + DU(\boldsymbol{x})(\boldsymbol{y}-\boldsymbol{x}) + o(\boldsymbol{y}-\boldsymbol{x})$ where $o(\boldsymbol{z})$ represents a function $h : TX \to \mathbb{R}^n$ s.t. $\lim_{\boldsymbol{z}\to\boldsymbol{0}} h(\boldsymbol{z})/|\boldsymbol{z}| = \boldsymbol{0}$,

$$DU(\boldsymbol{x}) = \begin{pmatrix} \frac{\partial U(1,0,\boldsymbol{x})}{\partial x_0^1} & \cdots & \frac{\partial U(1,0,\boldsymbol{x})}{\partial x_{l_{max}}^m} \\ \vdots & \vdots & \vdots \\ \frac{\partial U(m,l_{max},\boldsymbol{x})}{\partial x_0^1} & \cdots & \frac{\partial U(m,l_{max},\boldsymbol{x})}{\partial x_{l_{max}}^m}. \end{pmatrix} \tag{7}$$

When $\boldsymbol{x}$ satisfies Equation (5), all optimal strategies are selected, and non-optimal strategies are not selected in $\boldsymbol{x}$. Equation (6) implies that, in the neighborhood of $\boldsymbol{x}$, a small deviation among optimal strategies will be penalized with utility loss. In other words, if a small number of players begin to follow another strategy profile $\boldsymbol{y}$, they are defeated by the majority who follow the strategy profile $\boldsymbol{x}$. These conditions are satisfied if each type of player has a unique type-specific optimal strategy. When a single type has multiple best replies in the equilibrium and all other types have a unique type-specific optimal strategy, and if each of the multiple best replies is selected and has a self-defeating externality, these conditions are satisfied.

4.1 Evolutionary Stability and Dynamics

Is the RTESS a good criterion for the stability of information congestion? The answer depends on whether dynamics covered by the RTESS are fit for advertising competition across industries. As typical examples, an RTESS is locally asymptotically stable under any impartial pairwise comparison and best response dynamics (Theorem 8.4.7 in Sandholm (2010)).

Advertisers can estimate or observe the intensity of congestion (ϕ/N) in advertising. Thus, they can directly select the best strategy for the situation. Therefore, we consider that this dynamic, known as the *best response dynamic*, is valid in the context of advertising.

Our policy discussion focuses on the best response dynamics and does not discuss policies for more general dynamics. There are two reasons for this approach. First, our goal is to demonstrate the difficulty of controlling information congestion under evolutionary dynamics. Thus, it is sufficient to show that the control is difficult under the best response dynamics. Second, under impartial pairwise comparison dynamics and more general evolutionary dynamics, it is relatively difficult to find a simple and general interpretation on the senders' reactions to the policies. In particular, the required speed of cost changes will depend on exogenous parameters. Our following results show that, at least under the specific dynamic, we can control information congestion if we spend time and other resources. There would exist more robust policies for general dynamics, but we save these for future research.

$t \in [0, \infty)$ denotes continuous time in the economy. At $t = 0$, an exogenous initial condition $\boldsymbol{x}(0) \in X$ exists, and the system follows the best response dynamics. $\boldsymbol{x}(t)$ denotes one of the possible states at t under the best response dynamics for a given initial condition $\boldsymbol{x}(0)$. When describing a static state, we omit the time notation.

The best response dynamic for type r is given by[22]

Definition 3. *Best Response Dynamic:* $\dot{x}^r \in d(r)M^r(U(r, \boldsymbol{x})) - x^r$

[22]We follow the notation in Sandholm (2010). Gilboa and Matsui (1991) propose the best response dynamics and analyze the cyclically stable sets of strategy profiles ($\boldsymbol{x}$ in this project) that have a trajectory based on the best response dynamics from any other strategy profile in the set.

where

$$M^r(U(r,\boldsymbol{x})) = \underset{y^r \in \Delta^r}{\text{argmax}}(y^r)' \begin{pmatrix} U(r,0,\boldsymbol{x}) \\ \vdots \\ U(r,l_{max},\boldsymbol{x}) \end{pmatrix} \tag{8}$$

is the maximizer correspondence for type r and

$$\Delta^r = \left\{ y^r \in \mathbb{R}_+^{l_{max}+1} : \sum_{i \in L} y_i^r = 1 \right\}. \tag{9}$$

is the set of mixed strategies for type r. In our model, each $r \in R$ follows the best response dynamic given above.

4.2 Existence of Suboptimal Stable Equilibrium

Even in a simple setting, our model has multiple equilibria, including a suboptimal and locally stable equilibrium. For simplicity, we focus on the case of homogeneous senders ($R = \{1\}$). This specific setting is sufficient to show the suboptimal stable equilibrium.

Figure 1 shows the best responses of senders for each $\phi/N(\boldsymbol{x})$, which represents the probability that a unit message is received, with the restriction that the maximum number of sending messages is 5. (see Appendix A.1 for details.) The set of optimal strategies for senders at given $\boldsymbol{x}$ is either a single strategy or the pair of two consecutive numbers[23] (see Appendix A.1 for the proof.). Therefore, when the ratio of capacity to senders' population (ϕ/D) is given, we can calculate the possible range of ϕ/N for each pair of strategies. We combine the ranges and Figure 1 to derive Figure 2.

The curve in Figure 2 shows the equilibria for each γ/π when $D = \phi$. The orange rectangle represents multiple equilibria when $0.2 = \gamma/\pi$. Each point (A and C) is an RTESS[24] (see Appendix A.2 and A.3 for the proof.). The arrows in Figure 2 explain the direction of the dynamics at ϕ/N when we apply the best response dynamics to the specific state $\boldsymbol{x}$ in which $N = N(\boldsymbol{x})$ and all senders select the strategies that use one of two consecutive numbers like sending a message once or twice. When we consider a point above the curve, the dynamics gradually reduce N. When we consider a point below the curve, the dynamics gradually increase N. Appendice A.2 and A.3 provide the details about the stability and the derivation of Figure 2.

[23] In this study, we define that consecutive numbers are the sequence of (non-negative) integers like 0,1,2,3... without intervals. When we say two consecutive numbers, we imply a pair of two integers such as (0,1), (1,2) and (3,4).

[24] The circle (B) represents an equilibrium without stability.

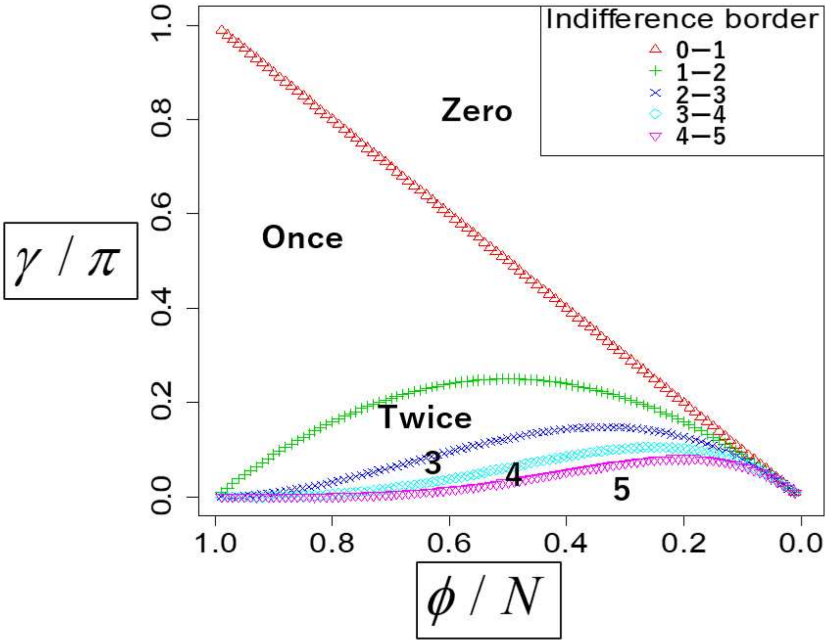

Figure 1: Best Response Diagram

To define Pareto efficiency for a continuum of players, we introduce the following distribution of senders' utility in $\boldsymbol{x} \in X$. For any $r \in R$ and any $j \in \mathbb{R}$, we define

$$F^r(j, \boldsymbol{x}) = \sum_{l \in L} x_l^r \mathbb{1}(U(r, l, \boldsymbol{x}) \leqq j). \tag{10}$$

$F^r(j, \boldsymbol{x})$ is the number of type r senders whose utility does not exceed j in $\boldsymbol{x}$. Since each player in each type is anonymous, for $\boldsymbol{x}, \boldsymbol{x'} \in X$, when $F^r(j, \boldsymbol{x}) \geqq F^r(j, \boldsymbol{x'})$ for all $j \in \mathbb{R}$, and if there exists $j' \in \mathbb{R}$ s.t. $F^r(j', \boldsymbol{x}) > F^r(j', \boldsymbol{x'})$, we consider that $\boldsymbol{x}$ is inferior to $\boldsymbol{x'}$ for senders in type r.

Using F^r above, in this study, we define a Pareto efficient state as follows:

Definition 4. *$\boldsymbol{x} \in X$ is Pareto efficient if and only if there is no $\boldsymbol{x'} \in X$ s.t., for any $r \in R$ and for any $j \in \mathbb{R}$,*

$$F^r(j, \boldsymbol{x}) \geqq F^r(j, \boldsymbol{x'}) \tag{11}$$

and there exists at least a pair $r' \in R$ and $j \in \mathbb{R}$ s.t.

$$F^{r'}(j, \boldsymbol{x}) > F^{r'}(j, \boldsymbol{x'})^{25}. \tag{12}$$

[25] When $\boldsymbol{x} \in X$ satisfies these conditions in the definition, any $x' \in X$ cannot enable any portion of players to be better off, while maintaining the utility of other players. If $\boldsymbol{x} \in X$ does not satisfy these conditions, there exists

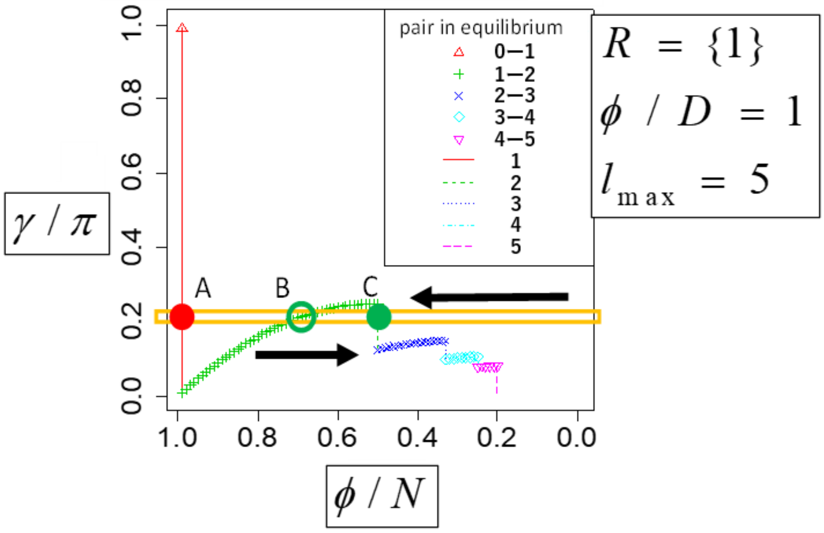

Figure 2: Bifurcation Diagram

In equilibrium C in the figure, all senders send their messages twice, and the probability that each receiver deals with these messages is $3/4$. This is Pareto inefficient compared with equilibrium A in which all senders send their messages only once, and all messages are dealt with by all receivers.

Proposition 1 provides a sufficient condition for the existence of such a suboptimal and locally stable equilibrium:

Proposition 1. *At least a (Pareto) suboptimal equilibrium is a regular Taylor evolutionarily stable state when* $l_{max} \geqq 2$, $R = \{1, 2, ...\}$, $d(1) = \phi$, $\pi(1)\frac{\phi}{l_{max}d(1)}(1 - \frac{\phi}{l_{max}d(1)})^{l_{max}-1} > l_{max}\gamma$ *and* $\gamma > \frac{\phi}{l_{max}d(1)} \max_{r \in R \setminus \{1\}} \pi(r)$ *.*

Proof: When all $d(1)$ senders of type 1 send messages l_{max} times, sending messages l_{max} times is the unique best strategy for senders of type 1. For other senders, sending no messages is strictly optimal. Thus, it is a regular Taylor evolutionarily stable state. However, if each type 1 sends a message while others do not, all type 1 senders gain the maximum benefit, and this state dominates the state mentioned above. ■

This section demonstrates that the model has multiple equilibria, and the suboptimal equilibrium is sometimes locally stable. In other words, the primitive structure generates suboptimal outcomes. Therefore, interventions may improve efficiency for this economy.

$\boldsymbol{x}' \in X$ such that a portion of players who attain better utility, while maintaining the others' utility. Therefore, we define a Pareto efficient state as $\boldsymbol{x} \in X$ satisfying these conditions. Later, when we discuss the efficiency of equilibria, we use this criterion in the original game with the original γ, not with the modified γ'_z.

5 Policy

In this section, we seek policies to achieve a stable Pareto efficient equilibrium. We assume that the policymaker does not know the senders' benefit distribution ($d(r)$). The externality in the game partially depends on the distribution, and the policymaker can gradually learn this in the policy process. In the two-step cost change, the reactions of players partially reveal the distribution, and the policymaker adjust the speed of the required cost change based on the reactions.

5.1 Approximate convergence

Our discussion in this section frequently utilizes the following basic characteristic of the best response dynamics.

Lemma 1. *Consider $t2 > t1$ and $\boldsymbol{x}(0) \in X$ and suppose that $\boldsymbol{x}(t)$ follows the best response dynamics. For each $t \in [t1, t2]$, if there exists $l' \in L$ s.t. $U(r, l, \boldsymbol{x}(t)) < U(r, l', \boldsymbol{x}(t))$, then the population of type r that selects the suboptimal strategy l has an upper-bound ($x_l^r(t) \leqq De^{-t+t1}$) at t.*

Proof: When $l \in L$ is strictly suboptimal at $t \in [t1, t2]$, $\dot{x}_l^r = -x_l^r \Leftrightarrow x_l^r(t) = x_l^r(t1)e^{-t+t1}$. Because D is the total number of senders, $x_l^r(t1) \leqq D$, and thus $x_l^r(t) \leqq De^{-t+t1}$. ■

If one strategy is strictly better than any other strategies for a long time, the population that selects the suboptimal strategy eventually converges to 0. In addition, for each suboptimal strategy, we have a simple upper-bound $x_l^r(t) \leqq De^{-t+t1}$, which depends on the total population D and $t - t1$ (the pass time after the intervention begins). For general evolutionary dynamics, the existence of such an approximate convergence for a strictly best strategy in a certain period is not guaranteed[26].

We introduce a new concept, called ϵ-convergence, to capture the approximate convergecne. Consider $x_l^r(t)$ (the population of type-r senders who select strategy l at t) where $r \in R$ and $l \in L$. $x_l^r(\lim)$ denotes the limit of $x_l^r(t)$ ($x_l^r(t) \rightarrow x_l^r(\lim)$ as $t \rightarrow \infty$). When we say that $x_l^r(t)$ is in ϵ-neighborhood of the limit $x_l^r(\lim)$, $|x_l^r(\lim) - x_l^r(t)| < \epsilon$. For this study, we say that $x_l^r(t)$ ϵ-converges at t when $x_l^r(t)$ is in ϵ-neighborhood of each possible limit, and if all possible $x_l'^r(t')$ satisfying $x_l'^r(t) = x_l^r(t)$ are ϵ-neighborhood of each limit for any $t' > t$[27].

[26] As a more general discussion, Bandhu and Lahkar (2022) discuss a family of evolutionary dynamics which satisfy the convergence for a strictly dominant strategy to analyze an evolutionary foundation of dominant strategy implementation.

[27] Gilboa and Matsui (1991) introduce ϵ-accessibility with which the system with the best response dynamics can move to a strategy profile (state $\boldsymbol{x}$ in this study) from ϵ-neighborhood of another strategy profile. We discuss implementations and later define ϵ-convergence after which each possible trajectory $x_l^r(t')$ after $t' > t$ has a limit (and stays in its ϵ-neighborhood of the limit after t). Unlike ϵ-convergence, ϵ-accessibility captures general movements like a circular movement.

When we consider a sequence of numbers $z(t)$ that converges to a number z as $t \to \infty$, for any $\epsilon > 0$, we find $\bar{t} > 0$ s.t., if $t > \bar{t}$, $|z - z(t)| < \epsilon$. In other words, when policymakers keep a strategy strictly optimal for each type r, and if the dynamic makes all type-r players eventually select a strictly optimal strategy, policymakers are certain of the ϵ-convergence where almost all (at least $d(r) - \epsilon$) players select the best strategies in finite time. Since R is a finite set, we can find the maximum of required time for each type. $T(\epsilon)$ denotes the maximum time for such an ϵ-convergence. In the following discussion, we assume the best response dynamics, which is why policymakers know $T(\epsilon)$ for ϵ-convergence when there exists a strictly best strategy.

5.2 Policymaker's Knowledge and Tools

The policymaker knows the population of senders D and capacity ϕ, an upper-bound of the required time for ϵ-convergence $T(\epsilon)$, minimum basic unit of population ω, and the feasible set of senders' benefits including maximum/minimum benefits $\pi_{max}, \ \pi_{min}$ and minimum benefit difference MD[28]. In addition, we assume that the policymaker can observe $N(\boldsymbol{x}(t))$, and can change the cost of sending a message γ. We denote the zth changed unit cost as γ'_z where z is an element of a finite set $Z = \{1, 2, \ldots, m+1\}$.

For Pareto efficiency, the duplication of messages matters. As seen in Figure 1, there exist two extreme situations ($\phi/N \to 1$ and $\phi/N \to 0$) where no senders have incentive to send multiple messages. Therefore, a policy which invites either of the cases is promising. Through this section, we discuss properties of such a policy, namely, *two-step cost change*[29].

In the following discussion, we utilize several characteristics of the best response dynamics. First, players move from a current strategy to another strategy if the payoff from another strategy is strictly better than the one from the current strategy. Second, because of the first characteristic, if the strictly dominant strategy exists for a type of player, the strategy is eventually selected by all players within the type. Third, conditional on playing dominant strategies, if another type has a strategy that strictly dominates all other strategies, this conditional dominant strategy is eventually selected by all players within the type. Further, from any original state, we can move to the state in which those types select the strictly dominant strategy or the conditional dominant strategy. If this realized state is the desirable state, or if we can find the policy from the realized state to the desirable state using the first charac-

[28]Instead of $T(\epsilon)$, ω, MD, policymakers can use exaggerated numbers like $MD > MD'$, and so we consider that this requirement is relatively weak.

[29]In the previous version of our working paper, named "Costly Advertising and Information Congestion", we discuss another policy that leads the system to $\phi/N \to 0$.

teristic, this whole process achieves the desirable state from any original state. Even if we restrict that the policymaker cannot wait for an infinite time for each step, we can approximately implement the process given above. At least under the best response dynamics, in finite time, our policy achieves ϵ-convergence, and the limit is a desirable outcome.

5.3 A Simple Negative Example for Static Pricing

We show that an additional cost per message sometimes impairs efficiency, as in the following example. Consider two types of senders $R = \{r_1, r_2\}$, $l_{max} = 2$, $\gamma = 0.9$, $\phi = 1$, $d(r_1) = 0.6$, $d(r_2) = 0.4$, $\pi(r_1) = 100$ and $\pi(r_2) = 1.6$. If $x_2^{r_1} = d(r_1)$ and $x_1^{r_2} = d(r_2)$ at $t = 0$, $\frac{\phi}{N(\boldsymbol{x}(0))} = 5/8$ and $\boldsymbol{x}(0)$ is a RTESS. If we change the cost from $\gamma = 0.9$ to $\gamma' = 1.6$, and all players follow the best response dynamics, r_1 senders keep $l = 2$ but r_2 senders eventually select $l = 0$. At the limit, $x_2^{r_1} = d(r_1)$ and $x_0^{r_2} = d(r_2)$. This equilibrium is inferior because, since the capacity is enough for all senders, there is a single Pareto efficient equilibrium where $x_1^{r_1} = d(r_1)$ and $x_1^{r_2} = d(r_2)$ in the original setting. In addition, any γ' (one-shot cost change) cannot achieve an evolutionary process which converges to the efficient equilibrium[30]. This points to the limitation of the traditional one-shot intervention for information congestion.

We generalize the implication from the example above.

Proposition 2. *Any additional cost per message cannot lead the system to a Pareto efficient state when the original game and $\boldsymbol{x}$ satisfies the following conditions: $l_{max} \geqq 2$, $R = \{1, 2, ...\}$, $D = \phi$, $\pi(1)\frac{\phi}{l_{max}d(1)}(1 - \frac{\phi}{l_{max}d(1)})^{l_{max}-1} > l_{max} \max_{r\in R\setminus\{1\}} \pi(r)$, and $\gamma > \max_{r\in R\setminus\{1\}} \frac{\phi}{l_{max}d(1)}\pi(r)$, $x^1_{l_{max}} = d(1)$ and $x_0^r = d(r)$ for any $r \in R \setminus \{1\}$.*

Proof: Because $\pi(1)\frac{\phi}{l_{max}d(1)}(1 - \frac{\phi}{l_{max}d(1)})^{l_{max}-1} > l_{max}\gamma$ when all type-1 senders send l_{max} messages, their strategy is strictly the best. Others have no incentive to send a message in this congested situation.

When we try a new cost $\gamma' \leqq \max_{r\in R\setminus\{1\}} \pi(r)$ on the congested situation, no senders have incentive to decrease their number of messages. If $\gamma' > \max_{r\in R\setminus\{1\}} \pi(r)$, type-1 senders may decrease the number of messages, but no other senders join to the competition[31]. Since there is a Paretor efficient state where all senders send a message and get the maximum benefit, other states are inefficient,■

[30] If $1.6 > \gamma' > 1.0$, a similar consequence happens. If $1.0 \geqq \gamma'$, the congestion cannot be relaxed. If $\gamma' > 1.6$, r_2 senders completely exit the market.

[31] If we apply any ascending or descending cost changes from γ to the example above, they cannot work by the same logic. We need higher costs to relax the congestion and lower costs to make the small-benefit senders active. Therefore, we introduce the two-step cost change.

5.4 Two-Step Cost Change and Loss Minimizing Equilibrium

Before introducing the exact policy, we first define an outcome, *Loss Minimizing (LM) equilibrium* with an arbitrary small upper-bound of loss $\bar{\sigma} \in (0, 1/2)$ s.t. $\pi_{min}(1 - \bar{\sigma}) > \gamma$ as follows:

Definition 5 (Loss Minimizing Equilibrium with a small $\bar{\sigma}$)**.** *An equilibrium $\boldsymbol{x}$ is a loss minimizing equilibrium if and only if the following conditions are satisfied:*

1. *All active senders send a single message ($x_l^r = 0$ if $l > 1$ $\forall r \in R$).*
2. *The benefit is larger than the cost for the active senders ($\pi(r) > \gamma''$ for any $r \in R$ s.t. $x_1^r > 0$).*
3. *$N(\boldsymbol{x}) = \min\{D, \phi/(1 - \sigma)\}$ where $\bar{\sigma} > \sigma \geqq 0$.*
4. *When there exists a type[32] $r \in R$ s.t. $\pi(r)\frac{\phi}{N(\boldsymbol{x})} = \gamma''$, $x_0^r > 0$ and $x_1^r > 0$.*

where γ'' is the cost considered by senders at the equilibrium. When we discuss the two-step cost change we define later, γ'' is γ'_z at the end of the two-step cost change.

We analyze the characteristics of LM equilibrium and later define the policy which achieves LM equilibrium. LM equilibrium satisfies the following good properties:

Proposition 3. *LM equilibrium $\boldsymbol{x} \in X$ with a small enough $\bar{\sigma}$ is a Pareto efficient state (with the original γ).*

Proof: Because we set $\bar{\sigma}$ s.t $\pi_{min}(1-\bar{\sigma}) > \gamma$, at $\boldsymbol{x}$, $\pi(r)\frac{\phi}{N(\boldsymbol{x})} > \gamma$ for any $r \in R$ s.t. $x_1^r > 0$. If any additional message appears, all active senders at $\boldsymbol{x}$ lose a part of their profits. If any active advertisers withdraw from the competition, the advertisers lose their profit because all active advertisers get positive profits at $\boldsymbol{x}$. When we change the state, either the total number N increases or an advertiser withdraws from the competition. Therefore, $\boldsymbol{x}$ is a Pareto efficient state. ■

Proposition 4. *LM equilibrium with a small $\bar{\sigma}$ is a regular Taylor evolutionarily stable state.*

Proof: see Appendix A.4.

The intuition of Proposition 4 is as follows. We consider a LM equilibrium in which marginal senders exist and are indifferent between sending or not sending a message (because, otherwise a LM equilibrium is an RTESS since all types have a type-specific unique best strategy). If a small portion of active marginal senders stop sending a message, the total message N decreases, and sending a message becomes more attractive for the marginal type. Further, a small portion of non-active marginal senders

[32] We call such a type *marginal type*.

begin to send a message, and the system returns to the original equilibrium. The opposite deviation invites a similar reaction. When a small fluctuation appears, the marginal senders move to the opposite. For other (non-marginal) types, a small fluctuation does not change their optimal strategy.

5.4.1 Two-Step Cost Change

We define *two-step cost change* which makes $\boldsymbol{x}(t)$ ϵ-converge to an LM equilibrium (with $\bar{\sigma}$) in a finite time. In the two-step cost change, the policymaker is required to allow a small congestion (represented by $\bar{\sigma}$) to make the equilibrium locally stable. Let $\frac{\phi}{\underline{N}} \in ((1-\bar{\sigma}), 1)$ s.t. $\pi_{max}(1-\frac{\phi}{\underline{N}}) < \pi_{min}$[33] and $\frac{1}{\frac{\phi}{\underline{N}}} - 1 < \frac{MD}{2\pi_{max}}$[34] denote the lower-bound of the target (inverse) congestion intensity after the policy. If $\frac{\phi}{\underline{N}}$ is close enough to 1, these conditions are satisfied. We need an arbitrary small buffer in the process and select $\bar{\delta}, \underline{\delta}$ s.t. $1 > \delta\frac{\phi}{\underline{N}} > \frac{\phi}{\underline{N}}$ for any $\delta \in [\underline{\delta}, \bar{\delta}]$.

To make the outcome locally stable, any optimal strategy in the equilibrium must be selected by a small portion of senders. To satisfy this condition while achieving ϵ-convergence in finite time, we utilize the minimum unit of population ω[35]. Define $\Pi = \{\pi(r)\min(1, \frac{\phi}{j\omega}) | r \in R, j \in \{0, ..., D/\omega\}$, and we require $\gamma_z \neq \pi$ for any $\pi \in \Pi$ and for any $z \in Z$.

We select the amount for each γ'_z in the following way. Let π_{z-1} denote the $z-1$th largest benefit type. First, $\gamma'_1 = \pi_{max} + 3$ or any arbitrary amount strictly larger than π_{max}. For any $z > 1$, $\gamma'_z = \delta\frac{\phi}{\underline{N}}\pi_{z-1}$ s.t. $\gamma'_z \neq \pi$ for all $\pi \in \Pi$. (In Appendix A.5, we prove the existence of δ and the set of γ_z. In short, because Π is a finite set and $(\underline{\delta}, \bar{\delta})$ is an infinite set, there exists δ.).

For convenience, we define

$$\bar{\epsilon} = (1 - 1/\delta)\left(\frac{\phi}{\underline{N}}\right)^{-1}\frac{\phi}{l_{max}}. \tag{13}$$

We define *two-step cost change* as follows:

Definition 6 (Two-Step Cost Change)**.**

[33] If this condition is satisfied and if $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ for all $t \in [t_z, t_{z+1}]$, no senders change their strategy from 1 to $l > 1$ under the best response dynamics. In the following discussion, we define the lowest γ_z s.t. $\gamma_z > \frac{\phi}{\underline{N}}\pi_{min}$. Therefore, to make senders with π_{max} avoid multiple messages, $\pi_{max}(1-\frac{\phi}{\underline{N}})\frac{\phi}{\underline{N}} < \frac{\phi}{\underline{N}}\pi_{min}$ is sufficient.

[34] If this condition is satisfied, and if $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ for all $t \in [t_z, t_{z+1}]$, for any $r \in R$, the range of possible expected gain from sending a message $[\pi(r)\frac{\phi}{\underline{N}} - \gamma_z, \pi(r) - \gamma_z]$ is narrow. Thus, there is only a single $r \in R$ s.t. $\pi(r) > \gamma'_z$ but $\pi(r)\frac{\phi}{\underline{N}} < \gamma'_z$ where γ'_z is defined later.

[35] This minimum unit is required because we assume that we can change the cost only a finite number of times. Without this minimum unit, we sometimes meet technical problems. For example, if $\pi(r)\min(1, \frac{\phi}{j\omega}) = \gamma$ where $j \in \{1, ..., D/\omega\}$ for $r \in R$, and if $D = d(r) = j\omega$, $\boldsymbol{x}$ s.t. $x_1^r = d(r)$ is an equilibrium but not RTESS because $x_0^r = 0$. If we know ω, we can avoid the finite set of such cases where continuous parameters coincide.

1. *Change the cost from γ to γ'_1, and define variables $z = 1$, $i = 1$ and $t_0 = 0$.*

2. a. *Wait for either*

$$(N(\boldsymbol{x}(t)) - l_{max}\epsilon_z) > \phi \tag{14}$$

or

$$(N(\boldsymbol{x}(t)) + l_{max}\epsilon_z) < \phi \tag{15}$$

and $T(\frac{\epsilon_z}{n}) < (t - t_{z-1})$ where $\bar{\epsilon} > \epsilon_z > 0$. (If neither appears forever, the limit is the intended outcome.) Define $t_z = (2t - t_{z-1})$. Update i s.t. $i = z$.

b. *Wait until $t = t_z$. If $N(\boldsymbol{x}(t)) - l_{max}\epsilon > \phi$ happens in $2.a.$, stop this process and define $t_{z+1} = \infty$. Otherwise, update z s.t. $z = i + 1$, change the cost from γ'_i to γ'_z and return to $2.a.$.*

The number of the required cost changes is at maximum finite $(n + 1)$, and the cost change is finished in finite time.

We claim the following main proposition.

Proposition 5. *From any initial state $\boldsymbol{x}(0)$, by the two-step cost change, $\boldsymbol{x}(t) \to \boldsymbol{x}^*$ and $\boldsymbol{x}^*$ is a LM equilibrium with $\bar{\sigma}$.*

Proof: See Appendix A.6.

The proof in Appendix A.6 is based on the following logic. Since we assume the best response dynamics, a strictly best strategy for type r would be selected by at least $d(r) - \epsilon$ amount of senders if the strategy has been strictly best for r from t' to t'' s.t. $T(\epsilon) = t'' - t'$. Consider $\phi < D$. Step 1 makes sending 0 messages strictly best for all $r \in R$, and eventually $(N(\boldsymbol{x}(t_1)) + l_{max}\epsilon_1) < \phi$ where $t_1 = T(\epsilon_1/n)$ and $\bar{\epsilon} > \epsilon_1 > 0$. In step 2, for a positive integer z, if we slightly decrease the cost from γ'_z to γ'_{z+1} s.t. $\gamma'_z - \gamma'_{z+1} < MD/2$ at t_z, at maximum a single type $r \in R$ begins to send a message. If $(N(\boldsymbol{x}(t)) + l_{max}\epsilon_z) < \phi$ happens, the amount of senders whose benefit is larger than γ_z is less than the capacity ϕ. Using this information, if we change the cost from γ_z to γ_{z+1}, all possible $N(\boldsymbol{x}(t))$ are lower than $\phi/\frac{\phi}{\underline{N}}$ after the cost change at t_z (Lemma 3 in Appendix A.6.). We repeat such a cost change and check the ϵ-convergence until $(N(\boldsymbol{x}(t)) - l_{max}\epsilon_k) \geqq \phi$ where k is a positive integer. If the condition appears, without any additional cost change, $\boldsymbol{x}(t)$ eventually enters a sufficiently close neighborhood of a LM equilibrium $\boldsymbol{x}^*$, and so it converges to $\boldsymbol{x}^*$. If both conditions never appear for γ_z, there exists the exact ϕ amount of senders whose benefit is larger than γ_z. As $t \to \infty$, in $\boldsymbol{x}(t)$, all such senders eventually select sending a message, and thus it converges to a LM equilibrium $\boldsymbol{x}^*$.

Proposition 5 means that we achieve LM equilibrium by the two-step cost change, regardless of both the initial condition and the distribution of senders' benefit. The policymaker does not need to know the distribution of senders' benefit and does not have to observe the number of active senders and the benefit of marginal senders in each equilibrium.

The two-step cost change for LM equilibrium has three weak points. First, we need time to implement the policy. Second, we assume that the distribution of senders' benefit is unchanged; however, this may not be realistic in the long term. Third, this policy may need an external budget.

The discussion of the two-step cost change and LM equilibrium indicates that, if we have enough time and can control the unit cost, we can fully utilize attention while avoiding an unnecessary amount of unsolicited messages, even if we cannot observe the private information of players, such as senders' benefit distribution, and cannot monitor the number of messages from each sender.

5.5 Evolutionary Implementation

The two-step cost change can be classified as an application of evolutionary implementation proposed by Sandholm (2002, 2005, 2007). When a policymaker observes players' behaviors only at an aggregate level, it is sometimes difficult to achieve a desirable state. For example, if the policymaker subsidies an action, the subsidy may invite too many types of players to select the action. Evolutionary implementation aims to solve inefficiencies in such anonymous situations by using evolutionary dynamics. In this study, we define evolutionary implementation as a policy which leads the target system, in which players can behave anonymously, to a desirable state by using evolutionary dynamics[36]. Our policy satisfies this definition.

However, The two-step cost change is completely different from the traditional approach, which uses a *potential function*. Sandholm (2002, 2005, 2007) analyzes the situation where the externality among players is symmetric. In the symmetric-externality case, there exists a price scheme on actions such that the price scheme equalizes the private cost of the action with its social cost. Under such a price scheme, each player gradually behaves in accordance with the policymaker's intention. In the congestion game with such a price scheme, Sandholm (2002) finds the potential function representing the total of players' utility. When such a potential function exists, standard evolutionary dynamics lead the system toward the direction in which the total utility increases. In addition, the negative externality

[36]Our definition is slightly different from the original discussion in Sandholm (2002). Sandholm (2002) selects a large set of evolutionary dynamics (called admissible dynamics), and the price scheme invites the social optimum as the limit of consequence of all dynamics in the set. The discussions in previous literature focus on the robust implication among wide ranges of dynamics.

in the congestion game makes the potential function strictly concave, and thus there exists a unique equilibrium with global stability. In a similar negative-externality setting, Sandholm (2005) considers the additional factor, idiosyncratic payoffs, which depend only on the type of players and the action. Sandholm (2007) discusses general settings where the externality is not necessarily negative. Because the externality can be complicated, multiple equilibria can appear. By applying stochastic dynamics, Sandholm (2007) shows that the price scheme makes the system almost always stay efficient in the long run. As a recent advancement, Lahkar and Mukherjee (2019, 2021) find a methodology that broadens the application range of the potential-function approach. Their methodology requires a type-independent externality based on the *aggregate strategy level* instead of a symmetric externality[37]. The current literature on the potential-function approach assumes that, before policies are designed, the policymaker knows/estimates the aggregate externality of each action at each state[38].

We focus on the situation where a policymaker does not know the externality at each state and, therefore, cannot apply the proposed approach with a potential function. Instead, our approach uses the unique characteristics of information congestion. In the two-step cost change, the initial step leads the system to a certain condition by creating a strictly dominant strategy for players. The second step makes the system move from the condition to the desirable equilibrium. This process is possible because the characteristic of the desirable equilibrium is well known. In our approach, the policymaker does not have to understand the details of externalities among players. Further, the policymaker may not know the details even after the policy ends. In addition, the number of required cost changes is finite. Our approach guarantees the global convergence but its outcome is typically not globally stable.

6 Conclusion

Under the primitive structure of advertising, advertisers select an excessive amount of unsolicited advertising in stable equilibrium even if the benefit of repetition is ignored. This structural inefficiency occurs because the private marginal cost of advertising does not take into account the decreasing performance in consumers' ability to process information. Traditional one-shot interventions do not work

[37]If an externality of each action is independent of the type of the player who performs the action, there exists a price which represents the externality of each action at each state. Lahkar and Mukherjee (2019) focus on a public goods game, and Lahkar and Mukherjee (2021) analyze general settings where a variable, called aggregate strategy level, can represent the impact of each state. A symmetric externality is a special case of type-independent externality where the impact of the externality on each type is identical. However, the impact of each state in Sandholm (2002, 2005, 2007) is not necessarily represented by a single variable. Therefore, both methods complement each other.

[38]We do not claim that this requirement is necessary for the potential-function approach because the process of policies can reveal the externality and the type distribution.

well, and we need costly interventions to efficiently utilize receivers' attention. Our discussion shows the difficulty of controlling advertising without an advanced structure.

Our main conclusion, the inefficiency of the primitive structure, potentially explains why we have seen recent advancements in advertising structures (e.g., Rust and Oliver 1994, Dahlen and Rosengren 2016). Typically, under new types of advertising, consumers are more active, and the communication between consumers and advertisers is (relatively) interactive[39]. Additionally, when advertisements are not well targeted to consumers, consumers regard them as nuisances and try to avoid such advertising via new technology/legislations[40]. These advanced structures/factors can overcome the limitation derived from information congestion.

This paper contributes to the field of information congestion. We analyze the theoretical model of information congestion in which heterogeneous senders can send multiple messages. We assume a finite number of senders' benefit types. Hence, the model fits the population game framework. We analyze equilibria by referring to the known concepts[41] of stability in the framework.

We find several new results which are useful for understanding information congestion. First, a suboptimal stable equilibrium sometimes appears. Thus, interventions are, at times, required to make the system efficient. Second, sometimes any static pricing, which previous literature supports, fails to achieve an efficient allocation. Due to this, we must think dynamic interventions. Third, the two-step cost change is effective, regardless of the initial condition and the distribution of the senders' benefit. It achieves loss minimizing equilibrium with local stability and Pareto efficiency. Our main results support the main policy implications in previous studies such as Van Zandt (2004) and Anderson and de Palma (2013). The primitive structure of advertising can cause efficiency loss in the allocation of limited attention. Interventions sometimes significantly improve the performance of advertising. However, we additionally note that the required interventions are costly and so primitive unsolicited advertising has a structural limitations.

Our results can be interpreted as "gleaning" through the many discussions that use a random assignment as a tie-breaking rule. For example, in a sealed auction, if the bidders believe that other bidders are submitting the same bid price as theirs, they may bid the same price multiple times by

[39]For example, behavior-based (conditional on consumption history) advertising in Shen and Villas-Boas (2018), advertising on nonretail platforms in Eliaz and Spiegler (2020) and skippable advertising and weak/strong conversion in Dukes and Liu (2023).

[40]For example, targeting advertising and ad avoidance in Johnson (2013), and Do Not Call policy in Goh et al. (2015). In particular, our discussion complements Johnson (2013) because both approaches get similar implications that the efficiency of advertising with simple structures is limited.

[41]For example, Regular Taylor evolutionary stable state is a sufficient condition of local stability under many types of dynamics.

borrowing their friends' names. When each bidder follows this logic, information congestion occurs. Our result supports an entry payment for auctions by revealing the repercussions that occur without the entry payment. In many mechanisms, senders can reveal their preferences, and we can achieve efficient allocation by using their preferences. Our study supplies the benchmark without the information regarding senders' (and receivers') preferences. Thus, this benchmark is useful to clarify the value of revealing/collecting private information in many studies with a random tie-breaking rule.

A Appendix

A.1 How to Derive Best Response Diagram (Figure 1)

Each curve in Figure 1 is the indifference boundary between two strategies of senders such as (0,1), (1,2) and (3,4)[42]. Consider a sender with type $r > 0$ and given $\boldsymbol{x}$ s.t. $N(\boldsymbol{x}) > \phi$. We can derive the indifference boundary between $(k, k+1)$ from the difference of the utilities as follows:

$$\begin{aligned} & U(r, k+1, \boldsymbol{x}) - U(r, k, \boldsymbol{x}) = 0 \\ \iff & \pi(r)\frac{\phi}{N(\boldsymbol{x})}\left(1 - \frac{\phi}{N(\boldsymbol{x})}\right)^k = \gamma \\ \iff & \frac{\gamma}{\pi(r)} = \frac{\phi}{N(\boldsymbol{x})}\left(1 - \frac{\phi}{N(\boldsymbol{x})}\right)^k. \end{aligned} \tag{16}$$

We only need to focus on the boundaries of two consecutive numbers because of the following lemma:

Lemma 2. *For a given $0 < \frac{\phi}{N(\boldsymbol{x})} < 1$, the best strategies for type r are either a single number or two consecutive numbers.*

Lemma 2 indicates that the set of best response strategies for each $\boldsymbol{x}$ includes at maximum two consecutive numbers. The other sets of strategies, such as $\{0, 2\}$ and $\{1, 3, 4\}$, cannot be a set of best strategies.

Proof of Lemma 2: Consider $0 < \phi/N < 1$. The utility function conditional on $\boldsymbol{x}$, denoted by $U_C(l)$, is strictly concave in $l \geqq 1$ if $\gamma \leqq \pi(r)\phi/N$. If and only if $\gamma < \pi(r)\phi/N$ is not satisfied, only 0 is the optimal strategy. When $\gamma = \pi(r)\phi/N$, $l = 1$ and $l = 0$ have the same utility ($= 0$) and both are optimal. When $\gamma < \pi(r)\phi/N$, $l = 1$ strictly dominates $l = 0$.

[42] We focus on the strategies $l < 6$ in Figure 1.

From now on, we focus on $l \geqq 1$ and $\gamma < \pi(r)\phi/N$. Since $U_C(l)$ is a single-variable function, when we focus on the discrete set of $l > 0$, we can simply do the convex extension. By the definition of strictly concave function, for any $t \in (0,1)$ and $j, k \geqq 1$,

$$tU_C(j) + (1-t)U_C(k) < U_C(tj + (1-t)k). \tag{17}$$

This indicates that, if l is a continuous number and the two strategies have the same utility, the third strategy with the same utility cannot exist. When we consider the subset of the strategy set, this implication holds. Thus, in the discrete case, we do not have to consider the case in which three strategies have the same utility.

In addition, because of (17), the two strategies with the same maximum utility are consecutive numbers. Suppose both $k1$ and $k2$ maximize the utility function, but they are not consecutive. Then, we can find at least an integer $k3$ between $k1$ and $k2$ and t such that $k3 = tk1 + (1-t)k2$. $U_C(k3)$ is larger than $U_C(k1) = U_C(k2)$ due to (17). This contradicts the assumption that $k1$ and $k2$ maximize the utility function.

A.2 How to Derive Bifurcation Diagram (Figure 2)

Consider homogeneous senders $R = \{1\}$. From the discussion in A.1, the set of optimal strategies in $\boldsymbol{x} \in X$ includes two consecutive numbers at maximum. We can then figure out the equilibria with the given parameters $\frac{\gamma}{\pi}$ and $\frac{\phi}{D}$ without complex calculations through the following way.

First, we can calculate the possible range of $\frac{\phi}{N(\boldsymbol{x})}$ with the pair of strategies. For instance, if $\frac{\phi}{D} = 1$, the pair (1,2) can achieve $\frac{1}{2} \leqq \frac{\phi}{N(\boldsymbol{x})} \leqq 1$. Second, from Figure 1, we know that the best response of senders in each $\frac{\phi}{N(\boldsymbol{x})}$ and $\frac{\gamma}{\pi}$. Third, by combining the boundaries in Figure 1 and the ranges, we can draw Figure 2 which illustrates all possible equilibria with the parameters.

A.3 Stability of Equilibria in Figure 2

Consider $\frac{\gamma}{\pi} = 0.2$ which is represented by the area in the orange rectangle in Figure 2. Figure 3 zooms in the orange rectangle. The arrows in Figure 3 illustrate the direction of movement in the system if we apply the best response or any impartial pairwise comparison dynamic to $\boldsymbol{x}$ in which $N = N(\boldsymbol{x})$ and all senders select the strategies in which the number of sending messages is either of two consecutive numbers like sending a message once and twice.

We check the stability of equilibria A and C in Figure 3. Consider the C's neighborhood $O =$

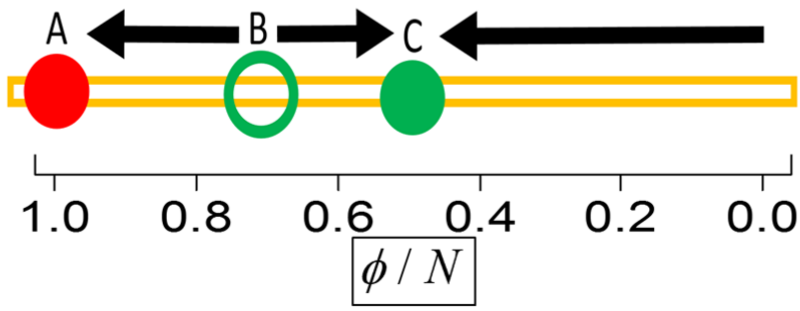

Figure 3: An Example of Stable Equilibria

$\{\boldsymbol{y} \in X : 0.4 < \phi/N(\boldsymbol{y}) < 0.6\}$. By the same logic in A.1, at least in $0.28 < \phi/N(\boldsymbol{y}) < 0.72$, the strategy of sending a message twice is the unique best response. Therefore, both (5) and (6) are satisfied. Further, equilibrium C is a regular Taylor evolutionarily stable state. By a similar logic, equilibrium A is a regular Taylor evolutionarily stable state, too.

A.4 Stability of LM equilibrium

We check whether a LM equilibrium meets the definition of Regular Taylor Evolutionarily Stable State (hereafter RTESS). For all $r, r' \in R,\ l, l' \in L$ and $\boldsymbol{x} \in X$, all of U's partial derivatives $\frac{\partial U(r,l,\boldsymbol{x})}{\partial x_{l'}^{r'}}$ exist and are continuous in X so that U is continuously differentiable. Therefore, U is Lipschitz continuous in X.

We defined RTESS in equations (5) and (6) using the definition adopted from Sandholm (2010). In LM equilibrium, if $D \leqq \phi$, there does not exist any marginal type. When each type has a unique optimal strategy, both (5) and (6) are satisfied, and such an equilibrium is a RTESS.

If $D > \phi$ and $\phi/N(\boldsymbol{x}) \leqq 1$, and if there does not exist any marginal type, we can apply a similar logic in the previous paragraph, and $\boldsymbol{x}$ is a RTESS. If there exists a marginal type, from the definition of LM equilibrium, the active senders with the minimum benefit among the active senders (hereafter senders with r_{amin}) are indifferent between sending no messages and sending a message. In addition, from the definition, $x_0^{r_{amin}} > 0$ and $x_1^{r_{amin}} > 0$. Therefore, (5) is satisfied.

When (5) is satisfied, the support of $\boldsymbol{x}$ includes all optimal strategies for each type. Then, the condition $(\boldsymbol{y} - \boldsymbol{x})'U(\boldsymbol{x}) = 0$ in (6) can be satisfied only if the support of $\boldsymbol{x}$ is weakly larger than the support of $\boldsymbol{y}$. Thus, we focus on the changes in r_{amin}'s strategy between sending a message or not. We denote such a change by $\boldsymbol{z}$. Instead of (6), we use the following condition from p282 in Sandholm (2010).

$$\boldsymbol{z}' DU(\boldsymbol{x})\boldsymbol{z} < 0 \ \forall \text{ nonzero } \boldsymbol{z} \in TX \cap \mathbb{R}^n_{L(\boldsymbol{x})} \tag{18}$$

where $L(\boldsymbol{x})$ is the support of $\boldsymbol{x}$, and $\boldsymbol{y} \in \mathbb{R}^n_{L(\boldsymbol{x})}$ indicates $\boldsymbol{y} \in \mathbb{R}^n$ and the support of y is a subset of $L(\boldsymbol{x})$.

Discuss $\boldsymbol{z}_{01} = \boldsymbol{e}_q - \boldsymbol{e}_s$ such that $\boldsymbol{e}_q/\boldsymbol{e}_s$ represents a unit population of type r_{amin} which sends 0 messages/ 1 message. ($\boldsymbol{e}_i$ is a standard basis of $\mathbb{R}^n$.). In this case, if

$$\frac{\partial U(r_{amin}, 0, \boldsymbol{x})}{\partial x_0^{r_{amin}}} - \frac{\partial U(r_{amin}, 0, \boldsymbol{x})}{\partial x_1^{r_{amin}}} < \frac{\partial U(r_{amin}, 1, \boldsymbol{x})}{\partial x_0^{r_{amin}}} - \frac{\partial U(r_{amin}, 1, \boldsymbol{x})}{\partial x_1^{r_{amin}}} \tag{19}$$

is satisfied, the equation (18) is satisfied.

$$\frac{\partial U(r_{amin}, 0, \boldsymbol{x})}{\partial x_0^{r_{amin}}} - \frac{\partial U(r_{amin}, 0, \boldsymbol{x})}{\partial x_1^{r_{amin}}} = 0 \tag{20}$$

and

$$\frac{\pi(r_{amin})\phi}{N(\boldsymbol{x})^2} = \frac{\partial U(r_{amin}, 1, \boldsymbol{x})}{\partial x_0^{r_{amin}}} - \frac{\partial U(r_{amin}, 1, \boldsymbol{x})}{\partial x_1^{r_{amin}}}. \tag{21}$$

Any other nonzero $\boldsymbol{z} \in TX \cap \mathbb{R}^n_{L(\boldsymbol{x})}$ is the scalar multiplication of $\boldsymbol{z}_{01}$ and the sign of $\boldsymbol{z}' DU(\boldsymbol{x})\boldsymbol{z}$ is unchanged. Therefore, LM equilibrium is a RTESS.

A.5 Existence of δ and γ'_z s.t. $\gamma'_z \neq \pi$ for any $\pi \in \Pi$

For an arbitrary $\delta \in (\underline{\delta}, \bar{\delta})$, we define $\gamma'_z(\delta) = \delta \frac{\phi}{N} \pi_{z-1}$ for any $z \in \{2, 3, ..., |Z|\}$. We claim that there exists $\delta'' \in (\underline{\delta}, \bar{\delta})$ s.t. $\gamma'_z(\delta'') \neq \pi$ for all $\pi \in \Pi$ and $z \in \{1, 2, 3, ..., z_{max}\}$.

Consider a $\delta \in (\underline{\delta}, \bar{\delta})$. If any $\delta'' \in (\underline{\delta}, \bar{\delta})$ satisfies the condition, the claim is correct. If the condition is not satisfied by δ, there exists at least a pair of $\pi \in \Pi$ and $z \in \{1, 2, 3, ..., z_{max}\}$ s.t. $\gamma'_z(\delta) = \pi$. Among the other pairs (π, z) s.t. $\gamma'_z(\delta) \neq \pi$, we can find the minimum difference $|\gamma'_z(\delta) - \pi| \geqq M$. Because $\gamma'_z(\delta)$ is differentiable and strictly increasing in δ, we can slightly decrease from δ to $\delta'' \in (\underline{\delta}', \delta)$ s.t. $M > |\gamma'_z(\delta) - \gamma'_z(\delta'')| > 0$ for any $z \in \{1, 2, 3, ..., z_{max}\}$. As a result, we find $\delta'' \in (\underline{\delta}, \bar{\delta})$ s.t. $\gamma'_z(\delta'') \neq \pi$ for all $\pi \in \Pi$ and $z \in \{2, 3, ..., |Z|\}$.

A.6 Proof of Proposition 5

We fix $\frac{\phi}{N}$ and δ which satisfy the required conditions (for a given $\bar{\sigma}$) explained in the main text. For given $\frac{\phi}{N}$ and δ, a set of γ'_z and $\bar{\epsilon}$ are uniquely decided. The timing of switching from γ'_z to γ'_{z+1}, denoted by t_z, depends on the reaction of $\phi/N(\boldsymbol{x}(t))$.

We would like to show $\boldsymbol{x}(t) \to \boldsymbol{x}^*$ and $\boldsymbol{x}^*$ is LM equilibrium (with $\bar{\sigma}$) if the two-step cost change is conducted. When $\phi \leqq D$, an equilibrium $\boldsymbol{x}^*$ is a LM equilibrium if and only if $\frac{\phi}{N(\boldsymbol{x}^*)}$ is 1 or almost 1 ($\frac{\phi}{N(\boldsymbol{x}^*)} > (1-\bar{\sigma})$), all active senders send a single message, and all best strategies for each type are utilized, and the benefit is larger than the cost for all active senders. Because of $\frac{\phi}{\underline{N}}$'s definition, $\frac{\phi}{\underline{N}} > (1-\bar{\sigma})$, and the first condition is satisfied if $\frac{\phi}{N(\boldsymbol{x}^*)} > \frac{\phi}{\underline{N}}$ and $\frac{\phi}{N(\boldsymbol{x}^*)} \leqq 1$.

After Step 1, since sending 0 messages is the strictly dominant strategy among all senders for γ_1', $(N(\boldsymbol{x}(t)) + l_{max}\epsilon_1) < \phi$ eventually happens in 2.a. Further, there exist at least $D - \epsilon_1$ senders who send 0 messages and at maximum ϵ_1 senders who are going to select 0 messages. In 2.b., at t_1, we change the cost from γ_1' to γ_2'. We return to 2.a.

Before checking the whole consequence of each process in Step 2, we want to confirm a fact about this process. Because of the definition of $\frac{\phi}{\underline{N}}$, if we keep $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$, for any $r \in R$, sending a message is strictly better than any $l > 1$. We claim that $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ for all $t \geqq t_1$ in this policy, and in the following paragraphs, we check this claim.

When we change the cost from γ_1' to γ_2' at t_1, $N(\boldsymbol{x}(t_1)) < \phi$. Under γ_2', because $\frac{\phi}{\underline{N}}$ is large enough, only senders with π_{max} have a larger benefit than γ_2'. For any other type, the unique optimal strategy is sending 0 messages regardless of $\boldsymbol{x}$, and ϵ-convergence happens. Later, as Lemma 3, we prove that, for any $z \in \{1, 2, 3, ..., z_{max}\}$ s.t. t_z exists, $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ for any $t \geqq t_z$ if we stop the cost change after t_z.

If we observe $N(\boldsymbol{x}(t_2)) + l_{max}\epsilon_2 < \phi$ at the end of 2.a, in the whole population D, there exist less than $N(\boldsymbol{x}(t_2)) + \epsilon_2$ amount of senders whose benefit is strictly larger than γ_2'. (Otherwise, some portion of senders in $(D - \epsilon_2)$ have to select the suboptimal strategy.) At t_2, we change the cost from γ_2' to γ_3'.

For any $z > 2$, consider the upper bound (hereafter $\bar{N}$) of $N(\boldsymbol{x}(t))$ after t_z if we ignore the further cost changes. There exists a marginal type ($z-1$th largest benefit type) $r' \in R$ s.t. $\gamma_z' > \pi(r') > \gamma_{z+1}'$. We suppose $d(r') > \pi(r')\phi/\gamma_{z+1}'$ because this setting would maximize the possible N. If we focus on $D - \epsilon_z$ with the optimal strategy for γ_z', any senders except type r' does not have any incentive to change their strategies under γ_{z+1}' as long as $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$. At t_z, $\frac{\phi}{N(\boldsymbol{x}(t_z))} > \frac{\phi}{\underline{N}}$.

After t_z, as long as $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$, only the marginal r' senders in $D - \epsilon_z$ and ϵ_z senders can increase messages under the best response dynamics. After t_z, as long as $\gamma_{z+1}' < \frac{\phi}{N(\boldsymbol{x}(t))}\pi(r')$, r' senders change their strategy from 0 to 1. Because we assume enough $d(r')$, if we ignore ϵ_z senders, the minimum inverse congestion ratio would be $\frac{\phi}{N}$ s.t. $\gamma_{z+1}' = \frac{\phi}{N}\pi(r')$, and thus the total amount of messages is $\frac{\pi(r')\phi}{\gamma_{z+1}'} = \phi/\left(\delta\frac{\phi}{\underline{N}}\right)$. Even if we consider ϵ_z, the total amount cannot be larger than

$\phi/\left(\delta\frac{\phi}{\underline{N}}\right) + l_{max}\epsilon_z$. Therefore, $\bar{N} = \phi/\left(\delta\frac{\phi}{\underline{N}}\right) + l_{max}\epsilon_z$ as long as $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ after t_z. From the definition of $\bar{\epsilon} > \epsilon_z$ and δ, $\bar{N} < \phi/\frac{\phi}{\underline{N}}$. $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ is satisfied at t_z, and the possible $\bar{N}$ is too small to break the condition from $\boldsymbol{x}(t_z)$ under the best response dynamics. In conclusion, $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ if $t \geqq t_z$ and if there are no additional cost changes.

We can repeat the same argument for each $[t_z, t_{z+1}]$ as long as $t_z < \infty$. Therefore,

Lemma 3. *In Step 2 after* t_1, $\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{\underline{N}}$ *under the best response dynamics.*

Next, we check whether $\boldsymbol{x}(t)$ converges to LM equilibrium in Step 2. Suppose that the cost change continues until t_k when we change the cost from γ_k to γ_{k+1}. If $N(\boldsymbol{x}(t_{k+1})) + l_{max}\epsilon_{k+1} < \phi$, there are not enough senders under γ_{k+1} to fill the capacity ϕ, and thus we move to the next step by changing the cost from γ_{k+1} to γ_{k+2}.

If $N(\boldsymbol{x}(t)) - l_{max}\epsilon_{k+1} > \phi$, this policy stops the cost change (and any additional intervention is not required). Because of Lemma 3, $\frac{\phi}{N(\boldsymbol{x}(t))} \geqq \frac{\phi}{\underline{N}}$. Since we take $\frac{\phi}{\underline{N}}$ s.t. $\pi_{max}(1/\frac{\phi}{\underline{N}} - 1) < \frac{MD}{2}$, there is only a single type r_{mar} whose optimal strategy fluctuates between 0 and 1 after the stop. Let N_{mar} denote the total amount of messages s.t. $\pi(r_{mar})\frac{\phi}{N_{mar}} = \gamma_{k+1}$. When $\frac{\phi}{N(\boldsymbol{x}(t))} < \frac{\phi}{N_{mar}}$, $\dot{x}_1^{r_{mar}} > 0$, and vice versa. Let $\epsilon'(t)$ in $t \in [t_k, \infty)$ denote the upper bound of the total population who selects suboptimal strategies in $r \in R - \{r_{mar}\}$[43].

As $t \to \infty$, at least $\sum_{r \in R - \{r_{mar}\}} d(r) - \epsilon'(t)$ senders select the unique optimal strategy for each type. Let $N_{-r_{mar}}$ denote the limit amount of messages from all $r \neq r_{mar}$. From the process of the two-step cost change, we know $N_{-r_{mar}} < \phi$[44]. Since $\epsilon'(t)$ is monotonically decreasing, the total amount of messages from the non-marginal types must be between $N_{-r_{mar}} - l_{max}\epsilon'(t)$ and $N_{-r_{mar}} + l_{max}\epsilon'(t)$. In addition, since $N(\boldsymbol{x}(t)) - l_{max}\epsilon_{k+1} > \phi$, we claim that

$$N_{-r_{mar}} + d(r_{mar}) > \phi. \tag{22}$$

This is because if $N_{-r_{mar}} + d(r_{mar}) \leqq \phi$, more than ϵ_{k+1} senders have to select the suboptimal strategy at t_{k+1}, but this is impossible.[45].

[43]Such $\epsilon'(t)$ exists because there exists a unique optimal strategy for each $r \in R - \{r_{mar}\}$.

[44]Otherwise, the process would stop before r_{k+1}.

[45]Even for the marginal type, the population selecting $l > 1$ cannot be larger than ϵ_{k+1}. When $N_{-r_{mar}} + d(r_{mar}) = \phi$, even if $d(r_{mar}) - \epsilon_{k+1}/n$ senders in r_{mar} selects 1 and all ϵ_{k+1} senders select l_{max}, the maximum total amount $N(\boldsymbol{x}(t_{k+1}))$ is lower than $N_{-r_{mar}} + d(r_{mar}) + l_{max}\epsilon_{k+1} = \phi + l_{max}\epsilon_{k+1}$. This is not enough for satisfying $N(\boldsymbol{x}(t_{k+1})) - l_{max}\epsilon_{k+1} > \phi$, and thus $N_{-r_{mar}} + d(r_{mar}) > \phi$.

If $d(r_{mar})$ is small s.t. $N_{mar} - N_{-r_{mar}} > d(r_{mar})$, there exists $\bar{t}$ s.t. for any $t > \bar{t}$,

$$\frac{\phi}{N(\boldsymbol{x}(t))} > \frac{\phi}{N_{mar}}. \tag{23}$$

First, since $N_{-r_{mar}} + l_{max}\epsilon'(t)$ is a strictly decreasing function in t, we can find t s.t. the possible total amount of messages from the non-marginal types would be arbitrarily close to $N_{-r_{mar}}$. Second, for the marginal type, the best strategy is either 0 or 1 (by Lemma 3). The other strategies are always strictly suboptimal for r_{mar}, and thus the possible maximum amount of messages converges to $d(r_{mar})$ as $t \to \infty$. Since $N_{mar} - N_{-r_{mar}} > d(r_{mar})$, we can find $\bar{t}$ s.t. (23) for any $t > \bar{t}$. Thus, after $t > \bar{t}$, for all type, there is a unique best strategy, and the system converges to a unique point $\boldsymbol{x}^* \in X$. This point satisfies the definition of LM equilibrium.

$d(r_{mar})$ s.t. $N_{mar} - N_{-r_{mar}} = d(r_{mar})$ cannot happen because of the definition of γ_z. When we decide γ_z, we make $\gamma_z \neq \pi$ for all $\pi \in \Pi$ where Π is the set derived from the minimum basic unit ω.

If there is $d(r_{mar})$ s.t. $N_{mar} - N_{-r_{mar}} < d(r_{mar})$, the system eventually reaches to $N_{mar} + l_{max}\epsilon'(t') \geqq N(\boldsymbol{x}(t)) \geqq N_{mar} - l_{max}\epsilon'(t')$ at a finite time after t'. For example, if $N(\boldsymbol{x}(t)) < N_{mar} - l_{max}\epsilon'(t')$, it must be broken in finite time. For the sake of contradiction, suppose the inequality remains forever. Because of the best response dynamics and because sending one message is the strict best strategy for r_{mar} (Lemma 3), $x_1^{r_{mar}}(t) \to d(r_{mar})$ as $t \to \infty$. $x_1^{r_{mar}}(t)$ is strictly increasing in t and bounded, and thus $x_1^{r_{mar}}(t)$ converges to a point. For the other type and the other strategies, because of a unique optimal strategy for each type, they converge to a point. Therefore, $\boldsymbol{x}(t)$ converges to a point denoted by $\boldsymbol{x}^* \in X$ where all marginal senders send a message. Thus, in $\boldsymbol{x}^*$, $N_{mar} < N(\boldsymbol{x}^*)$. However, under the best response dynamics, $\boldsymbol{x}(t)$ can converge to a point if and only if the point is a Nash equilibrium. When $N_{mar} < N(\boldsymbol{x}^*)$, $\boldsymbol{x}^*$ is not a Nash equilibrium because sending a message is suboptimal for r_{mar}, and thus we get the contradiction. We can apply the similar discussion to the case $N(\boldsymbol{x}(t)) > N_{mar} + l_{max}\epsilon'(t')$. Therefore, at finite time, $N_{mar} + l_{max}\epsilon'(t') \geqq N(\boldsymbol{x}(t)) \geqq N_{mar} - l_{max}\epsilon'(t')$ for any $t'(> t_k)$ happens.

A LM equilibrium $\boldsymbol{x}^*$ exists for γ_{k+1} and $d(r_{mar})$ s.t. $N_{mar} - N_{-r_{mar}} < d(r_{mar})$. Since there can exist at maximum a single marginal type in LM equilibrium, for any $r \in R - \{r_{mar}\}$, $x_1^r = d(r)$ if $\pi(r) > \gamma_{k+1}$, and otherwise $x_0^r = d(r)$ in $\boldsymbol{x}^*$. Thus, the total amount of messages from all non-marginal types is $N_{-r_{mar}}$. Suppose in $\boldsymbol{x}^*$, $x_1^{r_{mar}} = N_{mar} - N_{-r_{mar}}$ and $x_0^{r_{mar}} = d(r_{mar}) - x_1^{r_{mar}}$. This $\boldsymbol{x}^*$ satisfies the all requirements for a LM equilibrium if $d(r_{mar}) \neq N_{mar} - N_{-r_{mar}}$. Because of the definition of γ_z, $\pi \neq \gamma_z$ for all $\pi \in \Pi$, and thus $d(r_{mar}) \neq N_{mar} - N_{-r_{mar}}$. In addition, $\phi < N(\boldsymbol{x}^*) < \phi / \frac{\phi}{\underline{N}}$. Therefore, there exists a LM equilibrium $\boldsymbol{x}^*$ for γ_{k+1}. Because LM equilibrium is locally asymptotically stable under the best response dynamics, if $\boldsymbol{x}^*$ is a LM equilibrium, there exists a neighborhood O around $\boldsymbol{x}^*$ in which the system is attracted to $\boldsymbol{x}^*$.

As $t \to \infty$, $\epsilon'(t) \to 0$, and thus all of the other types selects the optimal strategy in a LM equilibrium $\boldsymbol{x}^*$ at the limit of $\boldsymbol{x}(t)$ ($N_{mar} - l_{max}\epsilon'(t) \to N_{mar}$, $N_{mar} + l_{max}\epsilon'(t) \to N_{mar}$). Except $x_1^{r_{mar}}$ and $x_0^{r_{mar}}$, all x_l^r converges to the value in $\boldsymbol{x}^*$. In addition, from the previous paragraph, $N(\boldsymbol{x}(t))$ approaches to any neighborhood of $N(\boldsymbol{x}^*) = N_{mar}$ at least in a finite time. Therefore, $\boldsymbol{x}(t)$ eventually enters O, where O is the neighborhood of LM equilibrium $\boldsymbol{x}^*$ which has the local asymptotic stability under the best response dynamics. Thus, $\boldsymbol{x}(t)$ converges to $\boldsymbol{x}^*$.

We show that, if $N(\boldsymbol{x}(t)) - l_{max}\epsilon \leqq \phi \leqq N(\boldsymbol{x}(t)) + l_{max}\epsilon$ for all $t \in [t_k, \infty)$ where ϵ satisfying $T(\frac{\epsilon}{n}) < (t - t_k)$ and $\bar{\epsilon} > \epsilon > 0$, the limit of $\boldsymbol{x}(t)$ is a LM equilibrium $\boldsymbol{x}^*$. This happens only if $\sum_{r \in R'} d(r) = \phi$ where $R' = \{r \in R | \pi(r) > \gamma_{k+1}\}$[46] (Otherwise, eventually the condition must be broken.). As same as the discussion for the case $N(\boldsymbol{x}(t)) - l_{max}\epsilon_{k+1} > \phi$, there exists a single type r_{mar} whose optimal strategy is fluctuated between 0 and 1. However, because $\pi(r_{mar}) > \gamma_{k+1}$, as $\epsilon \to 0$, there eventually exists a unique best strategy for each type. $\boldsymbol{x}(t) \to \boldsymbol{x}^*$, and in $\boldsymbol{x}^*$, $\phi = N(\boldsymbol{x}^*)$ and all active senders strictly select sending a message. Thus, $\boldsymbol{x}^*$ is a LM equilibrium.

When $k = 1$, if $N(\boldsymbol{x}(t_2)) + l_{max}\epsilon_2 < \phi$, there are no enough senders under γ_2' to fill the capacity ϕ, and thus we move to the next step by changing the cost from γ_2' to γ_3'.

If $N(\boldsymbol{x}(t_2)) - l_{max}\epsilon_2 > \phi$, this policy stops the cost change. In such a case, $\gamma_1' > \pi_{max} > \gamma_2'$ and $d(r) > \phi$ for $r \in R$ s.t. $\pi_{max} = \pi(r)$. For any other $r' \in R - \{r\}$, sending 0 messages is strictly dominant. Then, by the similar discussion for general $k+1$, the system eventually moves into the neighborhood of a LM equilibrium $\boldsymbol{x}^*$ where only r-type senders send a message and either $\pi_{max}\frac{\phi}{N(\boldsymbol{x}^*)} = \gamma_2'$ or $N(\boldsymbol{x}^*) = d(r)$ is satisfied.

If $N(\boldsymbol{x}(t)) - l_{max}\epsilon \leqq \phi \leqq N(\boldsymbol{x}(t)) + l_{max}\epsilon$ for all $t \in [t_1, \infty)$ where ϵ satisfying $T(\frac{\epsilon}{n}) < (t - t_1)$ and $\bar{\epsilon} > \epsilon > 0$, this implies $d(r) = \phi$ where $r \in R$ s.t. $\pi_{max} = \pi(r)$. We can apply the similar discussion for $k + 1$ again. When we consider r as the marginal type, because $\pi(r) > \gamma_2$, as $\epsilon \to 0$, there eventually exists a unique best strategy for each type. Thus, $\boldsymbol{x}(t) \to \boldsymbol{x}^*$ where $\boldsymbol{x}^*$ is a LM equilibrium.

In conclusion, Proposition 5 is proved.

Reference

Ali, S.N., 2018. Herding with Costly Information. Journal of Economic Theory 175, 713–729.

[46] From the definition, $\gamma_{k+1} \neq \pi(r)$ for any $r \in R$.

Anderson, S.P., de Palma, A., 2009. Information Congestion. RAND Journal of Economics, 40(4), 688–709.

Anderson, S.P., de Palma, A., 2012. Competition for Attention in the Information (Overload) Age. RAND Journal of Economics, 43(1), 1–25.

Anderson, S.P., de Palma, A., 2013. Shouting to Be Heard in Advertising. Management Science, 59, 1545–1556.

Anderson, S.P., Peitz, M., 2023, Ad Clutter, Time Use, and Media Diversity. American Economic Journal: Microeconomics 15, 227–270.

Bagwell, K., 2007. The Economic Analysis of Advertising. Handbook of Industrial Organization, 3, 1701–1844.

Bandhu, S., Lahkar, R., 2022. Evolutionary robustness of dominant strategy implementation. Economic Theory. https://doi.org/10.1007/s00199-022-01474-w

Baumol, W.J., Oates, W.E., 1971. The Use of Standards and Prices for Protection of the Environment. The Swedish Journal of Economics 73, 42-54.

Baumol, W.J., Oates, W.E., 1988. The Theory of Environmental Policy, 2nd ed. Cambridge University Press.

Butters, G., 1977. Equilibrium Distributions of Sales and Advertising Prices. Review of Economic Studies. 44(3), 465–491.

Callander, S., 2011. Searching for Good Policies. American Political Science Review 105, 643–662.

Callander, S., Foarta, D., Sugaya, T., 2022. Market Competition and Political Influence: An Integrated Approach. ECTA 90, 2723–2753.

Callander, S., Foarta, D., Sugaya, T., 2023. The Dynamics of a Policy Outcome: Market Response and Bureaucratic Enforcement of a Policy Change. American J Political Sci. https://doi.org/10.1111/ajps.12784

Cisco, 2019. Email: Click with Caution - How to protect against phishing, fraud, and other scams. https://www.cisco.com/c/dam/en/us/products/collateral/security/email-security/email-threat-report.pdf

Dahlen, M., Rosengren, S., 2016. If Advertising Won't Die, What Will It Be? Toward a Working Definition of Advertising. Journal of Advertising 45, 334–345.

Dijkstra, B.R., De Vries, F.P., 2006. Location choice by households and polluting firms: An evolutionary approach. European Economic Review 50, 425–446.

Dukes, Anthony J., Liu, Qihong, 2023. The Consumption of Advertising in the Digital Age: Attention and Ad Content. Management Science 70(4):2086-2106.

Eliaz, K., Spiegler, R., 2020. Incentive-Compatible Advertising on Nonretail Platforms. RAND

Journal of Economics 51, 323–345.

Eppler, M.J., Mengis, J., 2004. The Concept of Information Overload: A Review of Literature from Organization Science, Accounting, Marketing, MIS, and Related Disciplines. Information Society 20, 325–344.

Falkinger, J., 2007. Attention economies. Journal of Economic Theory 133, 266–294.

Falkinger, J., 2008. Limited Attention as a Scarce Resource in Information-Rich Economies. The Economic Journal 118, 1596–1620.

Gilboa, I., Matsui, A., 1991. Social Stability and Equilibrium. Econometrica 59, 859.

Goh, K-Y., Hui, K-L., Png, I.P.L., 2015. Privacy and Marketing Externalities: Evidence from Do Not Call. Management Science 61, 2982–3000.

Hammer, P., Riebe, E., Kennedy, R., 2009. How Clutter Affects Advertising Effectiveness. Journal of Advertising Research 49, 159–163.

Hefti, A. 2018., Limited Attention, Competition and Welfare. Journal of Economic Theory 178, 318–359

Hefti, A., Liu, S., 2020. Targeted information and limited attention. The RAND J of Economics 51, 402–420.

Hirshleifer, J., 1973. Where Are We in the Theory of Information?. American Economic Review, 63(2), Papers and Proceedings of the Eighty-fifty Annual Meeting of the American Economic Association (May, 1973), 31-39

Johnson, J.P., 2013. Targeted Advertising and Advertising Avoidance. The RAND Journal of Economics 44, 128–144.

Lahkar, R., Mukherjee, S., 2019. Evolutionary implementation in a public goods game. Journal of Economic Theory 181, 423–460.

Lahkar, R., Mukherjee, S., 2021. Evolutionary implementation in aggregative games. Mathematical Social Sciences 109, 137–151.

Malhotra, N.K., 1984. Reflections on the Information Overload Paradigm in Consumer Decision Making. Journal of Consumer Research, 10(4), 436-440.

Niehaus, P., 2011. Filtered Social Learning. Journal of Political Economy 119, 686–720.

Rao, J.M., Reiley, D.H. 2012. The Economics of Spam. Journal of Economic Perspectives, 26(3), 87-110.

Renault, R., 2015. Advertising in Markets. Handbook of Media Economics, 2, pp. 121–204.

Riebe, E., Dawes, J., 2006. Recall of Radio Advertising in Low and High Advertising Clutter

Formats. International Journal of Advertising 25, 71–86.

Rust, R.T., Oliver, R.W., 1994. The Death of Advertising. Journal of Advertising 23, 71–77.

Jinushi, J. 2023. Costly Advertising and Information Congestion, mimeo

Sandholm, W.H., 2002. Evolutionary Implementation and Congestion Pricing. The Review of Economic Studies 69, 667–689.

Sandholm, W.H., 2005. Negative Externalities and Evolutionary Implementation. The Review of Economic Studies 72, 885–915.

Sandholm, W.H., 2007. Pigouvian pricing and stochastic evolutionary implementation. Journal of Economic Theory 132, 367–382.

Sandholm, W.H., 2010. Population Games and Evolutionary Dynamics. MIT

Shen, Q., Villas-Boas, J.M., 2018. Behavior-Based Advertising. Management Science 64, 2047–2064.

Sims, C.A., 2003. Implications of Rational Inattention. Journal of Monetary Economics 50, 665–690

Stigler, G.J., 1961. The Economics of Information. Journal of Political Economy 69, 213–225.

Symantec, 2019. Internet Security Threat Report Volume 24. https://docs.broadcom. com/doc/istr-24-2019-en

Taylor, J., Kennedy, R., Sharp, B., 2009. Is Once Really Enough? Making Generalizations about Advertising's Convex Sales Response Function. Journal of Advertising Research 49, 198–200.

Vakratsas, D., Feinberg, F.M., Bass, F.M., Kalyanaram, G., 2004. The Shape of Advertising Response Functions Revisited: A Model of Dynamic Probabilistic Thresholds. Marketing Science 23, 109–119.

Van Zandt, T., 2004. Information Overload in a Network of Targeted Communication. RAND Journal of Economics, 35(3), 542–560.