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Present Bias, Cognitive Bias, and Illiquid Savings

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This paper analyzes consumption and savings of households, who may engage in myopic excess consumption due to present bias, but can also allocate funds into illiquid savings to self-control their consumption. In particular, I focus on situations where households have cognitive biases that lead them to be overconfident so that their present bias can be mitigated in the future, thus leading to inadequate self-control. To contend with this, the government can set a penalty rate for early withdrawals of semi-illiquid savings. The government can also set a subsidy rate for semi-illiquid and fully illiquid savings. These two policy variables are set socially optimally from a paternalistic perspective. In addition, I address cases where households are subject to mental accounting in their consumption with early withdrawals. The analysis will be conducted numerically by constructing a three-period model consisting of households with heterogeneous cognitive biases. Based on the model, I will examine the characteristics of household consumption and savings in different settings, their impact on welfare, the function of illiquid savings, and optimal policy.

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Abstract

This paper analyzes consumption and savings of households, who may engage in myopic excess consumption due to present bias, but can also allocate funds into illiquid savings to self-control their consumption. In particular, I focus on situations where households have cognitive biases that lead them to be overconfident so that their present bias can be mitigated in the future, thus leading to inadequate self-control. To contend with this, the government can set a penalty rate for early withdrawals of semi-illiquid savings. The government can also set a subsidy rate for semi-illiquid and fully illiquid savings. These two policy variables are set socially optimally from a paternalistic perspective. In addition, I address cases where households are subject to mental accounting in their consumption with early withdrawals. The analysis will be conducted numerically by constructing a three-period model consisting of households with heterogeneous cognitive biases. Based on the model, I will examine the characteristics of household consumption and savings in different settings, their impact on welfare, the function of illiquid savings, and optimal policy.

Keywords: Present bias, Cognitive bias, Illiquidity, Self-control, Mental accounting
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1. Introduction

The myopic behavioral characteristic in which one cannot resist the desire to consume impulsively is treated in economics as present bias. If economic agents are aware of such a characteristic, they can act proactively to mitigate its harmful effects. One example is self-control through commitment. Such forms of behavior are observed in a wide variety of decision making. Among them, I focus on self-control, in which households resolve to protect their reserves for retirement from early excess consumption by deliberately investing in, in other words “utilizing”, illiquid assets. This action raises one's utility by mitigating the excess consumption associated with present bias, while relinquishing the benefits derived from the liquidity of the assets. In the real world, private pensions in which households voluntarily participate are examples of such an investment. These various illiquid assets are referred to as "illiquid savings" and will be analyzed in this study.

Troublingly, however, not all households are accurately aware of their future present bias. Frequently the temptation appears to say, "I'll save money next month; let's just make this purchase now." Often, however, this type of consumption behavior does not change dramatically the following month. These households were overly confident that they would be able to curb their present bias next month. Such a misconception of their own behavioral characteristic is sometimes referred to as naiveté. In this paper, I refer to it more explicitly as cognitive bias. If households underestimate the magnitude of their future present bias, their self-control efforts will be inadequate. Thus, depending on the combination of the households' present bias and cognitive bias, the magnitude of self-control through the utilization of illiquid savings is likely to differ.

Turning to policy issues, the government could, from a paternalistic perspective, introduce economic policies that induce households to behave better. First, policies that subsidize illiquid savings (or reduce taxes) could be introduced to remedy the households' underutilization of illiquid savings. Second, to curb excess consumption based on early withdrawals of illiquid savings, a policy could impose an appropriate penalty for such early withdrawals. In the real world, these policy issues are important in designing pension plans that form part of social security.

In this paper, I construct a model that allows for a comprehensive analysis of these issues—specifically, a three-period economy model of households with heterogeneous cognitive biases. In each period, households have access to illiquid savings with constraints on withdrawals in addition to liquid savings in order to allocate their given total resources optimally to consumption. In this situation, households determine the amount of each saving and consumption in each period under present bias and cognitive bias. The government also optimally determines two policy variables (the aforementioned subsidy rate and penalty rate) using as objective functions the utility functions that would be assumed if households were under ideal circumstances unaffected by the biases. For the model, I analyze the behavior of households and the government by numerically solving the optimization problem.

Here, I will review some relevant literature. Strotz (1956) is the first that incorporated present bias of economic agents into the model and analyzed theoretically the function of commitment for self-control. Numerous studies followed, but Laibson (1997) had a major impact on the field by providing a lucid analysis of consumption-savings behavior by linking households' present bias, self-control, and the illiquidity of their savings. The analysis of naiveté, or the misperception by economic agents of their own future present bias, was greatly developed theoretically by O'Donoghue and Rabin (1999a, 1999b). On the empirical side, DellaVigna and Malmendier (2006) studied data on choice of contract form and supported the existence of cognitive bias.

Among the vast body of literature, Beshears *et al.* (2025) has the greatest relevance to this study in terms of analytical motivations and the style of model. They analyzed a two-period economy consisting of households with heterogeneous present bias in circumstances with multiple financial assets with different illiquidity. The government allocates an optimal amount of each financial asset to households and also sets an optimal penalty rate for early withdrawals. They identified how such a policy should provide illiquid savings in order to appropriately curb households' excess consumption due to their present bias.

In contrast, this study differs from Beshears *et al.* (2025) in four main respects. First, this study assumes that households have cognitive bias in addition to present bias; while Beshears *et al.* (2025) assumed heterogeneity in the magnitude of households' present bias, this study assumes heterogeneity in the magnitude of cognitive bias, while the present bias is assumed to be the same. Thereby, we delve into the effects of cognitive bias. The economic analysis of cognitive bias has been addressed in numerous previous studies, such as Ariely (2008). However, to my knowledge, there has been no previous attempt to quantitatively analyze the impact of cognitive bias in an analysis that utilizes illiquid savings as a means of self-control against present bias. Second, while Beshears *et al.* (2025) assumed that the size of various types of liquid savings is determined by the government, this study assumes that households determine the size of their savings as their own optimization problem. This allows the model to directly reflect the effect of households' self-control. It is interesting that differences in the magnitude of self-control appear due to the heterogeneity of cognitive biases. In light of a country's pension system, the model in Beshears *et al.* (2025) is highly consistent with many public pensions (basically, mandatory pensions), since it is the government, not households, that determines the amount of such pensions. On the other hand, for most private pensions available in a country, the government determines the instrumentality, such as the penalty rate for early withdrawals, while households determine the investment amount. In this respect, private pensions are highly consistent with the model in this study. In many developed countries, households utilize private pensions as a supplement to mandatory public pensions. In this sense, it is desirable to elucidate economic behavior toward private pensions in order to understand the marginal characteristics of household consumption-savings behavior. Note that in order to incorporate the element of households deciding the amount to invest on their own, the model in this study assumes three periods instead of two. Third, in this study, I also analyzed the effects of subsidy policies to encourage self-control by households. This analysis was made possible by incorporating the second point mentioned above into the model. Such subsidy policies have been incorporated in many pension plans in developed countries. The income tax deduction advantage with Japan's individual-type Defined Contribution pension plan (abbreviated as iDeCo) is one example. Therefore, it is highly significant to analyze the problem of the optimal subsidy rate. Fourth, I also analyzed cases in which mental accounting affects semi-illiquid savings. To say consumption using funds obtained through early withdrawals from semi-illiquid savings as long as the stipulated penalty has been paid is identical to consumption from other sources is not necessarily true. Households would be aware at the time they make semi-illiquid savings that it would be undesirable to withdraw from them early for consumption. In such a case, there is a possibility that excess consumption with early withdrawals will be curbed to some extent. This effect will be incorporated into the analysis.

The paper is organized as follows. Section 2 describes the full model, including household and government optimization problems. In addition, a relative evaluation index of welfare is introduced to quantitatively analyze the impact of the policy on household expected lifetime utility. I will also explain the settings of numerical analyses. Sections 3 through 5 report the results of the analysis. Section 3 presents the analysis assuming an environment in which households have access only to liquid savings and fully illiquid savings. Section 4 presents the analysis assuming an environment in which households have access only to liquid savings and semi-illiquid savings. Section 5 is an analysis that assumes an environment in which households have access to liquid savings, semi-illiquid savings, and fully illiquid savings. Finally, Section 6 is the conclusion.

2. Analysis Method

2.1 Model

2.1.1 Overview of the Model

This paper assumes a three-period model. At the beginning of period 1, each household basically receives an endowment of 100[1] and consumes it all during the three periods of its lifetime. Households have no income at any point in time other than the beginning. There are no labor or capital markets. At the end of period 1, households save what they did not consume in period 1 out of their endowments in up to three types of financial assets. The first is completely liquid savings, which can be withdrawn freely in periods 2 and 3. The second is savings with certain constraints on withdrawals in period 2, called semi-illiquid savings. These are savings that can be withdrawn in period 2 (hereafter referred to as early withdrawal) upon payment of a fixed-rate penalty determined in advance by the government.[2] The semi-illiquid savings can be withdrawn freely in period 3. The third is illiquid savings, which cannot be withdrawn in period 2 at all but can be withdrawn freely in period 3. In the context of a pension system, period 3, when both semi-illiquid and illiquid savings can be freely withdrawn, could be understood as the elderly period after retirement for households, while the first and second periods as the working years. Individual households experience idiosyncratic demand shocks (in other words, taste shocks) in period 2 of their working lives that cannot be forecasted in period 1.

Turning to real private pension plans with respect to the possibility of early withdrawal, for example, individual retirement accounts (IRAs) and corporate defined contribution plans (401k plans) in the United States can be interpreted as being similar to the nature of semi-illiquid savings of this study. On the other hand, Japan's iDeCo can be interpreted as similar to the nature of illiquid savings of this study.[3]

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[1] As discussed below, there are cases in which households receive subsidies from the government. In such cases, it can be assumed that the government finances the subsidy through a lump-sum tax at the initial point in time. Thus, the initial endowment will be of the magnitude of 100 minus the per capita tax. If the government receives revenue from early withdrawal penalty payment by households, the expected value of that revenue at the initial point in time is assumed to be added in advance to the initial endowment of 100.

[2] In the context of a pension plan, this corresponds to a cash withdrawal from the pension fund by partly cancelling the contract at a time prior to retirement.

[3] Individual retirement accounts (IRAs) and 401k plans in the U.S. allow early withdrawal upon payment of a 10% penalty. On the other hand, Japan’s iDeCo in principle does not allow early withdrawal.

Households make consumption-savings decisions to maximize their expected lifetime utility. However, their utility function contains the following two types of biases. First, all households have the same present bias in periods 1 and 2. As shown in Section 2.1.2, the parameter that determines its magnitude is β_1. It represents the degree of myopia in that utility from consumption in the future is given less weight than utility from current consumption. The smaller the value of β_1 is than 1, the stronger the myopia. Thus, strictly speaking, the degree of present bias should be expressed as $1-\beta_1$. However, for simplicity, present-bias parameter will be referred to as β_1. Second, in period 1, households expect that the present bias acting on them in period 2 is β_2 (where $\beta_1 \leqq \beta_2 \leqq 1$) rather than β_1. This implies that the degree of present bias will be lighter in the future, i.e., households are overconfident that they will be able to better control their myopic consumption. The magnitude of this misperception $(\beta_2 - \beta_1)$ is the cognitive bias. Households are heterogeneous only with respect to β_2, but homogeneous in other respects. Note that the government correctly knows that all households have a present bias of β_1 in period 2. In addition, households have uncertainty in period 1 about their own individual demand in period 2. Specifically, the period utility function in period 2 is multiplied by the stochastic variable θ (where $1-\lambda_\theta \leqq \theta \leqq 1+\lambda_\theta$) that is realized at the beginning of period 2. The constant λ_θ (where $0 \leqq \lambda_\theta < 1$) represents the magnitude of the uncertainty of this idiosyncratic demand shock θ. Households and the government are assumed to know information about λ_θ.

The government imposes a fixed-rate penalty on households' early withdrawals of semi-illiquid savings in period 2. One of the government's policies is to set the penalty rate in advance. The larger the penalty rate is, the more restrictive early withdrawals can be, making it possible to curb myopic consumption behavior more, while the households' responses to idiosyncratic demand shocks become more rigid. The government's policy optimally balances these tradeoffs. In addition, the government can provide a fixed-rate subsidy to households for making illiquid and semi-illiquid savings. Households have cognitive bias and present bias that reduce their lifetime utility ex post relative to what it would be without them. Since the government correctly recognizes, in period 1, the households' present bias to be realized in period 2, it can encourage households to utilize illiquid and/or semi-illiquid savings by providing subsidies to mitigate the adverse effects of their cognitive bias. The government's second policy is to set the subsidy rate in advance.

Thus, the government uses the above two policy instruments for the purpose of improving household expected lifetime utility. That is, the government determines the two policy variables by solving an optimization problem with the socially aggregated household expected lifetime utility without cognitive bias as the objective function. This can be interpreted as a paternalistic policy in the sense that the government complements household behaviors where they are not at their best. The following is a more specific description of the model.

2.1.2 Optimization Problems for Households

In period 1, households determine the amount of consumption in the current period and, at most, three types of savings by maximizing their expected lifetime utility as follows.[4]

$$\max_{\substack{c_1(\beta_2,\lambda_\theta),x_1(\beta_2,\lambda_\theta)\\ x_2(\beta_2,\lambda_\theta),x_3(\beta_2,\lambda_\theta)}} u\big(c_1(\beta_2,\lambda_\theta)\big) +$$

$$\beta_1\delta\cdot \mathrm{E}_1\Big[\theta\cdot u\Big(c_2\big(\beta_2,\theta,x_1(\beta_2,\lambda_\theta),x_2(\beta_2,\lambda_\theta),x_3(\beta_2,\lambda_\theta)\big)\Big)+\delta$$

$$\cdot\, u\Big(c_3\big(\beta_2,\theta,x_1(\beta_2,\lambda_\theta),x_2(\beta_2,\lambda_\theta),x_3(\beta_2,\lambda_\theta)\big)\Big)\Big],$$

$$\text{subject to}\quad c_1(\beta_2,\lambda_\theta)+x_1(\beta_2,\lambda_\theta)+x_2(\beta_2,\lambda_\theta)+x_3(\beta_2,\lambda_\theta)=100.$$

The expectation operator E_1 represents the expected value in period 1 of the following term with a stochastic variable θ. The probability distribution of θ is assumed to be uniform, depending only on the parameter λ_θ representing the magnitude of uncertainty. β_1 is a parameter with regard to present bias and δ is a discount rate for period utility. The interest rate is assumed to be zero. Generality is not lost due to this assumption. In the following, for notational simplicity, the arguments of endogenous variables, β_2 and λ_θ, may be omitted. Once again, the above optimization problem is written in simplified form,

$$\max_{c_1,x_1,x_2,x_3} u(c_1)+\beta_1\delta\cdot \mathrm{E}_1\big[\theta\cdot u\big(c_2(\theta,x_1,x_2,x_3)\big)+\delta\cdot u\big(c_3(\theta,x_1,x_2,x_3)\big)\big],$$

$$\text{subject to}\ \ c_1+x_1+x_2+x_3=100.$$

$c_2(\theta,x_1,x_2,x_3)$ and $c_3(\theta,x_1,x_2,x_3)$ in the above equation are determined by the households' optimization problem in period 2 given θ,x_1,x_2,x_3 as follows.

$$\max_{\alpha_1(\theta,x_1,x_2,x_3),\alpha_2(\theta,x_1,x_2,x_3)} \theta\cdot u\big(c_2(\theta,x_1,x_2,x_3)\big)+\beta_2\delta\cdot u\big(c_3(\theta,x_1,x_2,x_3)\big),$$

where $c_2(\theta,x_1,x_2,x_3)=\alpha_1(\theta,x_1,x_2,x_3)\cdot x_1+(1-\pi_2)\alpha_2(\theta,x_1,x_2,x_3)\cdot x_2,$

$$c_3(\theta,x_1,x_2,x_3)=(1-\alpha_1(\theta,x_1,x_2,x_3))\cdot x_1+\big(1-\alpha_2(\theta,x_1,x_2,x_3)\big)\cdot x_2+x_3.$$

$\alpha_1(\theta,x_1,x_2,x_3)$ and $\alpha_2(\theta,x_1,x_2,x_3)$ represent the ratios of liquid savings x_1 and semi-

[4] Here, the impact of taxes and the transfer of penalties to be collected is omitted in the initial endowment (100) in the budget constraint. See Section 2.1.3 for exact details.

illiquid savings x_2, respectively, to be withdrawn in period 2 for consumption. A penalty based on a fixed rate π_2 is collected by the government when withdrawing semi-illiquid savings x_2 in period 2. Illiquid savings x_3 cannot be withdrawn in period 2 at all. In period 3, consumption is made to exhaust each remaining saving. Note that households in period 1 sees it ideal to have no present bias in period 2. Therefore, the part related to the decision-making in the period 2 in the objective function of households in the period 1 does not include any present bias parameter. On the other hand, households expect in period 1 that their own behavior in period 2 will have a present bias β_2 although they do not want it in advance. That is, they are aware of the possibility that they may not necessarily choose their own ideal consumption behavior in the future. Therefore, the optimization problem in period 2 assumed in period 1 includes β_2 as a present bias in the future.

Hereafter, the period utility function of households is assumed to be of logarithmic form: $u(c) = \log c$. Under this assumption, the optimization problem for households in period 2 is solved analytically. Since the objective function is that

$$\theta \cdot \log c_2 + \beta_2\delta \cdot \log c_3 = \log(c_2^{\theta} \cdot c_3^{\beta_2\delta}),$$

we compute the first-order condition for maximizing the following:

$$F \equiv c_2^{\theta} \cdot c_3^{\beta_2\delta} = (\alpha_1 \cdot x_1 + (1-\pi_2)\alpha_2 \cdot x_2)^{\theta} \cdot ((1-\alpha_1) \cdot x_1 + (1-\alpha_2) \cdot x_2 + x_3)^{\beta_2\delta}.$$

Since all households preferentially utilize penalty-free liquid savings in period 2, in case that $\alpha_1 < 1$, $\alpha_2 = 0$ and the first-order condition for α_1 is satisfied. On the other hand, only when liquid savings are exhausted ($\alpha_1 = 1$), semi-illiquid savings can be utilized ($\alpha_2 \geqq 0$) and the first-order condition regarding α_2 is satisfied. Therefore,

$$\alpha_1 < 1, \qquad \frac{\partial F}{\partial \alpha_1} = 0, \qquad \alpha_2 = 0,$$

$$\text{or} \quad \alpha_1 = 1, \qquad \frac{\partial F}{\partial \alpha_2} = 0, \qquad 0 \leqq \alpha_2 \leqq 1.$$

Solving for the former FoC,

$$\alpha_1 = \frac{\theta(x_1 + x_2 + x_3)}{x_1(\theta + \beta_2\delta)}, \qquad \alpha_1 < 1, \qquad \alpha_2 = 0.$$

Solving for the latter FoC,

$$\alpha_1 = 1, \qquad \alpha_2 = \frac{\theta(1-\pi_2)(x_2 + x_3) - x_1\beta_2\delta}{x_2(1-\pi_2)(\theta + \beta_2\delta)}, \qquad 0 \leqq \alpha_2 \leqq 1.$$

The optimization problem for households in period 2 has now been solved. Based on this, next, the optimization problem for households in period 1 is addressed. To calculate the expected value in the objective function, we need the probability distribution of the stochastic variable θ. This is assumed to be a uniform distribution with an expected value of 1, a lower bound of $1-\lambda_\theta$, and an upper bound of $1+\lambda_\theta$ $(1-\lambda_\theta \leqq \theta \leqq 1+\lambda_\theta)$. The constant λ_θ representing the degree of uncertainty of θ satisfies $0 \leqq \lambda_\theta < 1$.

Households optimally choose the variable c_1, x_1, x_2, x_3 in period 1. We compute the optimal solution by performing a grid search on these variables.

2.1.3 Optimization Problems for the Government

As noted above, the government can have two policies: one that sets a penalty rate π_2 for early withdrawals of semi-illiquid savings, and one that sets a subsidy rate r_{sub} to be paid for illiquid and semi-illiquid savings.

For both policies, the government aims to maximize the average expected lifetime utility of the society which consists of households that are heterogeneous with respect to β_2. The government correctly recognizes that all households, regardless of their β_2, will in reality have a present bias β_1 in period 2 (where $\beta_1 \leqq \beta_2 \leqq 1$) in evaluating its objective function to be maximized. Specifically, the government makes accurate judgments about the variables that households choose in period 2 as the following α_{1pat} and α_{2pat}, unlike a priori expected values (α_1 and α_2) by the households themselves.

$$\alpha_{1pat}(\theta, x_1, x_2, x_3) = \frac{\theta(x_1 + x_2 + x_3)}{x_1(\theta + \beta_1 \delta)}, \qquad 0 \leqq \alpha_{1pat}(\theta, x_1, x_2, x_3) \leqq 1$$

$$\alpha_{2pat}(\theta, x_1, x_2, x_3) = \frac{\theta(1-\pi_2)(x_2 + x_3) - x_1 \beta_1 \delta}{x_2(1-\pi_2)(\theta + \beta_1 \delta)}, \qquad 0 \leqq \alpha_{2pat}(\theta, x_1, x_2, x_3) \leqq 1$$

$$c_{2pat}(\theta, x_1, x_2, x_3) = \alpha_{1pat}(\theta, x_1, x_2, x_3) \cdot x_1 + (1-\pi_2)\alpha_{2pat}(\theta, x_1, x_2, x_3) \cdot x_2$$

$$c_{3pat}(\theta, x_1, x_2, x_3) = (1 - \alpha_{1pat}(\theta, x_1, x_2, x_3)) \cdot x_1 + \left(1 - \alpha_{2pat}(\theta, x_1, x_2, x_3)\right) \cdot x_2 + x_3$$

Note that x_1, x_2, x_3 are the variables that depend on β_2, λ_θ and π_2. The right sides of α_{1pat} and α_{2pat} are in the form of β_2 replaced by β_1 on the right sides of α_1 and α_2, respectively, as described above.

For each of the two policies by the government, two different strategies can be assumed.

The first strategy improves households' utility by suppressing the effects of household cognitive bias and present bias in period 2, while excluding the effects of household present bias in period 1 from policy control. This strategy works primarily on household future consumption after period 2. For this reason, it is hereafter referred to as a future-consumption-control strategy. Given that both of the government's two policy variables act on illiquid and/or semi-illiquid savings, and that it is consumption after period 2 that is directly affected by these savings, this strategy is considered to be highly effective in principle.

The second strategy includes into policy control the effects of period 1's household present bias and thus improves household utility by controlling the effects of household cognitive bias and periods 1 and 2's present biases. This strategy works on current (period 1) as well as future consumption of households. For this reason, it is hereafter referred to as an all-consumption-control strategy. Neither of the two government policy variables directly affect current consumption, but they indirectly affect it through household budget constraints. Because of this indirectness, the policy variables would need to be relatively large to make this strategy effective. Details are reported in Section 3 and beyond.

A specific optimization problem for the two policy variables is shown below in turn.

2.1.3.1 Setting the Penalty Rate for Early Withdrawal of Semi-illiquid Savings

The government's optimization problem of determining the penalty rate π_2 for early withdrawal of semi-illiquid savings is as follows for a future-consumption-control strategy.

$$\max_{\pi_2} \int_{\beta_1}^{1} \left\{ u(c_1) + \beta_1 \delta \cdot \mathrm{E}_1 \left[\theta \cdot u\left(c_{2pat}(\theta, x_1, x_2, x_3) \right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, x_2, x_3) \right) \right] \right\} f(\beta_2) d\beta_2,$$

$$\text{subject to } \max_{c_1, x_1, x_2, x_3} u(c_1) + \beta_1 \delta \cdot \mathrm{E}_1 \left[\theta \cdot u\big(c_2(\theta, x_1, x_2, x_3)\big) + \delta \cdot u\big(c_3(\theta, x_1, x_2, x_3)\big) \right],$$

$$c_1 + x_1 + x_2 + x_3 = 100(1 + r_{dist}),$$

where $f(\beta_2)$ is the probability density function with respect to β_2. r_{dist} is the rate at which the government transfers to households in advance the expected amount of penalty collections they will get in period 2, of which the specific calculation is described below. Each of the endogenous variables c_1, x_1, x_2, x_3, and $\alpha_{1pat}, \alpha_{2pat}$, c_2, c_3, c_{2pat}, c_{3pat} are affected by π_2, although not explicitly stated in their arguments.

On the other hand, in scenarios where an all-consumption-control strategy is applied, the optimization problem is as follows.

$$\max_{\pi_2} \int_{\beta_1}^{1} \left\{ u(c_1) + \delta \cdot \mathrm{E}_1 \left[\theta \cdot u\left(c_{2pat}(\theta, x_1, x_2, x_3) \right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, x_2, x_3) \right) \right] \right\} f(\beta_2) d\beta_2,$$

$$\text{subject to } \max_{c_1, x_1, x_2, x_3} u(c_1) + \beta_1 \delta \cdot \mathrm{E}_1 \left[\theta \cdot u\big(c_2(\theta, x_1, x_2, x_3)\big) + \delta \cdot u\big(c_3(\theta, x_1, x_2, x_3)\big) \right],$$

$$c_1 + x_1 + x_2 + x_3 = 100(1 + r_{dist}).$$

The difference between the two problems above is whether the second term of the objective function (the term representing expected future utility) is subject to the parameter β_1 representing present bias. In scenarios where an all-consumption-control strategy without β_1 is applied, the balance between c_1 and a pair of c_{2pat} and c_{3pat} that the government aims to achieve is different from the balance that households' own objective function aims at. This implies a policy action to c_1 is included. On the other hand, in a scenario where a future-consumption-control strategy with β_1 is applied, the policy acts only on the balance between c_{2pat} and c_{3pat}. Note that we solve these optimization problems by a grid search on π_2.

The government's expected penalty income is distributed uniformly to households in advance. In this sense, money burning does not occur. Specifically, the government adds the expected income to the initial endowment of 100 at a fixed rate (r_{dist}) in period 1, where r_{dist} satisfies the following condition.

$$\int_{\beta_1}^{1} \mathrm{E}_1[\pi_2 \cdot \alpha_{2pat}(\theta, x_1, x_2, x_3) \cdot x_2] f(\beta_2) d\beta_2 = r_{dist} \int_{\beta_1}^{1} (c_1 + x_1 + x_2 + x_3) f(\beta_2) d\beta_2$$

$$= 100 r_{dist} \int_{\beta_1}^{1} f(\beta_2) d\beta_2.$$

Iterations are used to calculate the distribution rate (r_{dist}) above until convergence, so that the initial endowment at period 1 is $100(1 + r_{dist})$.

2.1.3.2 Setting the Subsidy Rate for Illiquid and Semi-illiquid Savings

If households are aware in advance of a future problem that reduces lifetime utility from excess consumption due to present bias, they will utilize semi-illiquid and/or illiquid savings as a commitment tool for self-control to limit its effects. However, households with cognitive bias do not always exercise sufficient self-control because they are only insufficiently aware of the future present bias. To remedy this situation through policy intervention, the government can provide incentives in the form of subsidies for households making semi-illiquid and illiquid savings.

We assume that the government provides subsidies to households in period 1 by applying the same subsidy rate r_{sub} to semi-illiquid savings x_2 and illiquid savings x_3.

The balances of semi-illiquid and illiquid savings after applying the subsidy would be denoted as sx_2 and sx_3 as follows.

$$sx_2 = (1 + r_{sub}) x_2,$$
$$sx_3 = (1 + r_{sub}) x_3.$$

In the analysis of the cases with subsidies, the aforementioned α_1 and α_2 are rewritten using sx_2 and sx_3.

$$\alpha_1 = \frac{\theta(x_1 + sx_2 + sx_3)}{x_1(\theta + \beta_2 \delta)}, \qquad 0 \leqq \alpha_1 \leqq 1,$$

$$\alpha_2 = \frac{\theta(1 - \pi_2)(sx_2 + sx_3) - x_1 \beta_2 \delta}{sx_2(1 - \pi_2)(\theta + \beta_2 \delta)}, \qquad 0 \leqq \alpha_2 \leqq 1.$$

As for the issue of the government optimally setting the subsidy rate r_{sub}, it is assumed that the government can take two different strategies: a future-consumption-control type and an all-consumption-control type. Specifically, the optimization problems for a future-consumption-control strategy and for an all-consumption-control strategy are, respectively, as follows.

$$\max_{r_{sub}} \int_{\beta_1}^{1} \left\{u(c_1) + \beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, sx_2, sx_3)\right)\right]\right\} f(\beta_2) d\beta_2,$$

$$\max_{r_{sub}} \int_{\beta_1}^{1} \left\{u(c_1) + \delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, sx_2, sx_3)\right)\right]\right\} f(\beta_2) d\beta_2,$$

subject to $\max_{c_1, x_1, x_2, x_3} u(c_1) + \beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_2(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_3(\theta, x_1, sx_2, sx_3)\right)\right],$

$$c_1 + x_1 + x_2 + x_3 = 100(1 - \tau),$$

where $c_{2pat}(\theta, x_1, sx_2, sx_3) = \alpha_{1pat}(\theta, x_1, sx_2, sx_3) \cdot x_1 + (1 - \pi_2)\alpha_{2pat}(\theta, x_1, sx_2, sx_3) \cdot sx_2,$

$$c_{3pat}(\theta, x_1, sx_2, sx_3) = (1 - \alpha_{1pat}(\theta, x_1, sx_2, sx_3)) \cdot x_1 + \left(1 - \alpha_{2pat}(\theta, x_1, sx_2, sx_3)\right) \cdot sx_2 + sx_3,$$

and $f(\beta_2)$ is the probability density function for β_2 as above. τ is the tax rate at which the government will finance the subsidies, and the specific calculations will be noted shortly. As in the penalty rate setting, the government knows that all households actually have a present bias β_1 in period 2. Based on this, the government makes accurate judgments about household withdrawal rates (the percentage of each savings allocated to consumption) in period 2 as the following α_{1pat} and α_{2pat} in the policy optimization.

$$\alpha_{1pat} = \frac{\theta(x_1 + sx_2 + sx_3)}{x_1(\theta + \beta_1\delta)}, \qquad 0 \leqq \alpha_{1pat} \leqq 1$$

$$\alpha_{2pat} = \frac{\theta(1 - \pi_2)(sx_2 + sx_3) - x_1\beta_1\delta}{sx_2(1 - \pi_2)(\theta + \beta_1\delta)}, \qquad 0 \leqq \alpha_{2pat} \leqq 1$$

Each endogenous variable (c_1, x_1, x_2, x_3, $\alpha_{1pat}, \alpha_{2pat}, c_2, c_3, c_{2pat}, c_{3pat}$) is affected by r_{sub}, although not specified as arguments. The optimization problem for the subsidy rate r_{sub} is also numerically solved by a grid search on r_{sub}.

It is assumed that the subsidy will be financed by a flat rate tax (τ) levied on the initial endowment of 100 in period 1, where τ satisfies the following condition.

$$\int_{\beta_1}^{1} r_{sub}(x_2 + x_3) f(\beta_2) d\beta_2 = \tau \int_{\beta_1}^{1} (c_1 + x_1 + x_2 + x_3) f(\beta_2) d\beta_2 = 100\tau \int_{\beta_1}^{1} f(\beta_2) d\beta_2.$$

Under this condition, the total subsidy equals the tax revenue, and thus, no negative money burning occurs. The initial endowment before obtaining the subsidy at period 1 is $100(1 - \tau)$ and iterations are used to calculate the tax rate (τ) above until convergence.

2.1.3.3 Other Settings for the Government's Optimization Problems

When the government implements two policies simultaneously, the optimization problem is the following for the two policy variables, the penalty rate π_2 and the subsidy rate r_{sub}. That is, in case of a future-consumption-control strategy,

$$\max_{\pi_2, r_{sub}} \int_{\beta_1}^{1} \left\{u(c_1) + \beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, sx_2, sx_3)\right)\right]\right\} f(\beta_2) d\beta_2,$$

and, in case of an all-consumption-control strategy,

$$\max_{\pi_2, r_{sub}} \int_{\beta_1}^{1} \left\{u(c_1) + \delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, sx_2, sx_3)\right)\right]\right\} f(\beta_2) d\beta_2.$$

In both cases, the common constraints below are imposed.

$$\max_{c_1, x_1, x_2, x_3} u(c_1) + \beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_2(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_3(\theta, x_1, sx_2, sx_3)\right)\right],$$

$$c_1 + x_1 + x_2 + x_3 = 100(1 + r_{dist})(1 - \tau), \text{ and}$$

$$\pi_2 > r_{sub}.$$

The last constraint is the no-arbitrage condition that precludes households from accumulating semi-illiquid savings for the purpose of obtaining subsidies and then profiting from the arbitrage of liquidating those savings by paying a penalty in the following period.

A part of the analyses below includes the optimization of the policy focusing on the utility of individual households with β_2 rather than optimizing the society-wide utility of heterogeneous households. The government's optimization problem in that case is as follows. In case of a future-consumption-control strategy,

$$\max_{\pi_2, r_{sub}} u(c_1) + \beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, sx_2, sx_3)\right)\right],$$

and in case of an all-consumption-control strategy,

$$\max_{\pi_2, r_{sub}} u(c_1) + \delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_{3pat}(\theta, x_1, sx_2, sx_3)\right)\right],$$

$$\text{subject to } \max_{c_1, x_1, x_2, x_3} u(c_1) + \beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_2(\theta, x_1, sx_2, sx_3)\right) + \delta \cdot u\left(c_3(\theta, x_1, sx_2, sx_3)\right)\right],$$

$$c_1 + x_1 + x_2 + x_3 = 100(1 + r_{dist})(1 - \tau),$$

$$\pi_2 > r_{sub}.$$

Each endogenous variable, such as c_1, x_1, x_2, x_3, depends on π_2, r_{sub}, and β_2, although they are not specified as arguments.

2.2 Measuring Improvement in Welfare Based on Relative Metrics

When illiquid and semi-illiquid savings exist, households can raise their welfare (expected lifetime utility) relative to when they do not exist. In addition, if the government provides adequate subsidies to making illiquid and semi-illiquid savings, households will raise their welfare compared to the case where the subsidies are not utilized. To quantitatively assess the extent of such an improvement in welfare, the measures that express expected lifetime utility in relative terms are defined as follows.

First, the level of expected lifetime utility of households in period 1 is defined in the following two ways, given individual demand uncertainty (λ_θ) and self-expected present bias (β_2) in period 2.

$$U_{fut}^{abs}(\beta_2,\lambda_\theta) = u\big(c_1^*(\beta_2,\lambda_\theta)\big) +$$

$$\beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}\big(\theta, x_1^*(\beta_2,\lambda_\theta), x_2^*(\beta_2,\lambda_\theta), x_3^*(\beta_2,\lambda_\theta)\big)\right) + \delta \cdot u\left(c_{3pat}\big(\theta, x_1^*(\beta_2,\lambda_\theta), x_2^*(\beta_2,\lambda_\theta), x_3^*(\beta_2,\lambda_\theta)\big)\right)\right],$$

$$U_{all}^{abs}(\beta_2,\lambda_\theta) = u\big(c_1^*(\beta_2,\lambda_\theta)\big) +$$

$$\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}\big(\theta, x_1^*(\beta_2,\lambda_\theta), x_2^*(\beta_2,\lambda_\theta), x_3^*(\beta_2,\lambda_\theta)\big)\right) + \delta \cdot u\left(c_{3pat}\big(\theta, x_1^*(\beta_2,\lambda_\theta), x_2^*(\beta_2,\lambda_\theta), x_3^*(\beta_2,\lambda_\theta)\big)\right)\right],$$

where, $c_1^*(\beta_2,\lambda_\theta)$, $x_1^*(\beta_2,\lambda_\theta)$, $x_2^*(\beta_2,\lambda_\theta)$, and $x_3^*(\beta_2,\lambda_\theta)$ represent the solution to the households' optimization problem in period 1.

$U_{fut}^{abs}(\beta_2,\lambda_\theta)$ is a utility measure when one accepts the existence of present bias β_1 in period 1. It is used when the government adopts a future-consumption-control strategy. On the other hand, $U_{all}^{abs}(\beta_2,\lambda_\theta)$ is a utility measure in case that the government aims for a state in which there is no present bias in period 1. It is used when the government adopts an all-consumption-control strategy. In both measures, consumption c_{2pat} and c_{3pat} are included on the right side, reflecting the fact that households actually have present bias β_1 in period 2 regardless of the size of the present bias as perceived by the household, β_2.

Next, assume an economy where illiquid and semi-illiquid savings are not available, and let $U_{fut}^{min}(\lambda_\theta)$ and $U_{all}^{min}(\lambda_\theta)$ be expected lifetime utilities under these circumstances, respectively. In such a case, since households do not have a commitment tool for self-control, their self-expected present bias (β_2) for the future does not affect the choice of c_1 and x_1. Therefore, the two measures can be defined as follows.

$$U_{fut}^{min}(\lambda_\theta) = u\left(c_1^*(\,\cdot\,,\lambda_\theta)\right) +$$

$$\beta_1\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}\big(\theta, x_1^*(\,\cdot\,,\lambda_\theta), 0{,}0\big)\right) + \delta \cdot u\left(c_{3pat}\big(\theta, x_1^*(\,\cdot\,,\lambda_\theta), 0{,}0\big)\right)\right],$$

$$U_{all}^{min}(\lambda_\theta) = u\left(c_1^*(\,\cdot\,,\lambda_\theta)\right) +$$

$$\delta \cdot \mathrm{E}_1\left[\theta \cdot u\left(c_{2pat}\big(\theta, x_1^*(\,\cdot\,,\lambda_\theta), 0{,}0\big)\right) + \delta \cdot u\left(c_{3pat}\big(\theta, x_1^*(\beta_2,\lambda_\theta), 0{,}0\big)\right)\right].$$

These are the expected lifetime utilities when there is no benefit at all from allocating funds into illiquid and/or semi-illiquid savings. Thus, in evaluation of welfare for future-consumption-control and all-consumption-control strategies, these two measures can be respectively interpreted as lower bounds for each case.

The upper bound is considered next. Assume an economy where illiquid and semi-illiquid savings are available. Let $U_{fut}^{max}(\lambda_\theta)$ be the expected lifetime utility if households have a present bias β_1 in period 1 but expect in advance that they will not have a present bias in period 2 ($\beta_2 = 1$) and in fact have no present bias at all in period 2. This measure can be interpreted as the upper bound on expected lifetime utility when one accepts that there is a present bias β_1 in period 1. Similarly, the expected lifetime utility of households with no present bias both in periods 1 and 2 (thus, $\beta_1 = \beta_2 = 1$) is denoted as $U_{all}^{max}(\lambda_\theta)$. This measure can be interpreted as the upper bound on expected lifetime utility when aiming for no present bias in period 1.

The levels of households' expected lifetime utility, $U_{fut}^{abs}(\beta_2, \lambda_\theta)$ and $U_{all}^{abs}(\beta_2, \lambda_\theta)$, can be transformed into the following relative measures, $U_{fut}^{rel}(\beta_2, \lambda_\theta)$ and $U_{all}^{rel}(\beta_2, \lambda_\theta)$ so that they are evaluated as the relative position from the upper and lower bounds defined above.

$$U_{fut}^{rel}(\beta_2, \lambda_\theta) = \frac{U_{fut}^{abs}(\beta_2, \lambda_\theta) - U_{fut}^{min}(\lambda_\theta)}{U_{fut}^{max}(\lambda_\theta) - U_{fut}^{min}(\lambda_\theta)} \times 100 \ (\%),$$

$$U_{all}^{rel}(\beta_2, \lambda_\theta) = \frac{U_{all}^{abs}(\beta_2, \lambda_\theta) - U_{all}^{min}(\lambda_\theta)}{U_{all}^{max}(\lambda_\theta) - U_{all}^{min}(\lambda_\theta)} \times 100 \ (\%).$$

In this paper, we refer to these measures as welfare improvement. By measuring these indicators, the effectiveness of the commitment to self-control and the policy effects of subsidies can be evaluated on a scale of welfare.

2.3 Numerical Analysis Settings

In the following numerical analysis, the discount rate δ for period utility is set as 0.9 and the parameter β_1 representing present bias is set as 0.6. The parameter β_2 representing cognitive bias is assumed to be uniformly distributed in the range $0.6 \leqq \beta_2 \leqq 1.0$. For the analysis, we use a discrete grid with an interval of 0.1 for β_2, reporting the results for four grid points of 0.9, 0.8, 0.7, and 0.6. Households where $\beta_2 = 0.6$ do not have any cognitive bias.

Solving the optimization problems is basically based on a grid search. For the optimization problem for households in period 1, a grid search is performed in the range [0, 100] with a grid interval of 0.1 for each of the variables c_1, x_1, x_2 and x_3, respectively. For the optimization problem for households in period 2, we exceptionally use the analytical solution, as described above. For the optimization problem of policy variables by the government, a grid search is conducted for the penalty rate π_2 and the subsidy rate r_{sub}, with the grid interval set to 0.01, respectively, to find the optimal solution.

For the parameter λ_θ regarding the uncertainty of idiosyncratic demand shocks in period 2, four cases: 0.0, 0.2, 0.4, and 0.6 are analyzed.[5] For θ, as mentioned above, the calculation is conducted based on an assumption of a uniform distribution in the range $[-\lambda_\theta, \lambda_\theta]$. In numerical calculations, θ is calculated on a discrete grid with 0.002 intervals. Numerical integration is used in calculating the expectation on θ.

3. Effects of Self-control through Illiquid Savings and Optimal Policy

In this section, we assume a situation in which households allocate resources not consumed in period 1 to liquid savings (x_1) and illiquid savings (x_3). The characteristic of illiquid savings not to be withdrawn early can be considered similar to the nature of, for example, Japan's iDeCo, as mentioned above. Under this situation, we will examine in turn the case without and with government subsidization.

3.1 Basic Analysis: No Subsidy Case

3.1.1 Effects of Self-control through Illiquid Savings

Figure 1 shows the results when households allocate total resources to current consumption (c_1) and two types of savings (x_1, x_3) as the optimization in period 1. Out of the total resources of 100, almost 49.4 are always used for consumption c_1, irrespective of λ_θ and β_2. Thus, the remaining 50.6 is saved separately in x_1 and x_3 as shown in Figure 1, depending on λ_θ, the uncertainty of demand in period 2, and β_2, the households' own expectations of period-2 present bias.

If there is no uncertainty in demand in period 2 ($\lambda_\theta = 0$), then x_1 and x_3 are identical for all households regardless of β_2. This is because the optimal value of consumption in subsequent periods (when there is no present bias in period 2) is determined in period 1. To prevent excess consumption over this optimal level, all households choose x_1 that is exactly equal to the optimal consumption in period 2 and allocate the remaining to x_3 to ensure that they go to consumption in period 3.

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[5] In the case where λ_θ is strictly zero, the allocation between x_2 and x_3 is indeterminate in a setting in the Section 5 where both x_2 and x_3 can exist, because there is no uncertainty about future demand. In this case, in displaying the results of the analysis, a minimum value (=0) for x_2 and a maximum value for x_3 shall be adopted.

On the other hand, as the uncertainty of demand (λ_θ) in period 2 increases, each household saves more x_1 and less x_3. One reason for this is that the larger λ_θ is, the greater the likelihood that the household will face demand that significantly exceeds its expectations in period 2, so it will increase x_1 to prepare for this. However, increasing x_1 also has the disadvantage of making it easier to engage in excess consumption in period 2. Within these tradeoffs, each household makes the optimal choice.

The magnitude of change in x_1 and x_3 associated with increased uncertainty (λ_θ) is more pronounced for households with β_2 close to 1. This is because households with β_2 close to 1 have greater cognitive bias and thus disregard the disadvantages associated with an increase in x_1. In other words, even though all households actually have a present bias of $\beta_1 = 0.6$ ($\leqq \beta_2$) in period 2, households with larger cognitive biases are more inaccurate in their perceptions and therefore misperceive the risk of welfare losses associated with excess consumption to be small even if they save more x_1.

3.1.2 Welfare Improvements from Illiquid Savings

Welfare improvements are shown in Figure 2. Each of the figures (a) and (b) respectively depicts the welfare improvements associated with future-consumption-control and all-consumption-control strategies under a relatively large uncertainty index (λ_θ) of 0.6. The dashed and dotted lines in the figures respectively represent actual welfare improvement that is realized ex post and welfare improvement that each household expects ex ante with cognitive bias. In both (a) and (b), welfare improvement is positive. Given that welfare improvement is zero by definition in the absence of illiquid savings, these results indicate that the presence of illiquid savings increases each household's welfare improvement through self-control. In addition, the larger the household cognitive bias (i.e., the larger β_2), the smaller the actual welfare improvement realized in period 2 and the larger the welfare improvement self-anticipated in period 1. This is due to the fact that such households are optimistically overconfident in period 1 that they will be able to curb excess consumption in period 2. This overconfidence leads to the choice of larger liquid savings in period 1, which results in higher excess consumption in period 2 and worsens welfare. Comparing (a) and (b), the overall level of welfare improvements is smaller in (b) than in (a). This is because households cannot easily suppress present bias in period 1 by using illiquid savings, and therefore it is not easy for them to approach the target state (i.e., no present bias in period 1) of the strategy in (b).

Figures (c) and (d) in Figure 2 respectively show welfare improvement corresponding to future-consumption-control and all-consumption-control strategies where the uncertainty index (λ_θ) is relatively small at 0.4. The features identified in (a) and (b) above are similarly observed in (c) and (d).

The comparison between (a) and (c) and between (b) and (d) reveals that higher uncertainty of demands in period 2 leads to both smaller actual welfare improvement and smaller welfare improvement based on prior self-assessment for each household. This is because greater demand uncertainty prompts households to retain more liquid savings in preparation for greater demand in period 2. As a result, the withdrawal of liquid savings in period 2 leads to more excess consumption.

The properties summarized above apply essentially to all unsubsidized cases in the sections that follow.

3.2 Cases with a Subsidy to Promote Illiquid Savings

Given that households with cognitive bias make insufficient utilization of illiquid savings (x_3) for self-control, the government may wish to implement policies to promote their utilization from a paternalistic perspective. In this section, we analyze a situation in which the government provides a fixed-rate subsidy to households making illiquid savings (x_3) in period 1. Note that in all the analyses on the effects of subsidies below in this paper, only results where the magnitude of demand uncertainty (λ_θ) is set as 0.6 are presented. The qualitative nature is the same for cases with different settings of λ_θ.

3.2.1 Optimal Setting of Subsidy Rate

The government sets the optimal subsidy rate with a view to maximizing household welfare. Figure 3 shows the results of deriving the optimal subsidy rate for individual households with different cognitive biases. Figures (a) and (b) show the subsidy rate optimized under future-consumption-control and all-consumption-control strategies, respectively. In both figures, households with larger β_2 are likely to have slightly larger optimal subsidy rates. This indicates that the larger the cognitive bias, the more desirable it is to provide greater incentives for making illiquid savings to correct it.

Comparing (a) and (b), the optimal subsidy rate is larger in (b) for all households. With respect to the optimal subsidy rate for the society as a whole, it is 0.05 for strategy (a) while it is 0.34 for strategy (b). Again, the optimal subsidy rate is larger for strategy (b) than for strategy (a). This is because, in addition to the objective of curbing excess consumption in period 2, the strategy in (b) also aims to curb excess consumption further in period 1, and thus requires a stronger incentive.

The level of the optimal subsidy rate (0.34) under strategy (b) is generally consistent with that of Japan's iDeCo (for taxable income between 9 million yen and 18 million yen) when considering that iDeCo has the same feature as a subsidy payment through the income tax deduction system for pension insurance premiums.

3.2.2 Savings and Welfare Improvements under the Optimal Subsidy Rate

This section presents the savings behavior and welfare improvements of each household when the subsidy rate is set at its optimal level with respect to the society as a whole, and analyzes how it changes compared to when there is no subsidy.

Figure 4 shows the savings behavior (x_1, x_3) of each household in period 1 with optimized subsidy rates under a future-consumption-control strategy in (a) and (b) and under an all-consumption-control strategy in (c) and (d), respectively, as shown by the solid lines. The dashed line shows for reference the results in the absence of subsidies analyzed in section 3.1. They show that for both strategies, the introduction of subsidies reduced liquid savings (x_1) and increased illiquid savings (x_3). Thus, it can be observed that subsidies incentivized households to make illiquid savings. However, there are differences in the amount of each saving, reflecting differences in household cognitive biases. Comparing (a) with (b), and (c) with (d), the inducement to illiquid savings is stronger in (c) and (d), reflecting the higher subsidy rate.

Figures 5 and 6 illustrate this situation in terms of welfare improvements. First, in Figure 5, the solid lines show welfare improvement for each household when the optimized subsidy rate is set under (a) a future-consumption-control strategy and (b) an all-consumption-control strategy, respectively. Compared to welfare improvement in the absence of the subsidy shown by the dashed line, welfare improvement for all households is higher in the presence of the subsidy, as the increase in illiquid savings strengthens the effect of controlling excess consumption in period 2. In general, welfare improvement for strategy (a) is about 10 percentage points higher with the subsidy than without the subsidy, and that for strategy (b) is about 20 percentage points higher with the subsidy than without the subsidy. A closer look at the differences across households in this regard reveals that households with relatively large cognitive bias show a slightly larger increase in welfare improvement due to the introduction of subsidies. As a result, under the optimal subsidy rate, the curve of welfare improvement according to households' β_2 is flattened.

Next, in Figure 6, the dashed and dotted lines represent actual welfare improvement that will be realized ex post and welfare improvement that each household expects under cognitive bias ex ante, respectively. Figures (a) and (b) are when there are optimal subsidy rates under future-consumption-control and all-consumption-control strategies, respectively.

Compared to Figure 2 above ((a) and (b); without subsidies), Figure 6 with subsidies shows relatively higher welfare improvement in both strategies (a) and (b), and in both ex post and ex ante. This reflects the effect of the increase in illiquid savings in curbing excess consumption in period 2.

When looking at the gap between actual welfare improvement realized ex post and welfare improvement expected under cognitive bias ex ante, the gap is smaller in Figure 6 than in Figure 2. This indicates that the introduction of optimal subsidies has the effect of mitigating the effects of cognitive bias. In fact, households with larger cognitive bias (i.e., those with larger β_2) have a larger reduction in the gap in Figure 6 than in Figure 2. In particular, in (b) of Figure 6, we see that the effect of cognitive bias almost disappears when a large subsidy rate is set under an all-consumption-control strategy.

4. Effects of Self-control through Semi-illiquid Savings and Optimal Policy

In this section, we assume a situation in which households can allocate resources not consumed in period 1 to liquid savings (x_1) and to semi-illiquid savings (x_2). The characteristics of the semi-illiquid savings that can be withdrawn early by paying penalty can be considered similar to the nature of IRAs and 401k plans in the U.S., for example, as described above. In the following, cases without and with government subsidies will be examined sequentially. In Section 4.3, the case in which mental accounting effects influence households with respect to semi-illiquid savings is also analyzed.

4.1 Basic Analysis: Without Subsidy

4.1.1 The Optimal Penalty Rate for Early Withdrawal

Results of deriving optimal values for penalty rates at early withdrawal for each individual household with different cognitive biases (β_2) are shown in Figure 7. Figures (a) and (b) show penalty rates optimized under future-consumption-control and all-consumption-control strategies, respectively.

For both strategies, households with larger cognitive bias tend to have smaller optimal penalty rates in an environment with greater uncertainty about future demand (e.g., $\lambda_\theta = 0.4, 0.6$). This reflects the fact that households with larger cognitive bias tend to increase their liquid savings and reduce their semi-illiquid savings in period 1 in preparation for the onset of large demand, because they are less worried in advance about excess consumption in period 2, as we will see below. Therefore, they spend from liquid savings for much of their excess consumption in period 2, and since they are less likely to make early withdrawals of their semi-illiquid savings, there is no need to set a large penalty rate to curb them. In addition, since there is still a small probability of early withdrawals, a smaller penalty rate may be less burdensome. On the other hand, if uncertainty about future demand is nonexistent ($\lambda_\theta = 0$) or very small, then contrary to the above, the optimal penalty rate is relatively small for households with no or very small cognitive bias ($\beta_2 = 0.6$). This is related to the fact that in the absence of demand uncertainty, as we will see below, all households can accurately predict demand in period 2 in advance and make liquid savings to meet it, regardless of the magnitude of cognitive bias, and therefore, liquid savings are almost identical. If the liquid savings are nearly identical, semi-illiquid savings will also be nearly identical, and thus the mechanism noted above for large demand uncertainty will not work. Looking at the details, households with smaller cognitive bias accurately predict that the magnitude of utility arising from future consumption they project in period 1 will be smaller due to future present bias. Based on this prediction, they slightly reduce consumption in period 1 and increase liquid savings to compensate. This makes the tendency for such households to make early withdrawals of semi-illiquid savings a little weaker, and thus the penalty rate to curb excess consumption can be relatively small. In this manner, the mechanism of optimal penalty rate for individual households is somewhat complex.

Next, Figure 8 shows the optimal penalty rate on early withdrawal for the society as a whole. Specifically, (a) and (b) show the penalty rates optimized under future-consumption-con

Both figures show that the larger the uncertainty in future demand (λ_θ), the larger the optimal penalty rate. The reason for this is similar to the mechanism for optimal penalty rate for individual households under large demand uncertainty.

In addition, if we take a broad view of both (a) and (b), the level of optimal penalty rate is about 0.4 when there is no demand uncertainty, and it is in the range of 0.5 to just under 0.6 when there is demand uncertainty. These levels are intuitively quite large. The reason for this is that the optimal levels are large enough to almost completely avoid early withdrawal of semi-illiquid savings. In fact, as shown in Figure 9, under the optimal penalty rate, the expected value of early withdrawal of semi-illiquid savings in period 2 as calculated in period 1 is very small in either case.

4.1.2 Effects of Self-control through Semi-illiquid Savings and Welfare improvements

This section presents the savings behavior of individual households when the penalty rate on early withdrawals of semi-illiquid savings is set optimally for society as a whole.

Figure 10 shows the savings (x_1, x_2) made by each household under the optimal penalty rate. Figures (a) and (b) are under a future-consumption-control strategy, and Figures (c) and (d) are under an all-consumption-control strategy, respectively.

Comparing the three cases (a) and (b), (c) and (d) in Figure 10, and (a) and (b) in Figure 1, which are the results when illiquid savings rather than semi-illiquid savings are available, the amounts of liquid savings (x_1) and (semi-)illiquid savings (x_2 or x_3) are almost identical in all cases, respectively.

Generally speaking, semi-illiquid savings differ from the illiquid savings analyzed in Section 3 in terms of their nature in that they can be withdrawn early in period 2 if a penalty is paid. However, as noted above, the optimal penalty rate chosen is of a magnitude that almost all early withdrawals are avoided by households. In such a case, almost all semi-illiquid savings would be allocated to consumption in period 3. Thus, the consumption-savings behavior of households using semi-illiquid savings (x_2) is very similar to that of the households using illiquid savings (x_3) as discussed in Section 3. In other words, in the situation discussed in this section, where semi-illiquid savings do not coexist with illiquid savings, it is optimal to set a sufficiently high penalty rate for semi-illiquid savings which would increase the function as a commitment tool for self-control to curb excess consumption, while sacrificing the function to respond to idiosyncratic demand shocks. That is, it is optimal for semi-illiquid savings in this section to perform almost the same function as that of illiquid savings when existing alone (as discussed in Section 3).

The welfare improvement (figures omitted) from utilizing these semi-illiquid savings is almost the same as that from utilizing illiquid savings in Section 3 (shown in Figure 2).

4.2 With Subsidy to Promote Semi-illiquid Savings

This section, as in section 3.2, considers government policies to promote utilization of semi-illiquid savings by households from a paternalistic perspective. While section 3.2 dealt with an optimization problem with subsidy rate as the policy variable, this section simultaneously optimizes two policy variables, subsidy rate and penalty rate.

4.2.1 Optimal Setting of Subsidy Rate and Penalty Rate

Subsidy rate and penalty rate that optimize the expected lifetime utility for society as a whole were calculated. The results for each of future-consumption-control and all-consumption-control strategies are shown in Table 1 (for the case $\lambda_\theta = 0.6$).

Table 1 shows that the optimal subsidy rate for a future-consumption-control strategy is 0.07, while that for an all-consumption-control strategy is 0.30, with the latter demonstrating stronger incentivizing effects. These are similar to the optimal subsidy rates (for the society as a whole) to illiquid savings in section 3.2, which were 0.05 and 0.34, respectively.

On the other hand, the optimal penalty rate is 0.47 for a future-consumption-control strategy and 0.46 for an all-consumption-control strategy, which are roughly the same. These levels are about 0.1 lower than the 0.58 and 0.57 observed in the case without subsidies as analyzed in section 4.1. This difference arises because the amount of semi-illiquid savings when there are subsidies is larger than when there are no subsidies which increases the likelihood of early withdrawals in period 2. A higher penalty rate increases the cost to fund truly necessary demand while, at the same time, it could discourage excess consumption. The penalty rate, which can be optimally determined by the tradeoff between the two, is relatively low in the case where subsidies increase the amount of semi-illiquid savings.

4.2.2 Savings and Welfare Improvements under Optimal Subsidy and Penalty Rates

This section presents savings behavior and welfare improvements of individual households when the subsidy rate and penalty rate are set to their optimal values for society as a whole. The differences from the case without subsidies and from Section 3.2 (the case with illiquid savings) are analyzed.

Figure 11 shows the savings (x_1 and x_2, for the cases where $\lambda_\theta = 0.6$), made by each household in period 1, where (a) and (b) are under a future-consumption-control strategy and (c) and (d) are under an all-consumption-control strategy, respectively, with the optimal subsidy rate and penalty rate, as shown in solid lines. The dashed lines show for reference the results in the absence of subsidies analyzed in section 4.1. These show that in both strategies, the introduction of subsidies basically decreases liquid savings (x_1) and increases semi-illiquid savings (x_2) (as discussed later, for households with small β_2 in Figure 11 (a) and (b), this is not the case). Thus, it can be observed that the incentives provided by subsidies boosted semi-illiquid savings as a whole. Moreover, an all-consumption-control strategy has a stronger effect on this boost than a future-consumption-control strategy. These characteristics are basically similar to those observed for illiquid savings in section 3.2.

On the other hand, with regard to the effects of subsidies through a future-consumption-control strategy, the following points differ between the results on semi-illiquid savings in this section and the results on illiquid savings in section 3.2.

In the case of semi-illiquid savings (Figure 11 (a) and (b)), contrary to those with larger bias, households with no (or very small) cognitive bias (e.g.; β_2= 0.6) increase their liquid savings and decrease their semi-illiquid savings with the introduction of subsidy. This is related to the fact that in the absence of subsidies, households are less likely to withdraw their semi-illiquid savings early for excess consumption because of the high penalty rate, but in the presence of subsidies, households were more likely to do so because of a lower penalty rate. The households with β_2=0.6 have no cognitive bias at all and are therefore accurately aware in advance of the possibility of such excess consumption in period 2, and they increase their liquid savings in favor of the route that allows them to consume without paying the penalty. Such a phenomenon is not observed in the analysis of illiquid savings (Figures 4 (a) and 4 (b)), where early withdrawal is not possible. It is also not observed in the analysis of semi-illiquid savings for households with larger cognitive bias.

Next, Figures 12 and 13 illustrate this situation in terms of welfare improvements. Figure 12 shows the welfare improvement for each household under the optimized penalty rate and subsidy rate with (a) a future-consumption-control strategy and (b) an all-consumption-control strategy, respectively, shown as solid lines. Compared to welfare improvement in the absence of subsidies, shown by the dashed line, welfare improvement increases for many households. This is because households are more effective in curbing excess consumption in period 2 due to the increase in their semi-illiquid savings when subsidies are available. The exception to this is that, as noted above, under a policy based on a future-consumption-control strategy, households with no (or very small) cognitive bias do not increase their semi-illiquid savings even with this subsidy, and thus welfare improvement does not improve.

The results in this section with those in Section 3.2 on illiquid savings are compared here with respect to the level of welfare improvement for each household after the introduction of subsidies. In particular, under a future-consumption-control strategy, welfare improvement levels are somewhat lower for semi-illiquid savings (Figure 12 (a)) compared to illiquid savings (Figure 5 (a)). The increase in welfare improvement associated with the introduction of subsidies is also smaller overall for semi-illiquid savings under a future-consumption-control strategy. These are due to the fact that with semi-illiquid savings, the effect of self-control is partially offset by the possibility of excess consumption with early withdrawals. With an all-consumption-control strategy, on the other hand, there is no significant difference between the welfare improvement for semi-illiquid savings (Figure 12 (b)) and illiquid savings (Figure 5 (b)). This is because the disadvantage of semi-illiquid savings (excess consumption with early withdrawals) and the advantage (flexible response to demand uncertainty with early withdrawals) are roughly balanced.

In Figure 13, the actual welfare improvement that is realized ex post is indicated by the dashed line, and the welfare improvement that each household expects under the cognitive bias ex ante is indicated by the dotted line. Figures (a) and (b) are the results under future-consumption-control and all-consumption-control strategies for the optimal subsidy rate and penalty rate.

Comparing actual welfare improvement realized ex post with welfare improvement expected under cognitive bias ex ante, the gap between them is larger for households with larger cognitive bias, under both strategies (a) and (b). Also, when we compare the case with semi-illiquid savings (Figure 13) to illiquid savings (Figure 6), that gap is relatively large for semi-illiquid savings, especially under an all-consumption-control strategy. It is because households with significant cognitive bias are a priori overconfident that they will not make as large early withdrawals in period 2 for excess consumption with regard to the increased semi-illiquid savings under the presence of subsidies.

4.3 With Mental Accounting Effects

Even though semi-illiquid savings (x_2) are contractually allowed to be withdrawn early (in period 2) with some costs, households intend, in period 1 (i.e.; early working age), to use such savings for consumption in period 3 (i.e.; distant future after retirement). Therefore, they exercise self-control by deliberately choosing savings that carry a penalty for early withdrawal. In this situation, if households end up making early withdrawals in period 2 against their intention in period 1, they may feel a sense of guilt in addition to bearing the penalty cost. In other words, households may make a psychological distinction for the fund, depending on whether its source is liquid or semi-illiquid savings, when making excess consumption in period 2. Such a possibility is known in various fields as mental accounting.

This section analyzes the effects of such mental accounting that act upon households. I assume a model where the present bias parameter in the utility maximization of households in period 2 is β_1, as in the basic model, for consumption using liquid savings x_1 (here, the households misidentify this parameter as β_2 in period 1), while it is β_3 for consumption using early withdrawals of semi-illiquid savings x_2 (where $\beta_1 < \beta_3 \leqq 1$). In this model, present bias is weakened by the mental accounting effect to the extent that β_3 is larger than β_1. Note that households are assumed to perceive the parameter β_3 as $\beta_{32} \equiv \max(\beta_2, \beta_3)$ in period 1. In our study, β_3 is set as a homogeneous variable among households while β_{32} is a heterogeneous variable as β_2 is heterogeneous.

Specifically, the model treats this mechanism as below. While variable α_1, which represents the proportion of liquid savings x_1 to be used for consumption in period 2, remains the same as shown in Section 2, variable α_2, which represents the proportion of semi-illiquid savings x_2 to be used for consumption in period 2, is perceived by households in period 1 as follows.

$$\text{When } \alpha_1 = 1, \qquad \alpha_2 = \frac{\theta(1-\pi_2)(x_2+x_3) - x_1\beta_{32}\delta}{x_2(1-\pi_2)(\theta+\beta_{32}\delta)}, \qquad 0 \leqq \alpha_2 \leqq 1.$$

Since it is $\beta_{32} \geqq \beta_2$ by definition, households perceive the extent to which they use x_2 myopically in period 2 to be less than that in the case of x_1 (for which the self-perception of present bias is β_2). In other words, households themselves recognize the existence of mental accounting effects. In more detail, households with β_2 larger than β_3 have a cognitive bias of the magnitude of $\beta_2 - \beta_3$ in period 1, while households with β_2 equal to or less than β_3 correctly recognize, in period 1, the size of the parameter β_3 to be realized in period 2.

The government correctly recognizes, in period 1, that all households will in reality have a present bias β_3 with regard to using semi-illiquid savings for consumption in period 2. Thus, the government makes accurate judgments about the variables representing the proportion of savings to be used for consumption in period 2 as the following α_{1pat} and α_{2pat}, unlike the a priori expected values (α_1 and α_2 above) by the households themselves.

$$\text{When } \alpha_{1pat} = 1, \qquad \alpha_{2pat} = \frac{\theta(1-\pi_2)(x_2+x_3) - x_1\beta_3\delta}{x_2(1-\pi_2)(\theta+\beta_3\delta)}, \qquad 0 \leqq \alpha_{2pat} \leqq 1.$$

The formula of α_{2pat} is of the form that β_{32} in the denominator of the formula of α_2 is replaced by β_3.

In the analysis incorporating mental accounting effects, only these points are modified in the basic model described in Section 2. The following two cases are reported in order, one in which β_3 is 1 and the other in which it is 0.8.

4.3.1 With Extremely Strong Mental Accounting Effects

This section analyzes the case where $\beta_3 = 1.0$. In the setting, consumption in period 2 with early withdrawal of semi-illiquid savings is limited to the level determined by intertemporal optimization under an ideal environment with no present bias at all. In other words, in this setting, the effects of mental accounting are extremely strong, and thus semi-illiquid savings are not used for myopic excess consumption in period 2 whatsoever. The following are the results of the numerical analysis.

To begin, the basic scenario without subsidies is examined. Looking at the results with the penalty rate set optimally, the results for both future-consumption-control and all-consumption-control strategies are identical. (For this reason, they are displayed as common results for both in Figures 14 and 15).

Depending on λ_θ, which represents demand uncertainty, and β_2, a parameter of present bias on liquid savings, each household saving is shown in Figure 14. Characteristically, as demand uncertainty in period 2 increases, households increase their semi-illiquid savings and decrease their liquid savings in preparation for it. This tendency is more pronounced for households with smaller cognitive bias. It is because households are aware in advance that holding a large amount of liquid savings could lead to excess consumption in period 2, while at the same time they understand that they can avoid excess consumption even if they have semi-illiquid savings due to the mental accounting effect. In other words, households accurately understand the self-control effect of semi-illiquid savings is complete. The results here are in contrast to the case without mental accounting effects analyzed in section 4.1 (Figure 10 (a) and (b)), where households reduced semi-illiquid savings and increased liquid savings as a response to increased demand uncertainty.

Thus, if $\beta_3 = 1.0$, semi-illiquid savings are utilized as a buffer against demand uncertainty because they are not consumed excessively. The expected value of early withdrawals for x_2 is therefore a distinctly positive amount, as shown in Figure 15. This contrasts with the case in section 4.1, where its expected value is almost zero.

Note that, in such a case, it is better that the optimal penalty rate is lower. (The optimal rate chosen is 0.01, the lower limit in our numerical analysis). It is because while there is no reason to increase the penalty rate because no excess consumption from x_2 will occur whatsoever, there will be a penalty payment associated with early withdrawal.

The actual welfare improvement realized ex post (dashed lines) and the expected welfare improvement under cognitive bias ex ante (dotted lines) are shown in Figure 16. For both strategies (a) and (b), the level of ex post welfare improvements is clearly higher than in the absence of mental accounting effects shown by dashed lines in Figure 12 (a) and (b). Thus, when households have a strong mental accounting effect ($\beta_3 = 1.0$) on their semi-illiquid savings, welfare improvement increases significantly through its utilization.

Next, the scenario in which the government introduces a subsidy to promote semi-illiquid savings (for the case that $\lambda_\theta = 0.6$) under these circumstances is analyzed. It is assumed that the government optimizes both penalty rate and subsidy rate simultaneously, as in section 4.2. The results presented in Table 2 show that the optimal subsidy rates are 0.03 and 0.12 for future-consumption-control and all-consumption-control strategies, respectively. These levels are less than half of the optimal subsidy rates in Table 1 without mental accounting effects (0.07 and 0.30). It reflects the fact that households are aware in period 1 that semi-illiquid savings have a strong mental accounting effect, so they autonomously try to fully utilize semi-illiquid savings as a self-control tool even if the subsidy effect is not as strong. The optimal penalty rates are 0.04 and 0.13, respectively. In cases where the mental accounting effect is extremely strong, excess consumption does not occur even if early withdrawals are not penalized. However, as already noted, the no-arbitrage condition applies to prevent arbitrage transactions for the purpose of obtaining profits from subsidies. Thus, the penalty rate here is the lower bound that satisfies that condition.

Individual household savings when such subsidies are introduced are shown in Figure 17. Since the results under the government's two optimization strategies are qualitatively similar, only the future-consumption-control scenario is presented here.

This demonstrates that households increase allocation of funds into semi-illiquid savings following the introduction of subsidies. The extent of the increase is more pronounced for households with greater cognitive bias ($\beta_2 = 0.9, 0.8$). This is due to the fact that such households misperceive in period 1 that the degree of excess consumption in period 2 is not large, even if they utilize liquid savings more. Thus, they had insufficient self-control through semi-illiquid savings in the absence of subsidies. In fact, as shown in Figure 18, the increase in welfare improvements following the introduction of the subsidy is greater for households with larger cognitive bias.

4.3.2 With Moderate Mental Accounting Effects

This section analyzes the scenario where $\beta_3 = 0.8$. In this setting, consumption in period 2 with early withdrawal of semi-illiquid savings is limited to the level determined by intertemporal optimization under a relatively weak present bias ($\beta_3 > \beta_1 = 0.6$). Here, the effect of mental accounting is not as strong as in the setting in the previous section, but is still present to a certain extent. Under this environment, myopic excess consumption in period 2 by using semi-illiquid savings is less likely to take place than in the absence of mental accounting effects. In the following, we present the results of our numerical analysis.

To begin, the basic scenario with no subsidy is examined. The optimal penalty rates are shown in Figure 19. For both (a) a future-consumption-control strategy and (b) an all-consumption-control strategy, the optimal penalty rates are within the range of 0.2 to slightly above 0.4, depending on the degree of demand uncertainty. These rates are clearly lower than when mental accounting effects are absent (Figure 8), where the optimal penalty rates are within the range of 0.4 to 0.6. It reflects the fact that, due to the effect of mental accounting, excess consumption using semi-illiquid savings is unlikely to occur in period 2 even with a low penalty rate. In fact, the expected value of the amount of early withdrawals from semi-illiquid savings in period 2 is extremely small, as shown in Figure 20. Given these results, it can be considered that the optimal penalty rate is set at the lower end of the range at which early withdrawals can be adequately controlled, while also taking into account the effects of mental accounting. The results shown in Figure 20 are generally the same as those in Figure 9 in section 4.1. This confirms that the effect of mental accounting has reduced the level of the penalty rate that sufficiently prevents early withdrawals.

Each household's savings behavior under this penalty rate are shown in Figure 21, depending on λ_θ representing demand uncertainty and β_2 representing self-perception of present bias on liquid savings. Although this figure only illustrates the optimization case for a future-consumption-control strategy, qualitatively, the features described below apply equally well to the case of an all-consumption-control strategy.

The savings behavior shown in Figure 21 is very similar to the savings behavior when there are no mental accounting effects as analyzed in section 4.1 (Figure 10 (a) and (b)).

Comparing these results with those with strong mental accounting effects ($\beta_3 = 1.0$), there is a significant difference. When $\beta_3 = 1.0$, since it was certain that no excess consumption would arise from semi-illiquid savings, semi-illiquid savings were utilized instead of liquid savings as a buffer for demand uncertainty (as shown in Figure 14 above). As a result, while paying a minimally set penalty rate (0.01), early withdrawals increased when demand increased (as shown in Figure 15 above). The results presented in this section, on the other hand, are the opposite: the penalty rate is raised to prevent excess consumption, while liquid savings are utilized as a buffer for demand uncertainty. Such a mechanism is similar to that in the absence of mental accounting effects.

Regarding welfare improvements, the results (of which the figures are omitted) with moderate mental accounting effects in this section are also very close to those with no mental accounting effects (shown by the dashed lines in Figure 12 (a) and (b)). It is due to the fact that, as noted above, for both strategies, the penalty rate is set so that almost no early withdrawals of semi-illiquid savings in period 2 occur. Thus, differences in penalty rates between the two cases do not lead to differences in economic burdens, since most of the consumption in period 2 is only from liquid savings. Therefore, there is little difference in welfare improvements.

Next, we analyze the case in which the government introduces a subsidy to promote semi-illiquid savings (where $\lambda_\theta = 0.6$). As in the previous section, the assumption is that the government optimizes both the penalty rate and the subsidy rate simultaneously. The results presented in Table 3 show that the optimal subsidy rates are 0.07 and 0.21 for future-consumption-control and all-consumption-control strategies, respectively. These levels are about twice as high as those with strong mental accounting effects (0.03 and 0.12). This is because when the mental accounting effect is moderate, households do not necessarily exercise sufficient self-control through semi-illiquid savings if the subsidy is low. To note, the optimal subsidy rates in the absence of households' mental accounting effects as seen in Table 1 of section 4.2 are 0.07 and 0.30. The result in this section aligns with that of a future-consumption-control strategy, while the result for an all-consumption-control strategy falls midway between this and the scenario with strong mental accounting.

The optimal penalty rates here are 0.08 and 0.22, respectively. As in the previous section, these are the lower limits at which the no-arbitrage conditions are established to prevent arbitrage transactions for the purpose of obtaining subsidies. Compared to the optimal penalty rates (0.47 and 0.46) in the absence of household mental accounting effects in section 4.2, these levels are considerably lower. On the other hand, compared to the optimal penalty rates when mental accounting effects are strong (0.04 and 0.13 in Table 2), the results here are higher.

The savings of individual households when such subsidies are introduced are shown in Figure 22. Since the results under the government's two optimization strategies are qualitatively similar, only the results for a future-consumption-control case are presented.

Figure 22 shows that households significantly increase allocation of funds into semi-illiquid savings following the introduction of subsidies. This reflects the fact that in the absence of subsidies, households primarily utilize liquid savings as a buffer against demand uncertainty in period 2, but with the introduction of subsidies, they change to primarily utilizing semi-illiquid savings as a buffer against such uncertainty. Such a change is consistent with the fact that the optimal penalty rates fall substantially with the introduction of subsidies, from 0.43 and 0.41 to 0.08 and 0.22, respectively, as shown in Table 3. This decrease significantly reduces the financial burden on households associated with early withdrawals. When there are moderate mental accounting effects, the decline in the penalty rate will result in a certain amount of excess consumption funded by semi-illiquid savings (with the present bias parameter $\beta_3 = 0.8$), but to a smaller extent than in the situation with excess consumption funded by liquid savings (with the present bias parameter $\beta_1 = 0.6$). Given these facts, the welfare improvement of each household increases significantly with the introduction of subsidies. Figure 23 shows the specific changes in the welfare improvement of each household.

Figure 23 shows that when the mental accounting effect is moderate, the increase in welfare improvement of each household following the introduction of subsidies is larger than when the mental accounting effect is very strong (Figure 16) or, conversely, when that effect is absent (Figure 12).

5. Effects of Self-control through Both Semi-illiquid and Illiquid Savings and Optimal Policy

This section discusses a situation in which households can allocate resources not consumed in period 1 to three kinds of assets: liquid savings (x_1), semi-illiquid savings (x_2), and illiquid savings (x_3). As in section 4, a penalty rate determined as the optimal policy is applied to early withdrawals of semi-illiquid savings.

The main issue in this section is how households with different cognitive biases utilize the two types of illiquid savings for self-control to curb excessive saving. The following will in turn analyze the base case in which there is no subsidy, the case with optimal subsidies, and the case in which mental accounting effects act on households with respect to semi-illiquid savings.

5.1 Basic Analysis: Without Subsidy

In the base case with neither subsidies nor mental accounting effects, the optimization problem in which households allocate their initial endowments to current consumption and three types of savings (x_1, x_2, x_3) in period 1 is examined. The results show that the utilization of semi-illiquid savings (x_2) is zero for any households regardless of β_2 under any demand uncertainty (λ_θ) and with any penalty rate. Thus, the optimal penalty rate on early withdrawals of x_2 is indeterminate.

Due to the utilization of only two types of savings (x_1, x_3), their specific amounts are completely identical to the results in section 3.1 (Figure 1) which assumes only x_1 and x_3 are available. The results for welfare improvements are also identical to those in Figure 2 in section 3.1.

The reason why funds are not allocated to semi-illiquid savings at all is as follows. Semi-illiquid savings are less effective than illiquid savings as a tool of commitment for self-control to curb excess consumption in period 2, since the latter do not allow early withdrawals. Moreover, as a buffer to deal with demand uncertainty, semi-illiquid savings cannot beat liquid savings which have no penalty. In other words, although the characteristics of semi-illiquid savings are intermediate between the other two types of savings, they do not hold a competitive edge. This results in semi-illiquid savings not being utilized at all.

5.2 With Subsidy to Promote Semi-illiquid and Illiquid Savings

In this section, as in Sections 3.2 and 4.2, a situation in which the government offers subsidies to encourage households to utilize semi-illiquid savings and illiquid savings from a paternalistic perspective is examined. The government simultaneously optimizes two policy variables, subsidy rate and penalty rate.

5.2.1 Optimal Setting of Subsidy Rate and Penalty Rate

The optimization results for the two policy variables, subsidy rate and penalty rate, by future-consumption-control and all-consumption-control strategies, respectively, are shown in Table 4 (for the case $\lambda_\theta = 0.6$).

Table 4 shows that the optimal subsidy rate for a future-consumption-control strategy is 0.04, while that for an all-consumption-control strategy is 0.21. Compared to the optimal subsidy rates of 0.05 and 0.34 for each strategy on illiquid savings in Section 3.2 and 0.07 and 0.30 on semi-illiquid savings in Section 4.2, the optimal subsidy rates in this section are both relatively low. It is because the targets of the subsidies here are broad: both illiquid and semi-illiquid savings.

On the other hand, the optimal penalty rate is 0.20 or higher for a future-consumption-control strategy and 0.22 for an all-consumption-control strategy. For a future-consumption-control strategy, if the penalty rate is lower than this rate, a semi-illiquid savings balance will be generated and welfare will decline from excess consumption through early withdrawals, but if the penalty rate is equal to or higher than this rate, no semi-illiquid savings will be generated as shown in Figure 24 (b) below. With an all-consumption-control strategy, the optimal penalty rate is determined in the tradeoff between controlling excess consumption and dealing with demand uncertainty. Compared to the optimal penalty rates (0.47 and 0.46) for each strategy in Section 4.2, the optimal penalty rates here are both lower. In Section 4.2, since illiquid savings did not exist, it was desirable to set the penalty rate higher so that the nature of semi-illiquid savings would be closer to those of illiquid savings in that they would function effectively as a tool for self-control of households. On the other hand, the role played by semi-illiquid savings in this section differs from that in Section 4.2 because of the presence of illiquid savings.

5.2.2 Savings and Welfare Improvements under the Optimal Subsidy and Penalty Rates

This section presents saving behavior and welfare improvement of each household under the optimal subsidy rate and penalty rate for society as a whole (when $\lambda_\theta = 0.6$).

Figure 24 shows in solid lines the savings (x_1, x_2, x_3) of each household in period 1 under a future-consumption-control strategy in (a), (b), and (c) and those under an all-consumption-control strategy in (d), (e), and (f), respectively. The dashed lines refer to the results in the absence of the subsidies analyzed in Section 5.1. Figure 25 shows the welfare improvements under a future-consumption-control strategy in (a) and those under an all-consumption-control strategy in (b).

Under a future-consumption-control strategy, the balance of semi-illiquid savings (Figure 24 (b)) of any households is zero, as discussed earlier. Thus, their savings behavior in Figure 24 (a) and (c) are very similar to when only liquid and illiquid savings are available (shown in Figure 4 (a) and (b)) in Section 3.2 (but not exactly the same due to the slightly different setting of the optimal subsidy rate). Specifically, the introduction of subsidy reduces liquid savings and increases illiquid savings for each household. The effect of self-control through illiquid savings is greater for households with smaller cognitive bias. The welfare improvement shown in Figure 25 (a) is also very similar to that shown in Figure 5 (a) in Section 3.2.

On the other hand, under an all-consumption-control strategy, the introduction of subsidies significantly reduces liquid savings (Figure 24 (d)) for each household, funds are allocated to increase semi-illiquid savings (Figure 24 (e)) and illiquid savings (Figure 24 (f)). Households with larger cognitive bias are characterized by a relatively larger increase in semi-illiquid savings and a smaller rise in illiquid savings. It is because the perception of the tradeoff between the advantages (response to demand uncertainty through early withdrawal) and disadvantages (possibility of excess consumption) of semi-illiquid savings differs depending on the size of the cognitive bias. That is, households with larger cognitive bias tend to hold more semi-illiquid savings because they disregard the disadvantages. In addition, the level of welfare improvements shown in Figure 25 (b) increases substantially with the introduction of subsidies for each household. Overall, the level of welfare improvements with subsidies is similar to the level post introduction of subsidies when only liquid and illiquid savings are available as analyzed in Section 3.2 (Figure 5 (b)), and to when only liquid and semi-illiquid savings are available as analyzed in Section 4.2 (Figure 12 (b)). However, looking somewhat more closely, the welfare improvement for households with large cognitive bias ($\beta_2 = 0.9$) is slightly lower here than in Sections 3.2 and 4.2. It is because, given the relatively low optimal penalty rate in this section, households with large cognitive bias will exacerbate the welfare improvement by engaging in more excess consumption through early withdrawals of their semi-illiquid savings in period 2.

5.3 With Mental Accounting Effects

This section, as in section 4.3, assumes the existence of mental accounting effects in which the present bias (of which the relevant parameter β_3 is greater than β_1) is weakened when households utilize semi-illiquid savings (x_2) to consume in period 2. In this case, semi-illiquid savings are utilized, unlike the scenario in section 5.1. In the following, two cases are discussed: one when β_3 is 1 and the other when β_3 is 0.8.

5.3.1 With Extremely Strong Mental Accounting Effects

This section analyzes the scenario when $\beta_3 = 1.0$. In this setting, the effect of mental accounting is so strong that semi-illiquid savings are not used for myopic excess consumption in period 2 at all. For this reason, semi-illiquid savings are prominently utilized as a tool of self-control. The results of the numerical analysis are presented below in detail.

To begin, the base case with an optimal penalty rate and no subsidy is explored. Household consumption and savings are exactly the same for both cases under future-consumption-control and all-consumption-control strategies (for this reason, Figures 26 and 27 below are presented as common results for both strategies).

The savings of each household are shown in Figure 26, depending on λ_θ, which represents demand uncertainty, and β_2, a parameter of present bias on liquid savings. Most strikingly, as demand uncertainty increases in period 2, each household increases its semi-illiquid savings as a buffer and at the same time reduces its illiquid savings. On the other hand, for liquid savings, in the absence of demand uncertainty, households hold an amount equal to the deterministically forecasted size of consumption in period 2, but when demand uncertainty exists, households reduce their liquid savings slightly and increase their semi-illiquid savings. It is because households recognize in period 1 that holding liquid savings in excess of demand that will be realized in period 2 will result in excess consumption. To mitigate such excess consumption, households curb their liquid savings, and intend to use semi-illiquid savings by early withdrawal when a large demand is realized in period 2, even if they have to pay a small penalty. However, a closer look reveals that households with a fairly large cognitive bias ($\beta_2 = 0.9$) tend to disregard the possibility of excess consumption and prepare for a possible large demand realized in period 2 with liquid savings more, rather than with semi-illiquid savings which has a penalty. All of these characteristics are based on the households' understanding that they will not overconsume when holding a large amount of semi-illiquid savings due to strong mental accounting effects. This mechanism is the same as in Section 4.3.1. In this section, however, not only semi-illiquid savings but also illiquid savings are available, so the characteristics of overall savings behavior differ as described above.

As discussed, semi-illiquid savings are utilized as a buffer against demand uncertainty. Therefore, as shown in Figure 27, the expected value of early withdrawals of x_2 is distinctly positive for all households in the presence of uncertainty. It can be confirmed in Figure 27 that the greater the demand uncertainty, the more early withdrawals of x_2 are expected on average.

Note that the lower the penalty rate in this case, the better (in a numerical analysis, 0.01, the lower limit of the possible options, is the optimal rate). As mentioned earlier, because a not so small amount of semi-illiquid savings are withdrawn early in response to consumption demand, households favor less burden of paying the penalty. On the other hand, since holding semi-illiquid savings does not lead to excess consumption at all, there is no reason to raise the penalty rate. In other words, it is not the existence of penalties but that of mental accounting effects that allows households to self-control through semi-illiquid savings.

The actual welfare improvement realized ex post and the welfare improvement expected under cognitive bias ex ante are shown in Figure 28 (for the case $\lambda_\theta = 0.6$). Compared to the scenario with no mental accounting effects (dashed lines in Figure 2 (a) and (b), the common results of Section 5.1 and 3.1), the level of actual welfare improvement is clearly higher in the results of this section for both the two strategies (a) and (b). Specifically, for all households, the level of welfare improvement in this section is generally about 20 percentage points higher for (a) and just under 10 percentage points higher for (b). Thus, when households have a strong mental accounting effect on their semi-illiquid savings, their utilization leads to a significant increase in welfare improvement.

Next analyzed is the scenario with subsidies to promote semi-illiquid and illiquid savings (for the case $\lambda_\theta = 0.6$). The assumption here again is the government simultaneously optimizing both the penalty rate and the subsidy rate.

The results presented in Table 5 show that the optimal subsidy rates are 0.02 and 0.17 for future-consumption-control and all-consumption-control strategies, respectively. These levels are lower than the results (0.04 and 0.21) without mental accounting effects. It is because households recognize in period 1 that semi-illiquid savings have a strong mental accounting effect, so they voluntarily utilize them as a commitment tool for self-control, even if the subsidy effect is not strong. The optimal penalty rates are 0.03 and

0.18, respectively. In cases where the mental accounting effect is very strong, excess consumption does not occur even without penalizing early withdrawals. However, as noted above, no-arbitrage conditions apply to prevent arbitrage transactions aimed at making profits from subsidies. The penalty rate here is the lower limit that satisfies that condition. They are qualitatively similar to the analysis with subsidies in section 4.3.1.

Individual household savings when such subsidies are introduced are shown in Figure 29. It illustrates that with the introduction of subsidies each household increases its semi-illiquid and illiquid savings and decreases its liquid savings. The extent of the change depends on the subsidy rate and the households' cognitive bias.

Specifically, in the case of a future-consumption-control strategy ((a), (b), and (c)), the change in amount of each savings due to the introduction of subsidies is very small for most households because of the small subsidy rate. However, only households with a large cognitive bias ($\beta_2 = 0.9$) have a larger change with the introduction of the subsidy than other households, because they utilize more liquid savings in the absence of subsidies. On the other hand, in the case of an all-consumption-control strategy ((d), (e), and (f)), the change in the amount of each savings is relatively large because of the large subsidy rate. As a result of such a change in savings behavior, the welfare improvement for each household is shown in Figure 30. Again, the increase differs between (a) and (b) depending on the magnitude of the subsidy rate.

5.3.2 With Moderate Mental Accounting Effects

This section analyzes the scenario when $\beta_3 = 0.8$. In this setting, mental accounting effects are present to some extent, although not as strong as in the previous section. Since the degree to which semi-illiquid savings are used for myopic excess consumption in period 2 is less than in the absence of mental accounting effects, semi-illiquid savings function to a certain extent as a tool of self-control. Therefore, households will still utilize semi-illiquid savings under this setting. Below are the results of the numerical analysis.

To begin, the base case without subsidies is explored. The savings behavior of each household is shown in Figure 31, depending on λ_θ, demand uncertainty, and β_2, a parameter of present bias on liquid savings. Figures (a) through (c) are under a future-consumption-control strategy and (d) through (f) are under an all-consumption-control strategy. The results under both strategies are quite similar to each other. However, a closer comparison of the two scenarios, one with an all-consumption-control strategy and the other with a future-consumption-control strategy, reveals that the former shows a slightly lower consumption (of which the figures are omitted) and liquid savings, a slightly higher semi-illiquid savings, and a slightly lower illiquid savings in period 1.

This reflects the fact that the optimal penalty rates are set slightly differently for the two strategies, as seen below, and that an all-consumption-control strategy places more emphasis on consumption in the second and subsequent periods than on consumption in period 1. Another feature common to both strategies is the difference in savings behavior according to household cognitive bias. Specifically, households with larger cognitive bias ($\beta_2 = 0.9, 0.8$) do not utilize semi-illiquid savings regardless of the magnitude of demand uncertainty because they falsely believe that they can control present bias in period 2 without resorting to the mental accounting effects of semi-illiquid savings. In contrast, households with smaller cognitive biases ($\beta_2 = 0.7, 0.6$) understand that even moderate mental accounting effects are helpful in controlling present bias in period 2. Therefore, they utilize semi-illiquid savings both as a tool for self-control and as a buffer against demand uncertainty. In fact, the expected amount, shown in Figure 32, of semi-illiquid savings withdrawn early in period 2 confirms that households with small cognitive bias significantly utilize them in case demand uncertainty is large ($\lambda_\theta = 0.4, 0.6$). In contrast, households with large cognitive bias utilize mainly liquid savings as a buffer against demand uncertainty. Indeed, the greater the demand uncertainty and the larger the cognitive bias, the more households increase their liquid savings and, conversely, reduce their illiquid savings.

It is interesting that when mental accounting effects are moderate, household savings behavior differs significantly depending on the magnitude of the cognitive bias. In reality, household mental accounting effects are likely to be of moderate strength rather than at full strength at $\beta_3 = 1.0$. Thus, it suggests that when governments make decisions about relevant policies, they should be aware of the impact of households' heterogeneity with respect to cognitive bias on policy effects.

The optimal penalty rate is shown in Figure 33. As indicated in Figure 31, when demand uncertainty is relatively small ($\lambda_\theta = 0.0, 0.2$), no households utilize semi-illiquid savings. Thus, since the penalty rate on early withdrawals has no economic impact, the optimal penalty rate is undetermined. On the other hand, when demand uncertainty is relatively large ($\lambda_\theta = 0.4, 0.6$), the optimal penalty rates are 0.04 and 0.03 for each λ_θ under a future-consumption-control strategy as shown in (a) and 0.02 and 0.01 under an all-consumption-control strategy as shown in (b). For both strategies, the penalty rates are somewhat smaller when $\lambda_\theta = 0.6$. It reflects the fact that, because greater demand uncertainty increases the need of some households for early withdrawals of their semi-illiquid savings, lower penalties increase their welfare. The slight difference in the optimal penalty rates in (a) and (b) also generates the differences in savings behavior described above.

Note that the optimal penalty rate here is generally lower than in Section 4.3.2. Although the mental accounting effect is moderate in both cases, the absence of illiquid savings in Section 4.3.2 necessitated an increase in the penalty rate for semi-illiquid savings as the only commitment tool for self-control to prevent excess consumption. In contrast, as analyzed in this section, one can utilize illiquid savings, as well as semi-illiquid savings, for self-control. Therefore, semi-illiquid savings can play another important role as a buffer against demand uncertainty in addition to a role for preventing excess consumption. This is why the overall optimal penalty rate is lower here.

Welfare improvement is shown in Figure 34, which reflects the influence of the difference in savings behavior according to household cognitive bias, as we discussed earlier as the common feature of the two strategies (a) and (b). Specifically, households with large cognitive bias ($\beta_2 = 0.9, 0.8$) do not take advantage of mental accounting effects ($\beta_3 = 0.8$) of semi-illiquid savings, and thus the welfare improvement realized ex post is relatively low, which is almost the same as in the case without mental accounting effects (the result in section 5.1 as shown by the dashed lines in Figure 2 (a) and (b)). In contrast, households with smaller cognitive bias ($\beta_2 = 0.7, 0.6$) take advantage of the mental accounting effect of semi-illiquid savings, so the welfare improvement realized ex post is clearly higher than when there are no mental accounting effects. However, the level of welfare improvement here is not as high as when mental accounting effects are very strong as shown in Figure 28.

Next, the scenario in which the government introduces a subsidy to promote semi-illiquid and illiquid savings under the same circumstances (when $\lambda_\theta = 0.6$) is analyzed. The assumption is, as in the previous section, that the government optimizes both the penalty rate and the subsidy rate simultaneously. The results presented in Table 6 show that the optimal subsidy rates are 0.04 and 0.20 for future-consumption-control and all-consumption-control strategies, respectively. The levels are somewhat higher compared to when mental accounting effects are strong (Table 5), where the values are 0.02 and 0.17. It reflects the fact that, if the subsidy is low, households with large cognitive bias do not sufficiently exercise self-control through semi-illiquid savings when moderate mental accounting effects are moderate. On another note, the optimal subsidy rates in the absence of household mental accounting effects as seen in Table 4 in Section 5.2 are 0.04 and 0.21. The results in this section are roughly in line with those for both future-consumption-control and all-consumption-control strategies. The optimal penalty rates in this section are 0.05 and 0.21, respectively. As in the previous section, this level is the lower bound at which no-arbitrage conditions are satisfied to prevent arbitrage for making profits from subsidies.

The savings of each household when such subsidies are introduced are shown in Figure 35, depending on the present bias parameter β_2 on liquid savings. Figures (a) to (c) are the results under a future-consumption-control strategy and (d) to (f) are the results under an all-consumption-control strategy.

The results under the two strategies are qualitatively similar to each other. Under both strategies, each household reduces its liquid savings and increases its semi-illiquid savings and illiquid savings following the introduction of the optimal subsidy. In particular, the increase in semi-illiquid savings and the decrease in liquid savings are significantly large for households with large cognitive bias ($\beta_2 = 0.9, 0.8$) that did not utilize semi-illiquid savings at all in the absence of subsidies. This is because they shifted funds from liquid savings to semi-illiquid savings as a buffer against demand uncertainty in period 2, following the introduction of subsidies.

In addition, a detailed comparison between the two scenarios with a future-consumption-control strategy and an all-consumption-control strategy shows more illiquid savings and less liquid savings in the latter, reflecting a larger subsidy rate. On the other hand, there is little difference between the two strategies in the amount of semi-illiquid savings. It is because, under the setting of this section, the primary role of semi-illiquid savings is to provide a buffer against demand uncertainty, so its requirement is almost the same for both strategies. In addition, there is also a small difference between the two strategies for consumption (c_1) in period 1 (the figure is omitted). Specifically, the all-consumption-control strategy has lower consumption in period 1. It is due to the greater incentive to obtain subsidies, which leads to reducing consumption in period 1 and increasing savings such as illiquid savings.

As a result of the savings behavior described above, welfare improvement for each household is shown in Figure 36. Again, the magnitude of the increase in welfare improvements differs between the two strategies, as shown in figures (a) and (b), according to the size of the subsidy rate. Households with larger cognitive bias that experienced a larger change in savings following the introduction of the subsidy have a larger increase in welfare improvements.

Figure 36 shows that when the mental accounting effect is moderate, the increase in welfare improvements of households following the introduction of the subsidy is larger than when the mental accounting effect is extremely strong (Figure 30) or, on the contrary, when there are no mental accounting effects (Figure 25). It is because when the mental accounting effect is extremely strong, the level of welfare improvements is already high even without the subsidy, and the increase in welfare improvements with the introduction of the subsidy is relatively limited. In the absence of mental accounting effects, the potency of semi-illiquid savings as a controlling mechanism to curb excess consumption is relatively weak, so that even if subsidies increase the utilization of these savings, the level of welfare improvements will not be as high as when mental accounting effects are present.

6. Conclusion

This paper has analyzed consumption and savings of households, who may engage in myopic excess consumption due to present bias, but can utilize illiquid and/or semi-illiquid savings to self-control their consumption. In particular, I have focused on situations where households have cognitive biases that lead them to be overconfident that their present bias can be mitigated in the future, thus leading to inadequate self-control. Scenarios in which the government sets a penalty rate for early withdrawals of semi-illiquid savings and also sets a subsidy rate for semi-illiquid and illiquid savings to promote them have been discussed. These two policy variables have been set socially optimally from a paternalistic perspective. In addition, I have addressed the scenarios where households are subject to mental accounting in their consumption with early withdrawals. The analysis has been conducted numerically by constructing a three-period model consisting of households with heterogeneous cognitive biases.

As a result of the analysis, the characteristics of household consumption and savings in different settings, their impact on welfare, the function of illiquid and semi-illiquid savings, and the optimal policy have been reported in detail in Sections 3 through 5. In the following, the results of this paper's analysis are organized into four points, with an awareness of policy implications, based on the premise that the pension system, which forms important part of social security, provides a function of the illiquid or semi-illiquid savings in this paper.

(1) Clear differences in household consumption-savings behavior and the resulting welfare improvement have been observed depending on the magnitude of their present bias and cognitive bias. It can be also confirmed that if government subsidies are provided optimally based on information on present bias and cognitive bias, the decline in welfare caused by these biases is significantly suppressed. The optimal subsidy rate has depended on the setting of the government's policy strategy. Specifically, the desired subsidy rate has differed significantly depending on whether the policy strategy focuses on mitigating present bias in the future and related cognitive bias, or on mitigating

present bias at the present time in addition to the aforementioned biases.

Therefore, in determining the subsidy rate as a policy matter, it is extremely important to have accurate information on households' present bias, cognitive bias (or their probability distribution), etc., as well as to clarify the policy objectives. For this reason, it is desirable to further conduct related empirical analyses.

(2) For semi-illiquid savings, which can be withdrawn before the due date if a penalty is paid, it has been revealed that their function differs significantly when coexisting with illiquid savings (Section 5), which cannot be withdrawn before the due date, and when not coexisting with illiquid savings (Section 4). In overview, when not coexisting with illiquid savings it has been socially best to set the penalty rate for early withdrawal of semi-illiquid savings relatively high. This has made semi-illiquid savings function more as a commitment tool for self-control to curb excess consumption, at the expense of their ability to respond to idiosyncratic demand shocks. In other words, it has been most effective to allow semi-illiquid savings to perform almost the same function as illiquid savings alone (Section 3). On the other hand, when coexisting with illiquid savings, it has been often socially best to set the penalty rate for early withdrawal relatively low. As a result, the main function of semi-illiquid savings has been to respond to idiosyncratic demand shocks, while their function as a commitment tool for self-control has been left mainly to illiquid savings.

Therefore, judgment on how semi-illiquid savings should be positioned for policy purposes and how the penalty rate should be set will depend largely on whether the system is designed to make fully illiquid savings available at the same time.

(3) The function of semi-illiquid savings has differed significantly depending on the degree to which the mental accounting effect is exerted on early withdrawal. The stronger the mental accounting effect is, the more households recognize the effectiveness of semi-illiquid savings as a commitment tool for self-control and voluntarily increase their utilization of semi-illiquid savings, and thus they are more inclined to utilize savings for the purpose of both controlling excess consumption and responding to idiosyncratic demand shocks. On the other hand, when mental accounting effects are absent (or very weak), the role of semi-illiquid savings has been found to be less differentiated from that of illiquid savings. For example, in the absence of subsidies, semi-illiquid savings have not been utilized at all when there are no mental accounting effects, even though both semi-illiquid and illiquid savings would have been available. It is because semi-illiquid savings in that situation are inferior to illiquid savings for the

purpose of households' self-control and inferior to liquid savings for the purpose of responding to idiosyncratic demand shocks.

Therefore, judgment on how semi-illiquid savings should be positioned for policy purposes and how the penalty rate should be set will largely depend on the evaluation of the effects of mental accounting on households. For this reason, further related empirical analysis is warranted.

(4) The optimal provision of subsidies has served to raise the utilization of illiquid and/or semi-illiquid savings, which would otherwise have been underutilized, to near proper levels. Looking at households individually, households with insufficient self-control in the absence of subsidies have been those with large cognitive bias. Thus, the larger the households' cognitive bias, the greater the increase in the welfare improvement associated with the introduction of the subsidy. This point is characteristic of the subsidy as a distributional policy.

Finally, I will point out three remaining issues in relation to this study. The first is, as noted above, the need to deepen the empirical analysis of present bias, cognitive bias, and mental accounting effects. It will allow for more appropriate judgments in determining relevant policies and institutions. Second, while this study allowed for heterogeneity only in the households' cognitive biases, further analysis should incorporate heterogeneity in present biases as well. Third, the impact of opt-in costs (i.e.; the various costs of making semi-illiquid and/or illiquid savings) should be analyzed. This study has analyzed the effect of mental accounting on early withdrawal, which can actually include not only the mental impact on households but also the effects of the hassle involved in making a mid-course cancellation. In this sense, this study has conducted a certain qualitative analysis of the effects of opt-out costs related to early withdrawal of semi-illiquid savings. On the other hand, it has not taken into account the impact of opt-in costs. It is inferred qualitatively that in the environments with opt-in costs the optimal subsidy rate will be higher to offset the effect than in the environments without the costs. More specific characteristics would be understood by an analysis that incorporates opt-in costs into the model.

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Figure 1

Savings in period 1 by households with β_2 with liquid savings (x_1) and illiquid savings (x_3)

(a) Liquid savings (x_1) | (b) Illiquid savings (x_3)

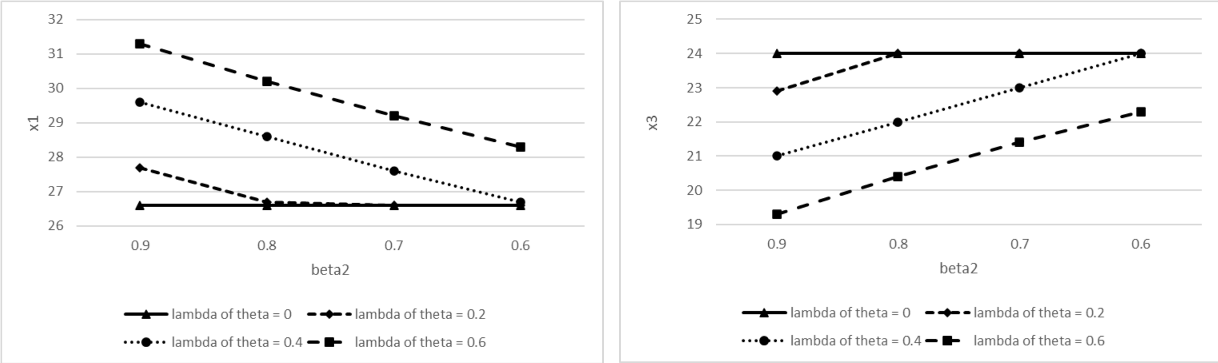


Figure 2

Welfare improvements of households with β_2 with liquid savings (x_1) and illiquid savings (x_3): Expected with a cognitive bias and realized actually

(a) Welfare improvement under future-consumption-control (λ_θ = 0.6)

(b) Welfare improvement under all-consumption-control (λ_θ = 0.6)

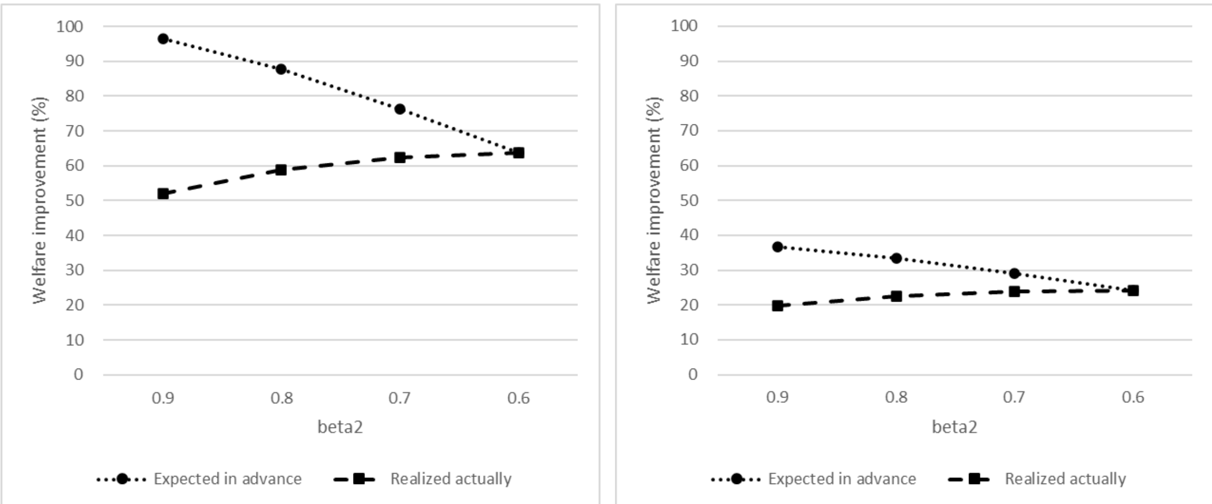
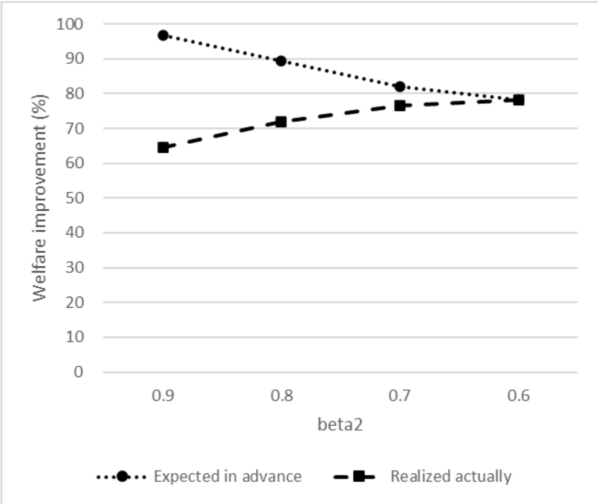


Figure 2 (continued)

(c) Welfare improvement under future-consumption-control (λ_θ = 0.4)



(d) Welfare improvement under all-consumption-control (λ_θ = 0.4)

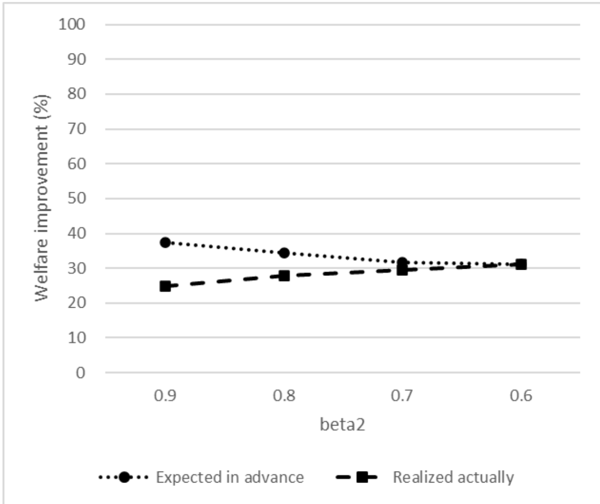
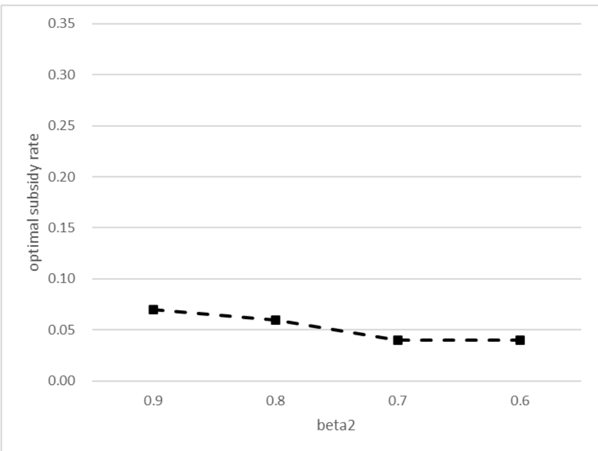


Figure 3

Optimal subsidy rate on illiquid savings (x_3) for households with β_2 (λ_θ = 0.6)

(a) Optimal subsidy rate under future-consumption-control



(b) Optimal subsidy rate under all-consumption-control

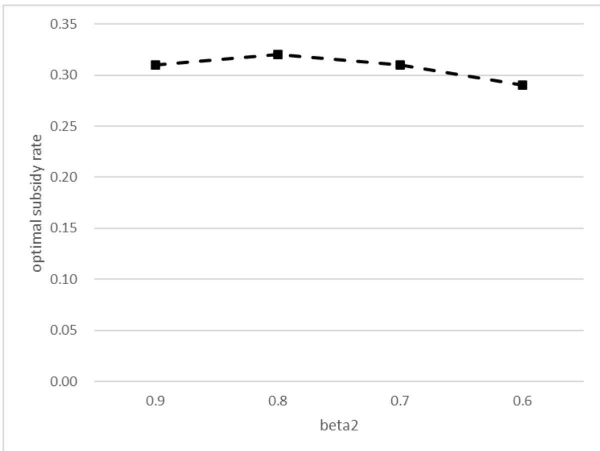
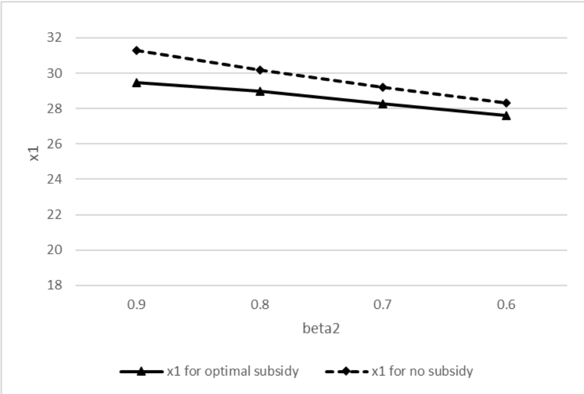


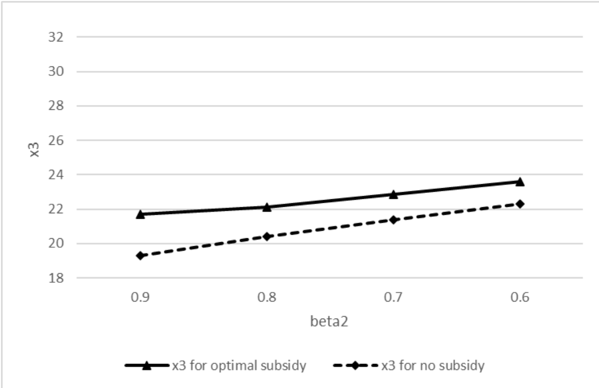
Figure 4

Savings in period 1 by households with β_2 with the optimal subsidy for the aggregate society and without subsidy (λ_θ = 0.6)

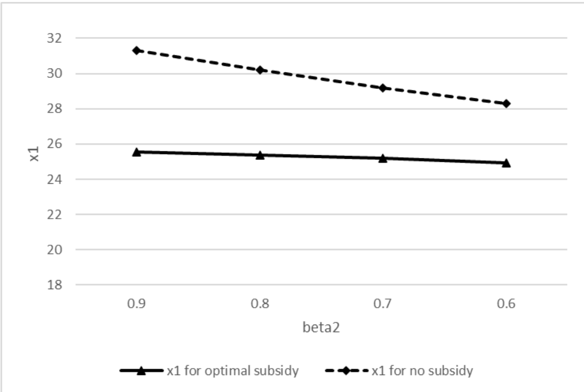
(a) Liquid savings (x_1)
under future-consumption-control



(b) Illiquid savings (x_3)
under future-consumption-control



(c) Liquid savings (x_1)
under all-consumption-control



(d) Illiquid savings (x_3)
under all-consumption-control

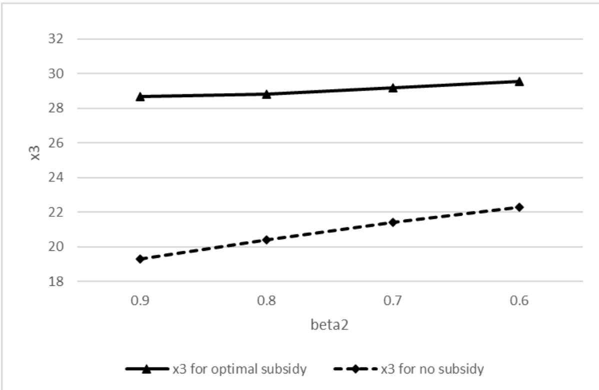
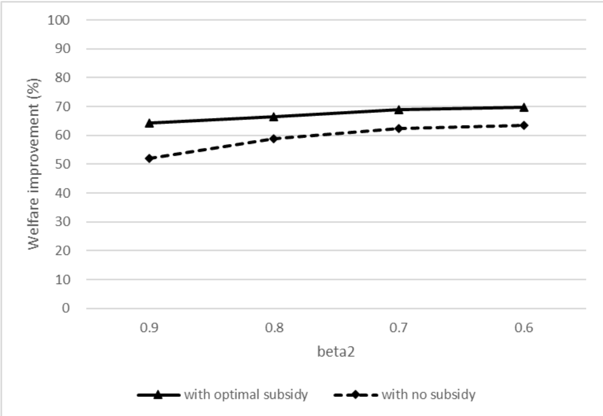


Figure 5

Welfare improvements of households with β_2 with the optimal subsidy for the aggregate society and without subsidy (λ_θ = 0.6)

(a) Welfare improvement under future-consumption-control



(b) Welfare improvement under all-consumption-control

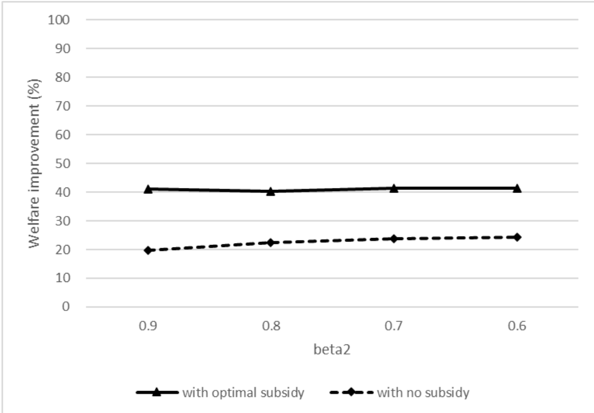
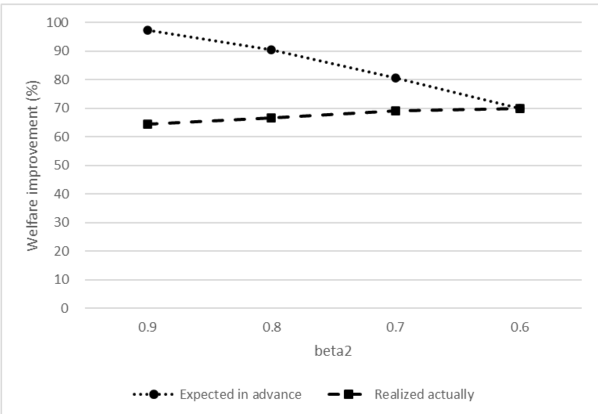


Figure 6

Welfare improvements of households with β_2 with the optimal subsidy for the aggregate society (λ_θ = 0.6): Expected with a cognitive bias and realized actually

(a) Welfare improvement under future-consumption-control



(b) Welfare improvement under all-consumption-control

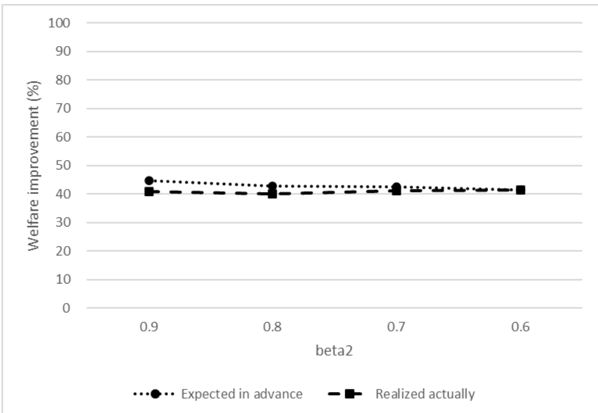
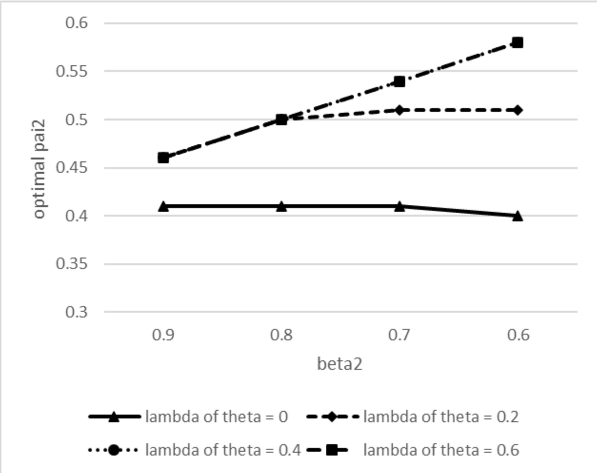


Figure 7

Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) for households with β_2

(a) Optimal penalty rate under future-consumption-control



(b) Optimal penalty rate under all-consumption-control

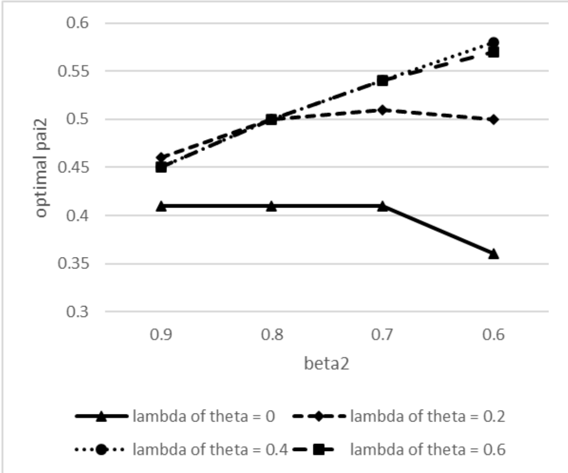
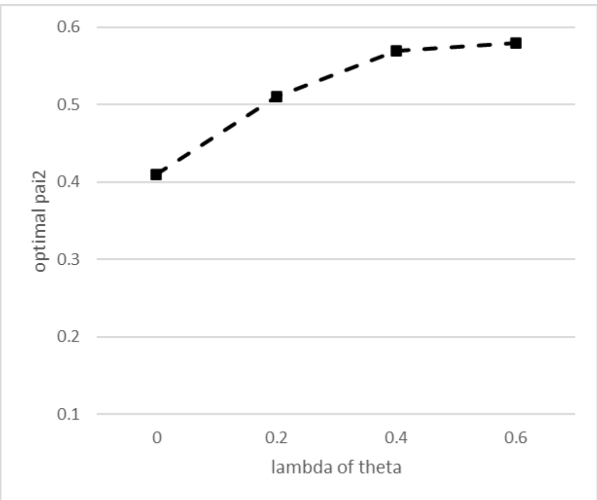


Figure 8

Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) for the aggregate society

(a) Optimal penalty rate under future-consumption-control



(b) Optimal penalty rate under all-consumption-control

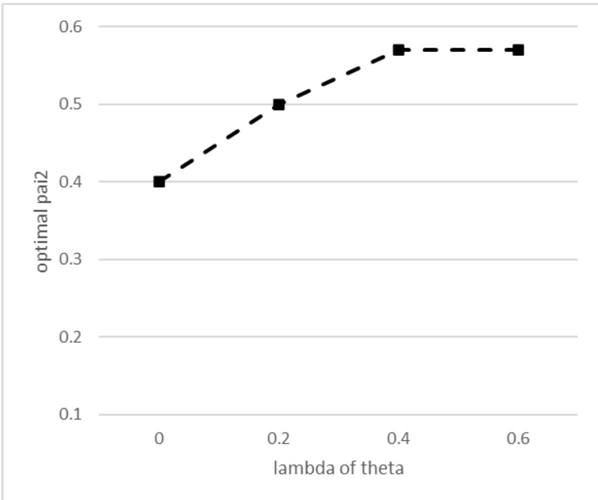
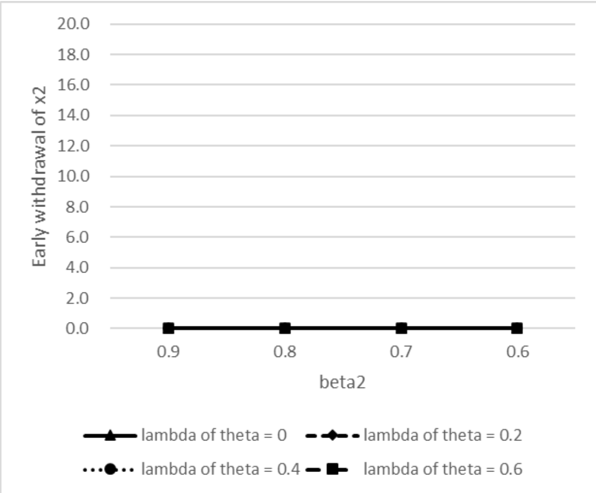


Figure 9

Expected early withdrawal of semi-illiquid savings in period 2 by households with β_2 with the optimal penalty rate for the aggregate society

(a) Under future-consumption-control



(b) Under all-consumption control

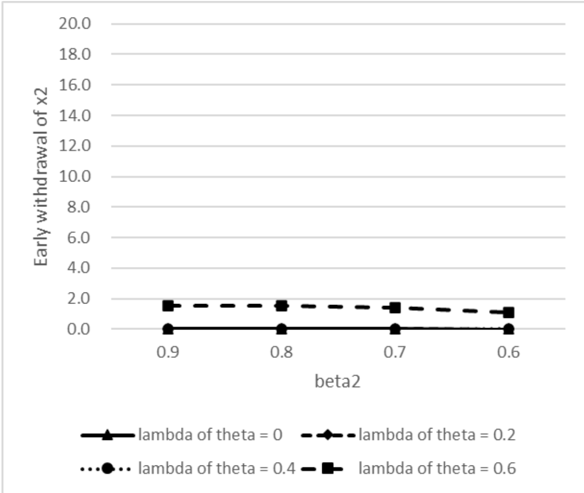
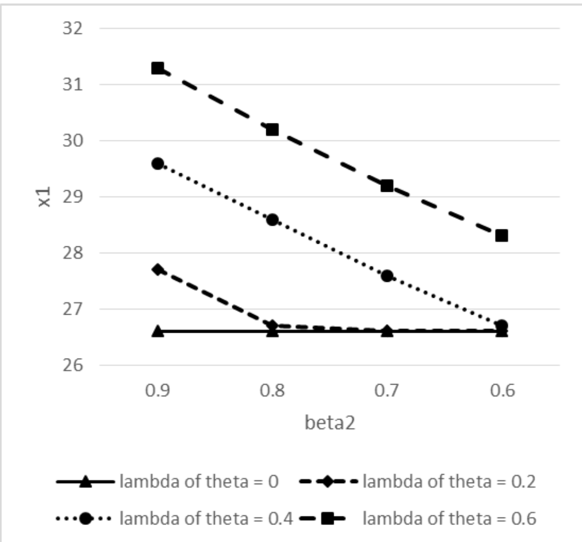


Figure 10

Savings in period 1 by households with β_2 with liquid savings (x_1) and semi-illiquid savings (x_2) with the optimized penalty rate

(a) x_1 under future-consumption-control



(b) x_2 under future-consumption-control

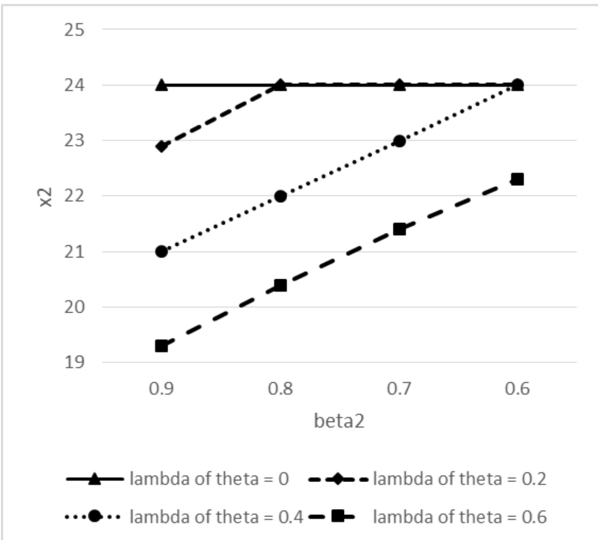
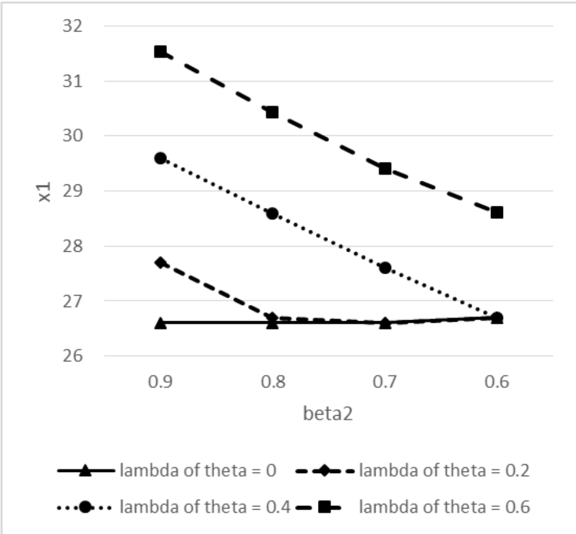


Figure 10 (continued)

(c) x_1 under all-consumption-control



(d) x_2 under all-consumption-control

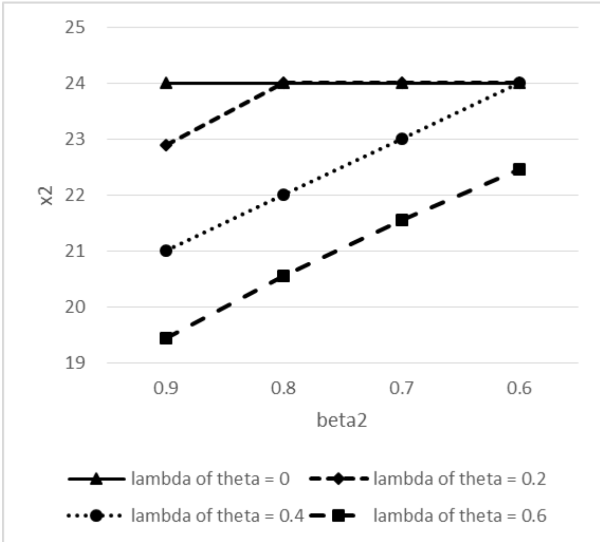


Table 1

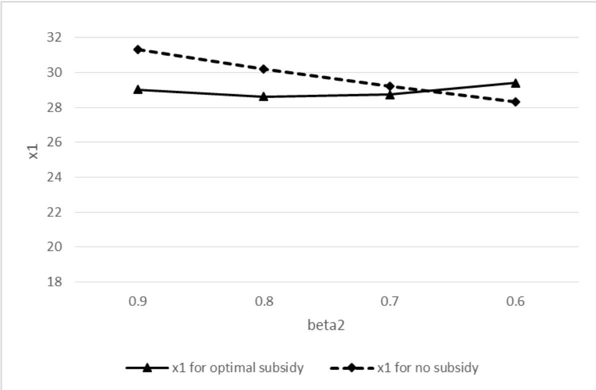
Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) and optimal subsidy rate for the aggregate society ($\lambda_\theta = 0.6$)

Type of optimization	Optimized policy variables	
	Penalty rate on early withdrawal	Subsidy rate on illiquid and/or semi-illiquid savings
Future-consumption-control	0.47	0.07
No subsidy case	0.58	---
All-consumption-control	0.46	0.30
No subsidy case	0.57	---

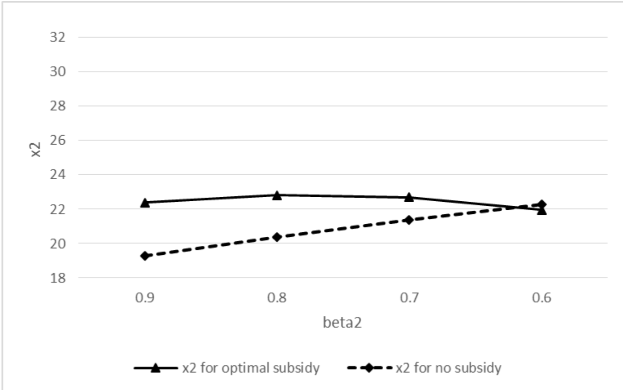
Figure 11

Savings in period 1 by households with β_2 with the optimal penalty and subsidy rates for the aggregate society and under no subsidy (λ_θ = 0.6)

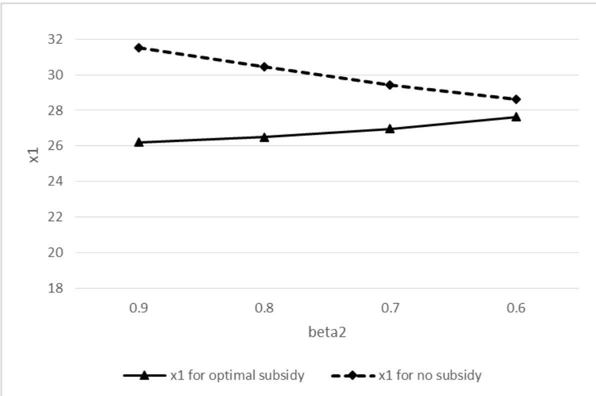
(a) Liquid savings (x_1)
under future-consumption controlling



(b) Semi-illiquid savings (x_2)
under future-consumption-control



(c) Liquid savings (x_1)
under all-consumption control



(d) Semi-illiquid savings (x_2)
under all-consumption control

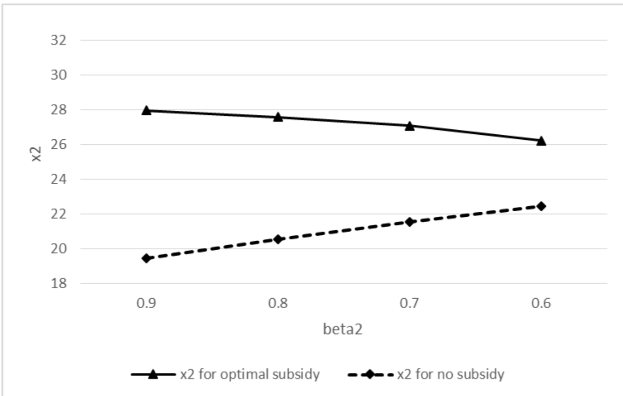


Figure 12

Welfare improvements of households with β_2 with optimal penalty and subsidy rates for the aggregate society and without subsidy (λ_θ = 0.6)

(a) Welfare improvement under future-consumption-control (b) Welfare improvement under all-consumption-control

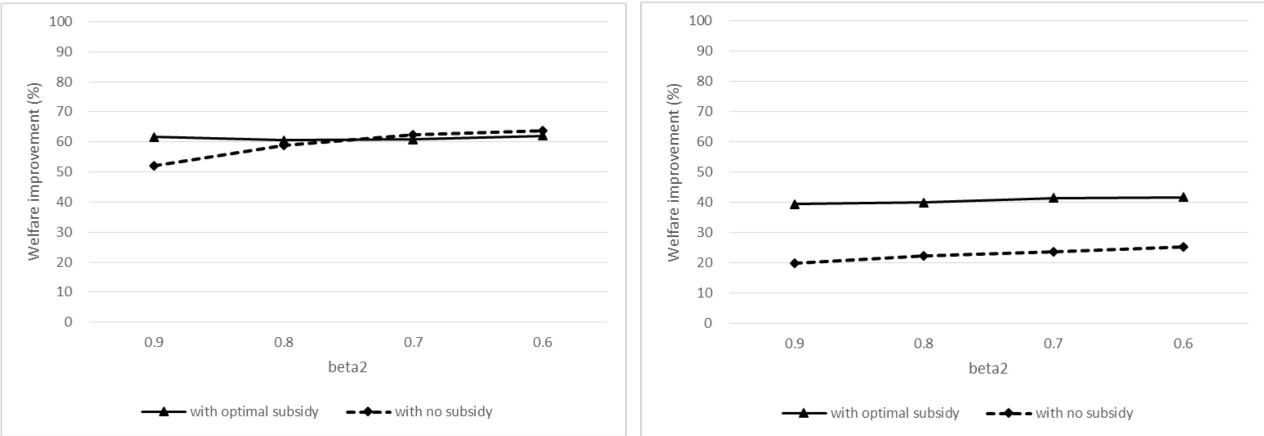


Figure 13

Welfare improvements of households with β_2 with the optimal penalty and subsidy rates for the aggregate society (λ_θ = 0.6): Expected with a cognitive bias and realized actually

(a) Welfare improvement under future-consumption-control (b) Welfare improvement under all-consumption-control

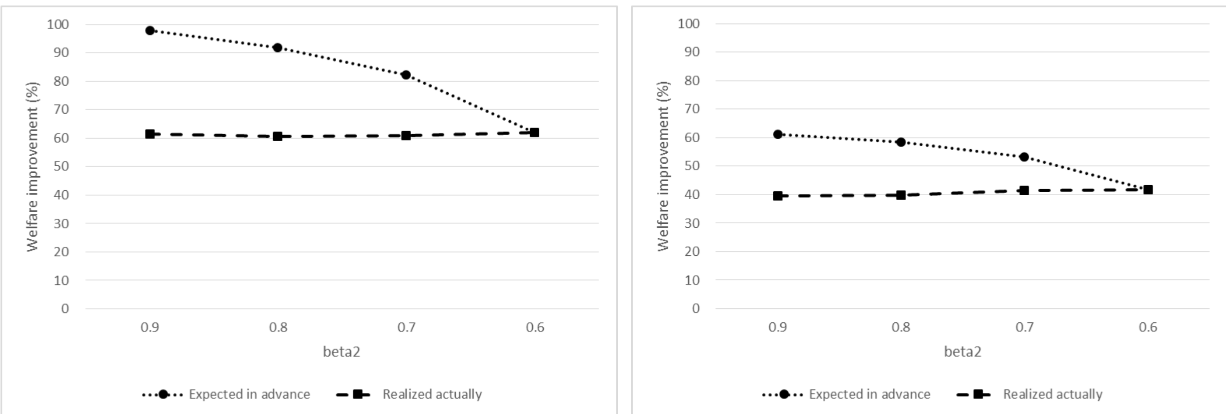
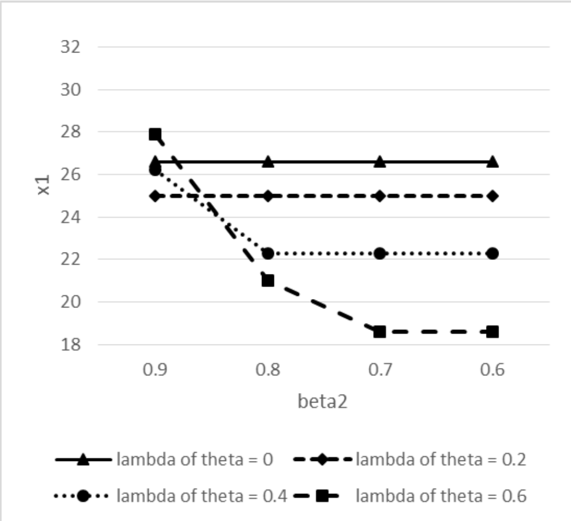


Figure 14

Savings in period 1 by households with β_2 and with strong mental accounting ($\beta_3 = 1.0$) on semi-illiquid savings: with the optimal penalty rate

(a) Liquid savings (x_1)



(b) Semi-illiquid savings (x_2)

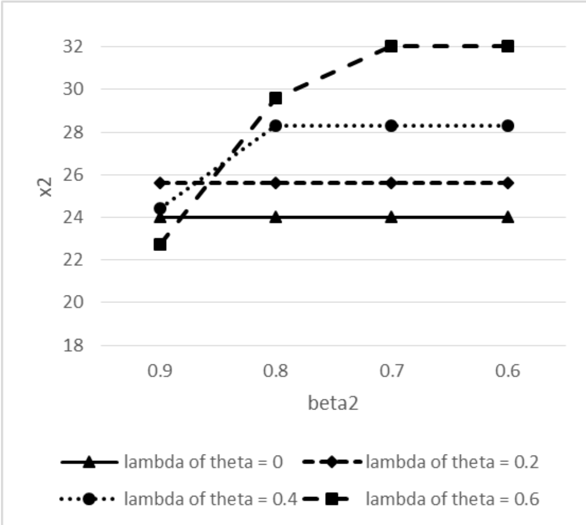


Figure 15

Expected early withdrawal of semi-illiquid savings in period 2 by households with β_2 and with strong mental accounting ($\beta_3 = 1.0$): with the optimal penalty rate

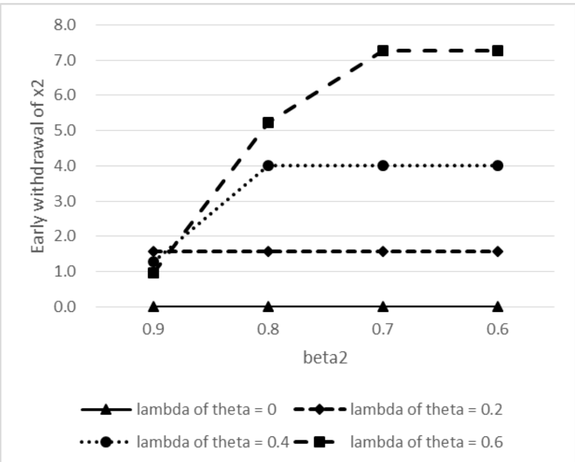
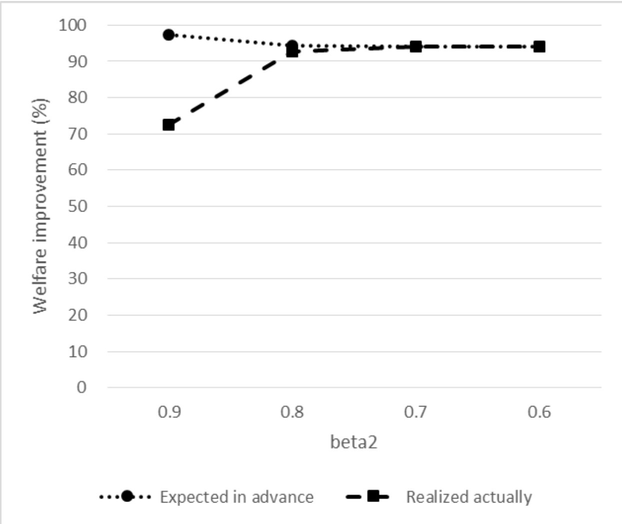


Figure 16

Welfare improvements of households with β_2 and with strong mental accounting ($\beta_3 =$ 1.0) on semi-illiquid savings: Expected with a cognitive bias and realized actually

(a) Welfare improvement under future-consumption-control (λ_θ = 0.6)



(b) Welfare improvement under all-consumption-control (λ_θ = 0.6)

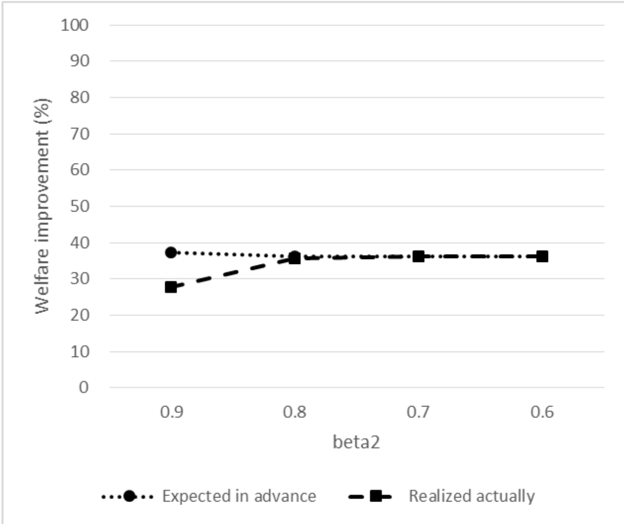


Table 2

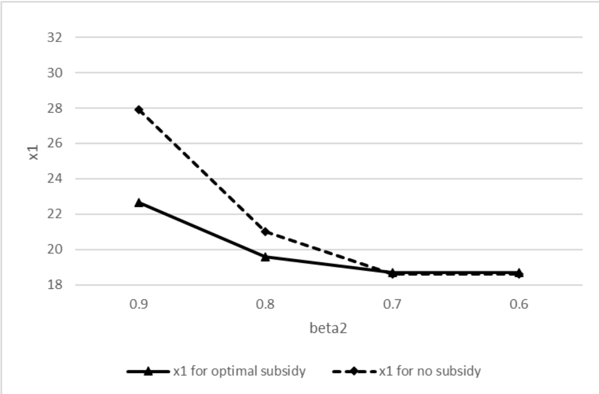
Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) and optimal subsidy rate for the aggregate society (λ_θ = 0.6) with strong mental accounting ($\beta_3 =$ 1.0)

Type of optimization	Optimized policy variables	
	Penalty rate on early withdrawal	Subsidy rate on illiquid and/or semi-illiquid savings
Future-consumption-control	0.04	0.03
No subsidy case	0.01	---
All-consumption-control	0.13	0.12
No subsidy case	0.01	---

Figure 17

Savings in period 1 by households with β_2 and with strong mental accounting ($\beta_3 = 1.0$) on semi-illiquid savings ($\lambda_\theta = 0.6$): with the optimal subsidy and penalty rates and under future-consumption-control strategy

(a) Liquid savings (x_1)



(b) Semi-illiquid savings (x_2)

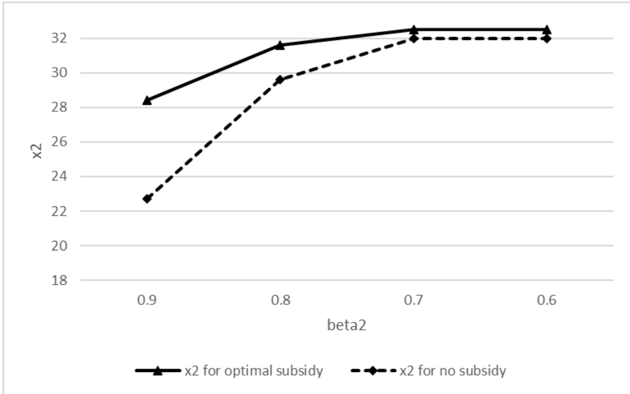
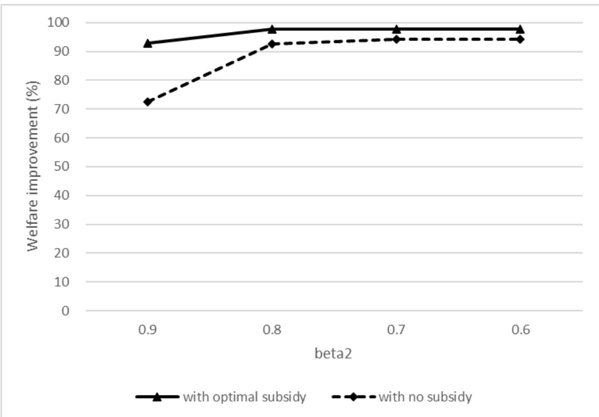


Figure 18

Welfare improvements of households with β_2 and with strong mental accounting ($\beta_3 = 1.0$) on semi-illiquid savings with the optimal subsidy and penalty rates

(a) Welfare improvement
under future-consumption-control
($\lambda_\theta = 0.6$)



(b) Welfare improvement
under all-consumption-control
($\lambda_\theta = 0.6$)

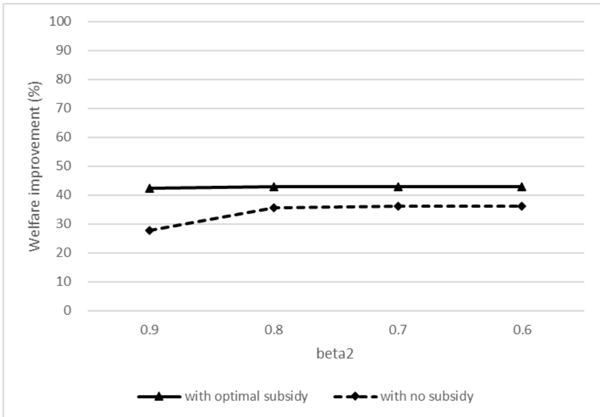
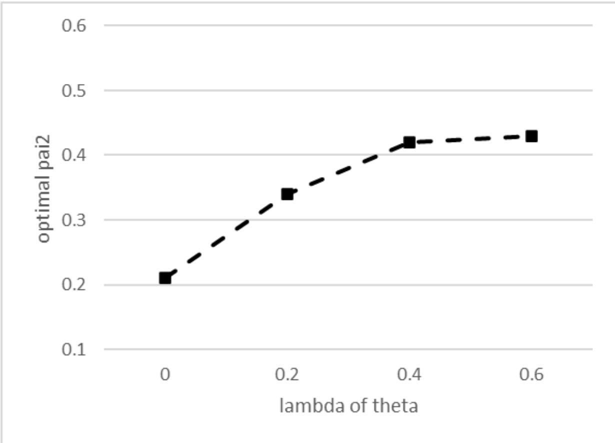


Figure 19

Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) for the aggregate society with moderate mental accounting ($\beta_3 = 0.8$)

(a) Optimal penalty rate under future-consumption-control



(b) Optimal penalty rate under all-consumption-control

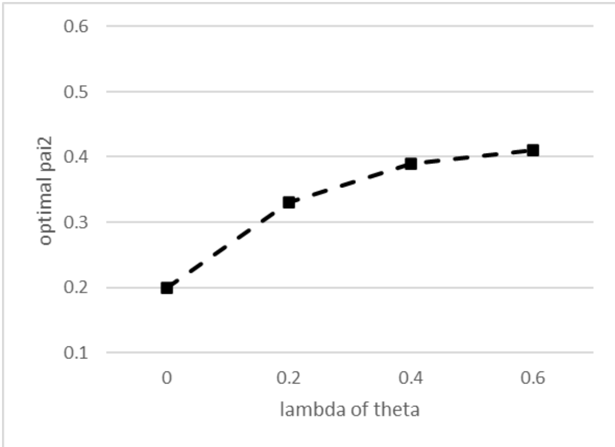
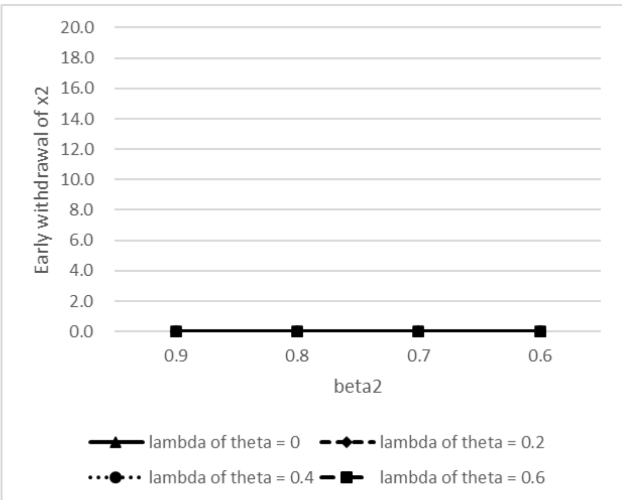


Figure 20

Expected early withdrawal of semi-illiquid savings in period 2 by households with β_2 and with moderate mental accounting ($\beta_3 = 0.8$) with the optimal penalty rate for the aggregate society

(a) Expected early withdrawal under future-consumption-control



(b) Expected early withdrawal under all-consumption-control

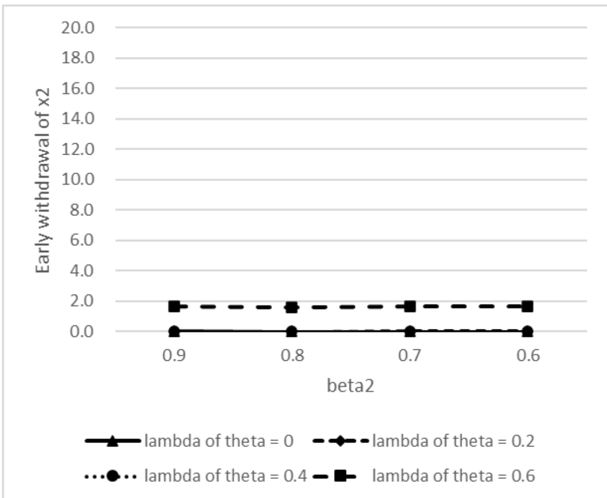
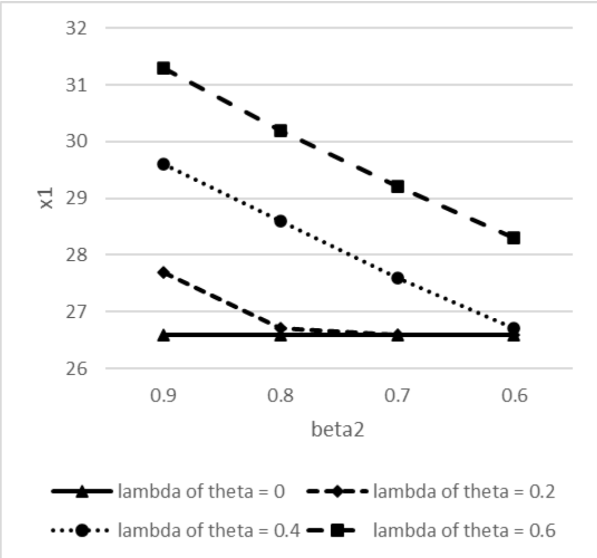


Figure 21

Savings in period 1 by households with β_2 and with moderate mental accounting (β_3 = 0.8) on semi-illiquid savings: with the optimal penalty rate under future-consumption-control strategy

(a) Liquid savings (x_1)



(b) Semi-illiquid savings (x_2)

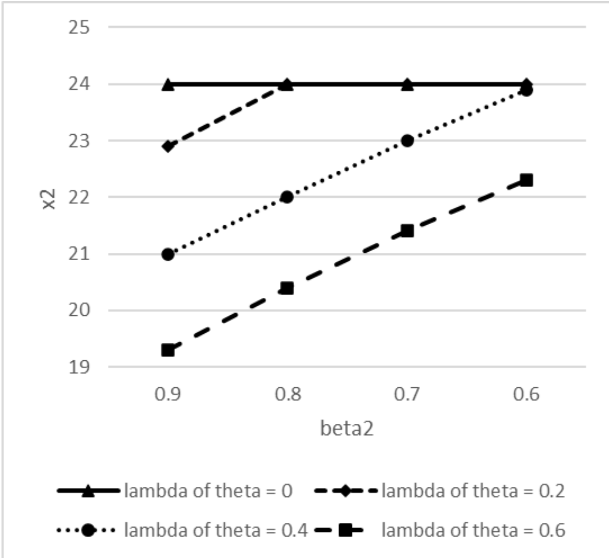


Table 3

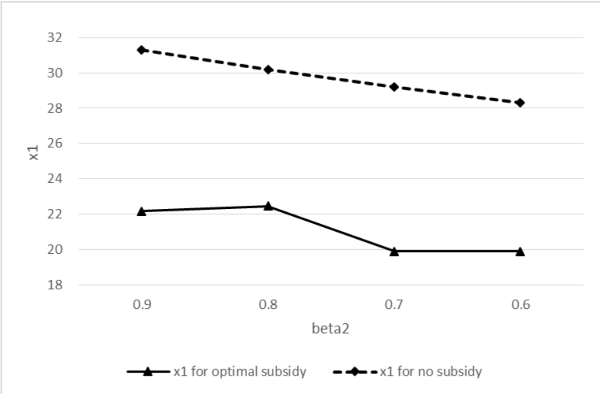
Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) and optimal subsidy rate for the aggregate society (λ_θ = 0.6) with moderate mental accounting (β_3 = 0.8)

Type of optimization	Optimized policy variables	
	Penalty rate on early withdrawal	Subsidy rate on illiquid and/or semi-illiquid savings
Future-consumption-control	0.08	0.07
No subsidy case	0.43	---
All-consumption-control	0.22	0.21
No subsidy case	0.41	---

Figure 22

Savings in period 1 by households with β_2 and with moderate mental accounting ($\beta_3 = 0.8$) on semi-illiquid savings ($\lambda_\theta = 0.6$): with the optimal subsidy and penalty rates and under future-consumption-control strategy

(a) Liquid savings (x_1)



(b) Semi-illiquid savings (x_2)

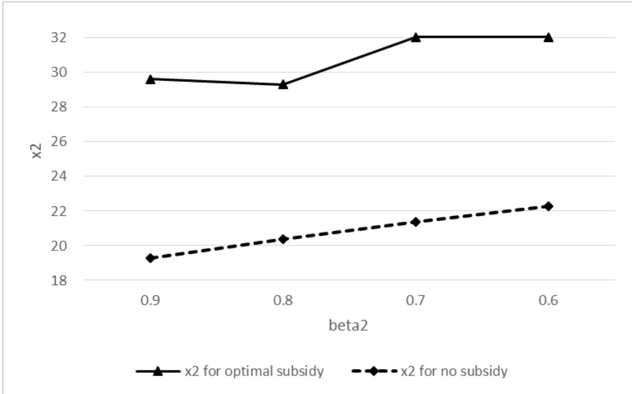
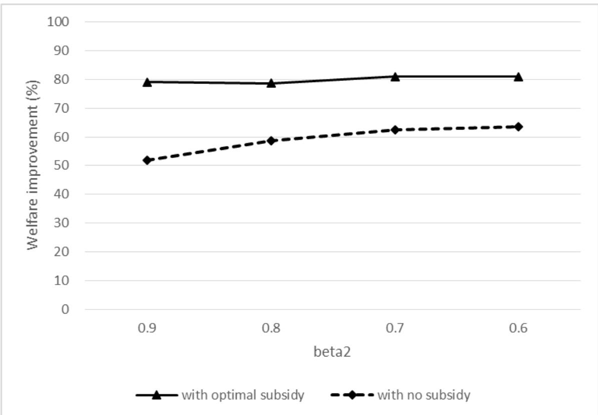


Figure 23

Welfare improvements of households with β_2 and with moderate mental accounting ($\beta_3 = 0.8$) on semi-illiquid savings under the optimal subsidy and penalty rates

(a) Welfare improvement under future-consumption-control ($\lambda_\theta = 0.6$)



(b) Welfare improvement under all-consumption-control ($\lambda_\theta = 0.6$)

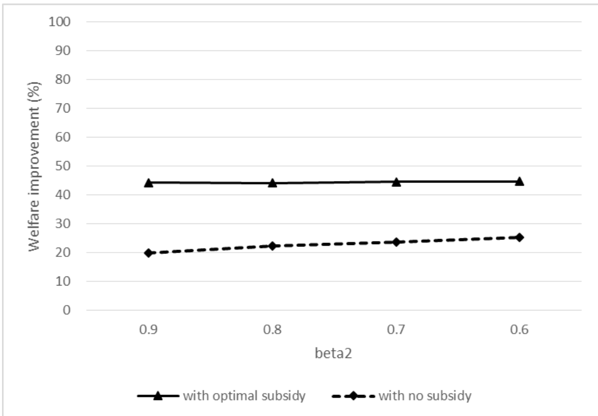


Table 4

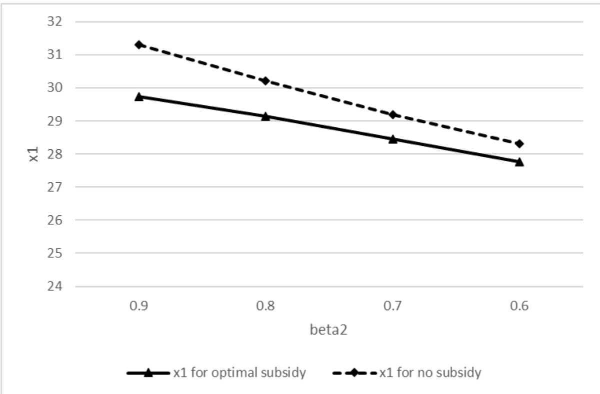
Optimal penalty rate on early withdrawal from semi-illiquid savings and the optimal subsidy rate for semi-illiquid and illiquid savings (λ_θ = 0.6)

Type of optimization	Optimized policy variables	
	Penalty rate on early withdrawal	Subsidy rate on illiquid and/or semi-illiquid savings
Future-consumption-control	0.20 over	0.04
No subsidy case	Indeterminate	---
All-consumption-control	0.22	0.21
No subsidy case	Indeterminate	---

Figure 24

Savings at period 1 by households with β_2 with the optimal penalty and subsidy rates and without subsidy (λ_θ = 0.6)

(a) Liquid savings (x_1)
under future-consumption-control



(b) Semi-illiquid savings (x_2)
under future-consumption-control

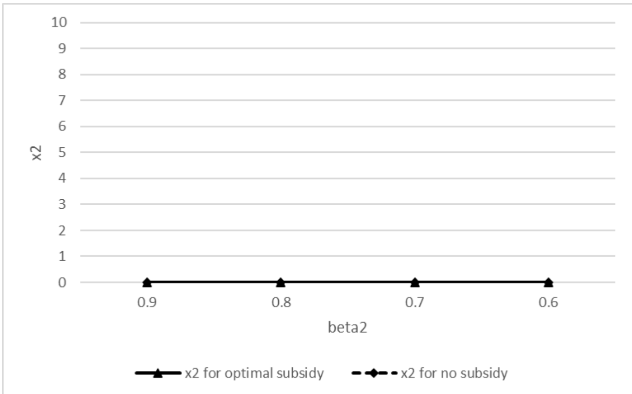
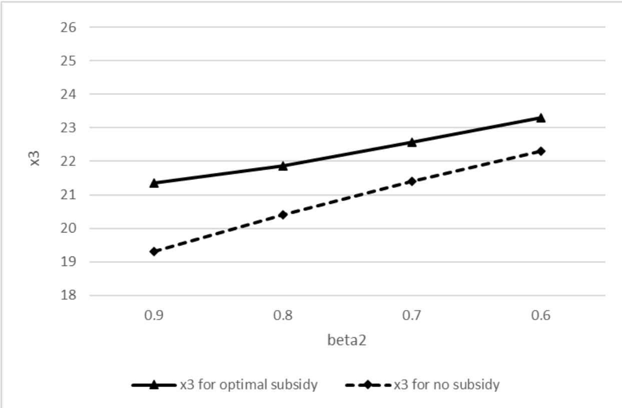
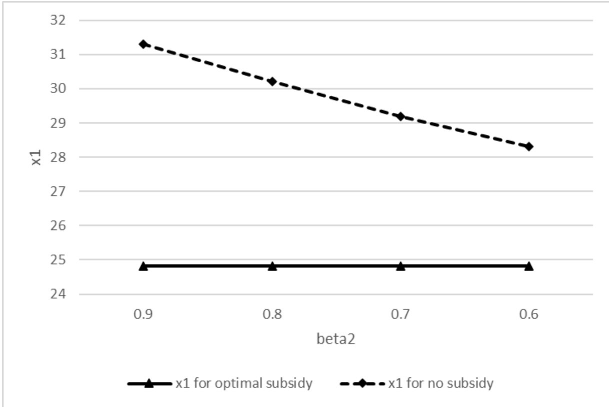


Figure 24 (continued)

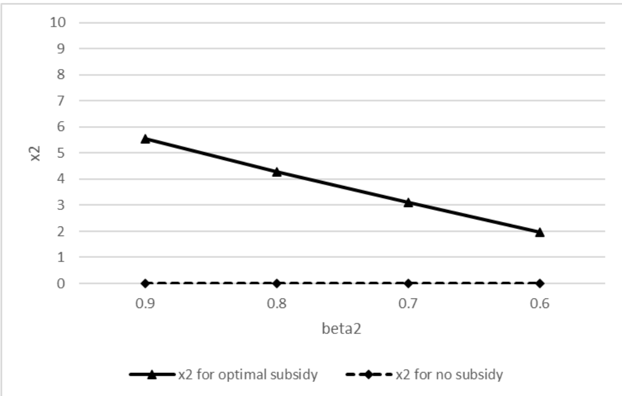
(c) Illiquid savings (x_3)
under future-consumption-control



(d) Liquid savings (x_1)
under all-consumption-control



(e) Semi-illiquid savings (x_2)
under all-consumption-control



(f) Illiquid savings (x_3)
under all-consumption-control

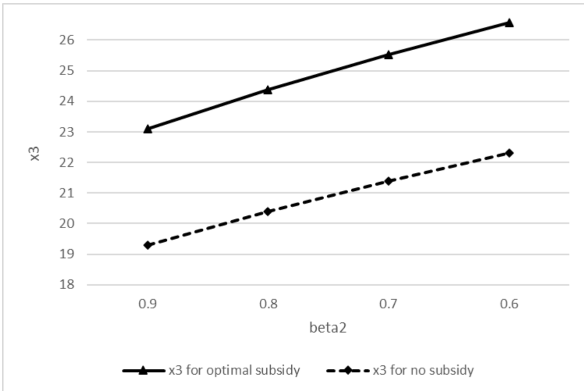


Figure 25

Welfare improvements of households with β_2 with the optimal penalty and subsidy rates and without subsidy ($\lambda_\theta = 0.6$)

(a) Welfare improvement under future-consumption-control

(b) Welfare improvement under all-consumption-control

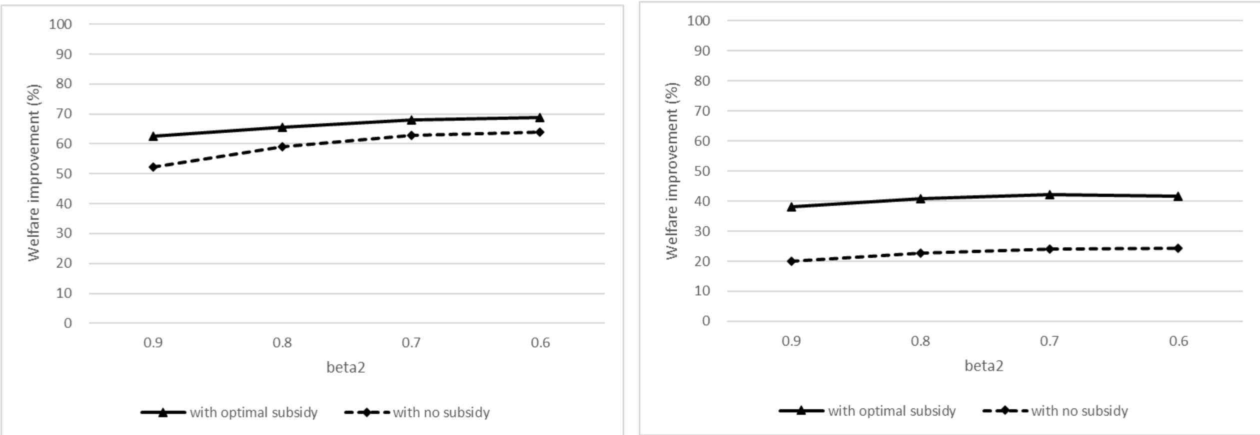


Figure 26

Savings in period 1 by households with β_2 and with strong mental accounting (β_3 = 1.0) on semi-illiquid savings: with the optimal penalty rate

(a) Liquid savings (x_1)

(b) Semi-illiquid savings (x_2)

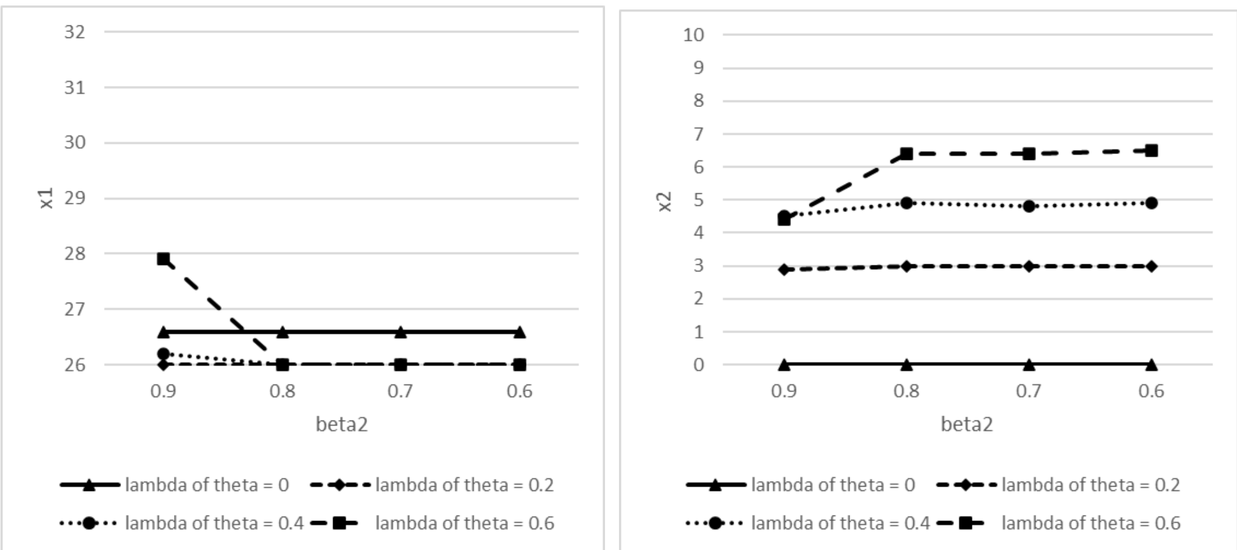


Figure 26 (continued)

(c) Illiquid savings (x_3)

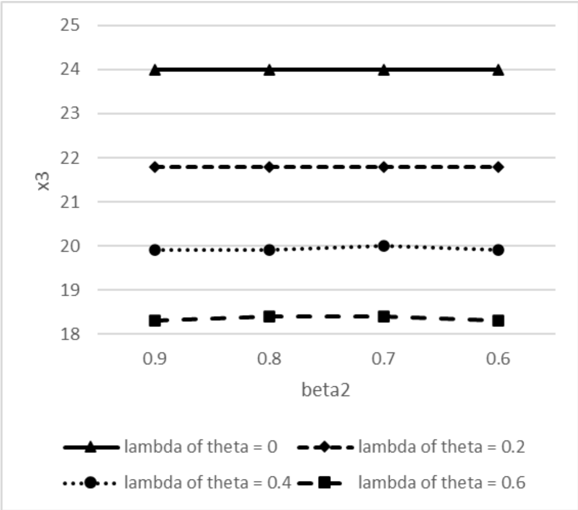


Figure 27

Expected early withdrawal of semi-illiquid savings in period 2 by households with β_2 and with strong mental accounting ($\beta_3 = 1.0$): with the optimal penalty rate

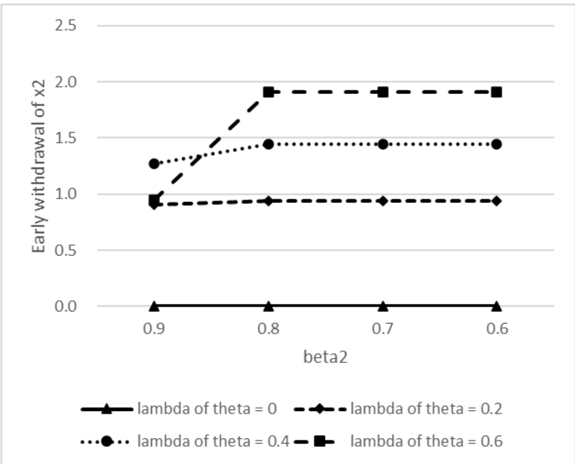
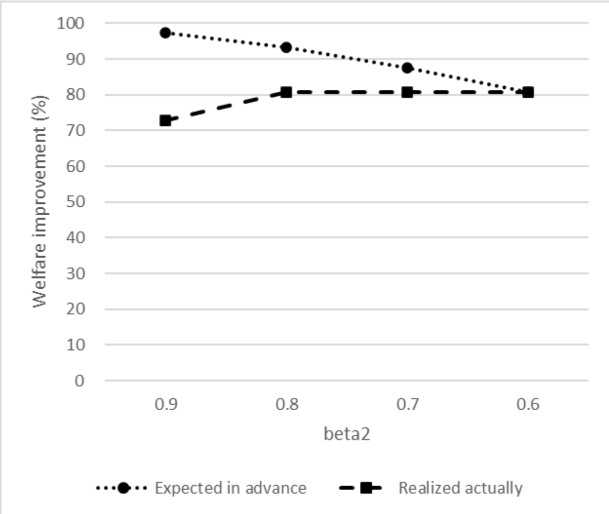


Figure 28

Welfare improvements of households with β_2 and with strong mental accounting ($\beta_3 =$ 1.0) on semi-illiquid savings: Expected with a cognitive bias and realized actually

(a) Welfare improvement under future-consumption-control (λ_θ = 0.6)



(b) Welfare improvement under all-consumption-control (λ_θ = 0.6)

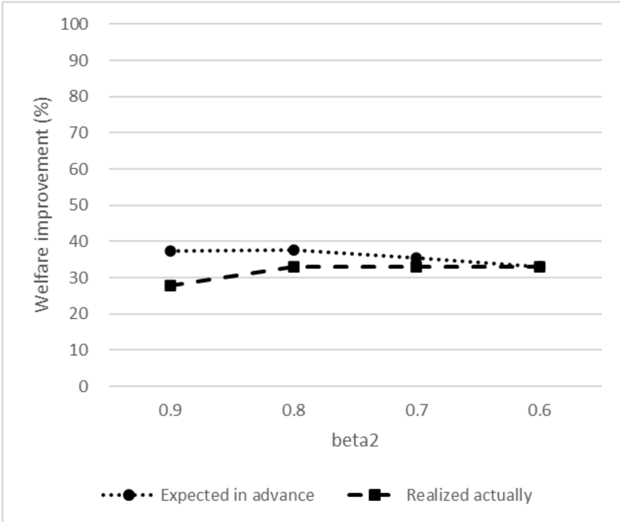


Table 5

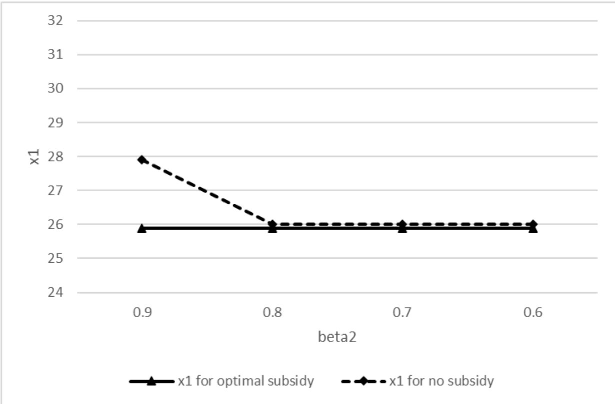
Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) and optimal subsidy rate for the aggregate society (λ_θ = 0.6) with strong mental accounting ($\beta_3 =$ 1.0)

Type of optimization	Optimized policy variables	
	Penalty rate on early withdrawal	Subsidy rate on illiquid and/or semi-illiquid savings
Future-consumption-control	0.03	0.02
No subsidy case	0.01	---
All-consumption-control	0.18	0.17
No subsidy case	0.01	---

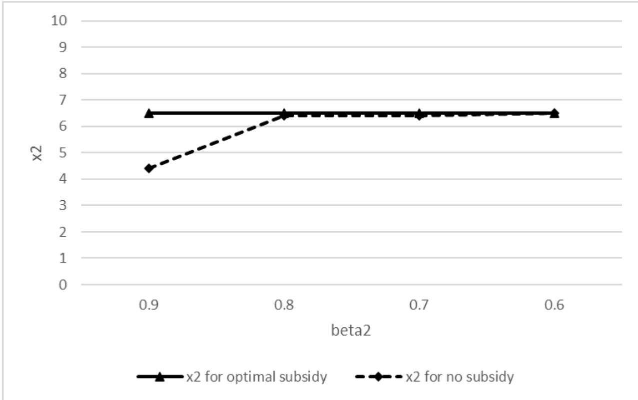
Figure 29

Savings in period 1 by households with β_2 and with strong mental accounting ($\beta_3 = 1.0$) on semi-illiquid savings ($\lambda_\theta = 0.6$): with the optimal subsidy and penalty rates

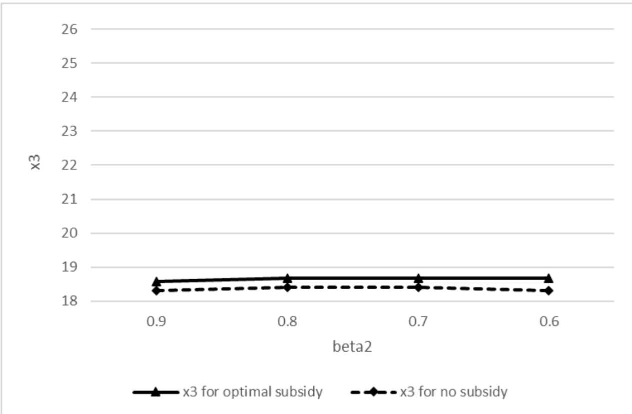
(a) x_1 under future-consumption-control



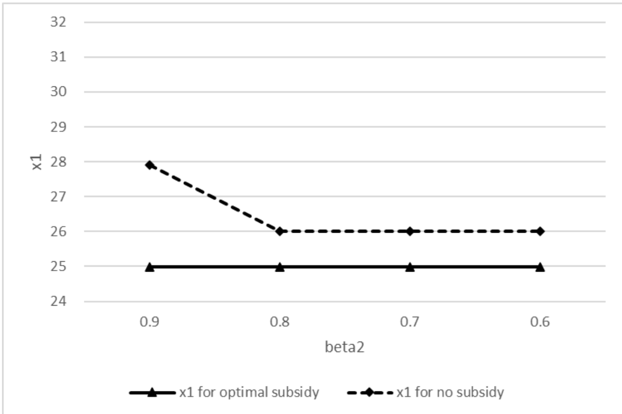
(b) x_2 under future-consumption-control



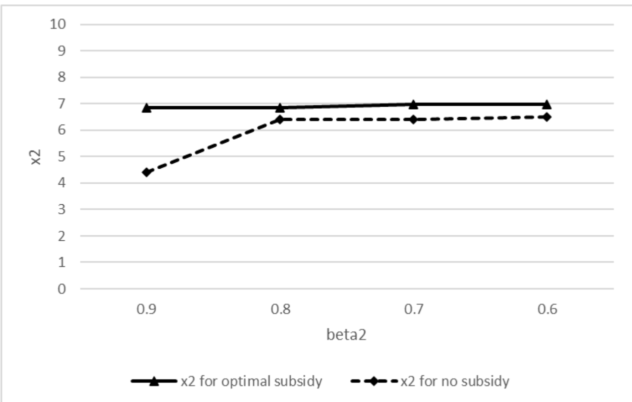
(c) x_3 under future-consumption-control



(d) x_1 under all-consumption-control



(e) x_2 under all-consumption-control



(f) x_3 under all-consumption-control

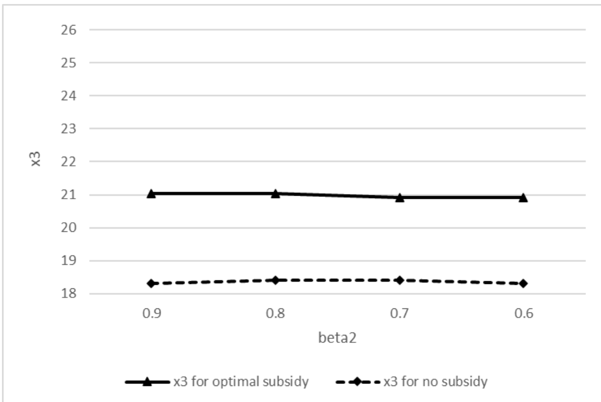


Figure 30

Welfare improvements of households with β_2 and with strong mental accounting ($\beta_3 = 1.0$) on semi-illiquid savings under the optimal subsidy and penalty rates

(a) Welfare improvement under future-consumption-control ($\lambda_\theta = 0.6$)

(b) Welfare improvement under all-consumption-control ($\lambda_\theta = 0.6$)

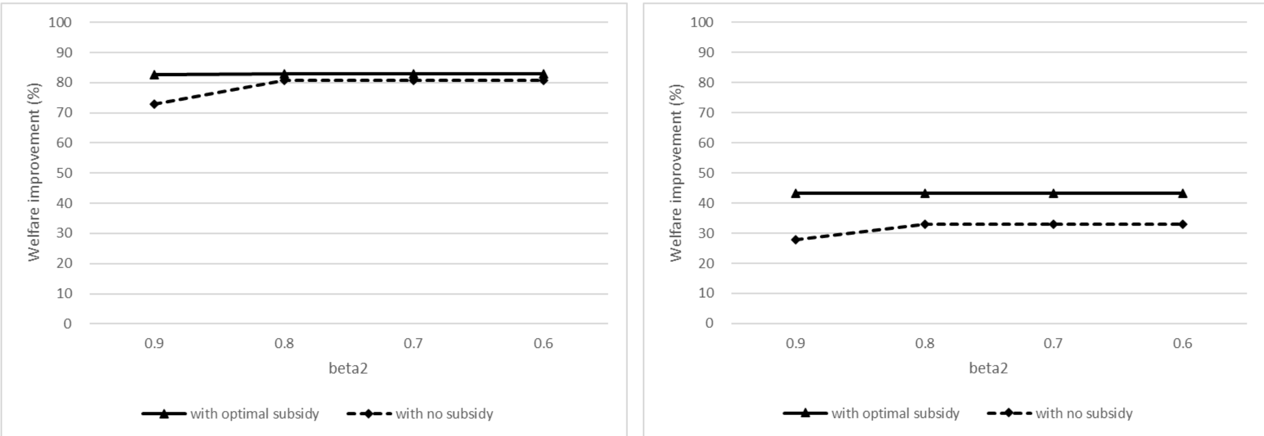


Figure 31

Savings in period 1 by households with β_2 and with moderate mental accounting ($\beta_3 = 0.8$) on semi-illiquid savings: with the optimal penalty rate

(a) x_1 under future-consumption-control

(b) x_2 under future-consumption-control

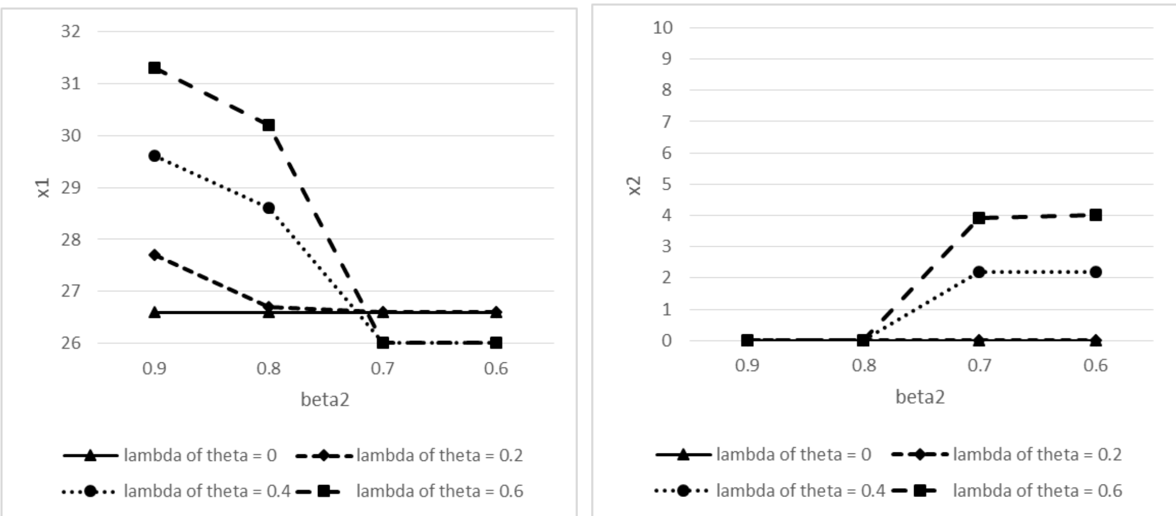
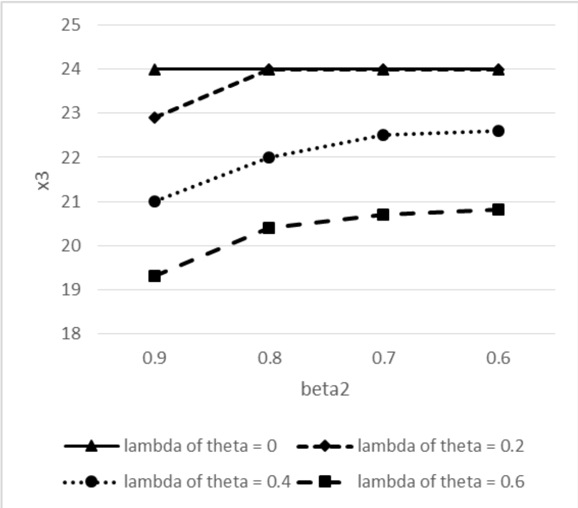
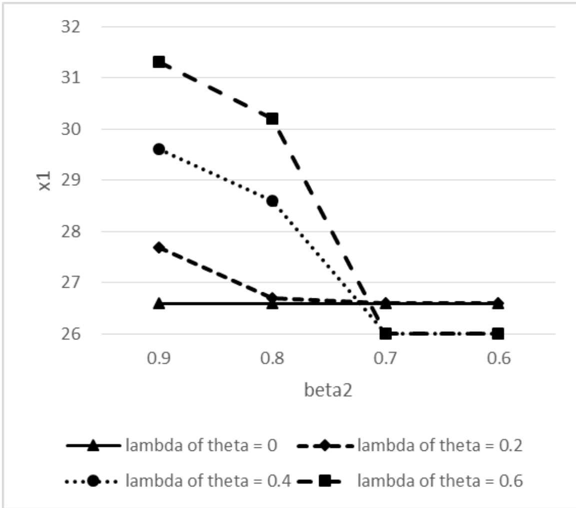


Figure 31 (continued)

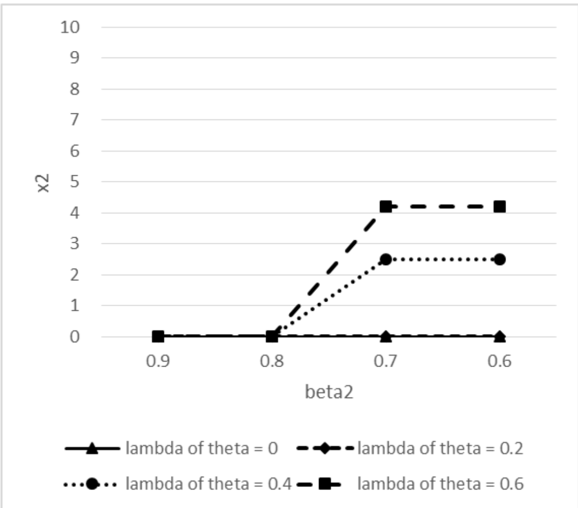
(c) x_3 under future-consumption-control



(d) x_1 under all-consumption-control



(e) x_2 under all-consumption-control



(f) x_3 under all-consumption-control

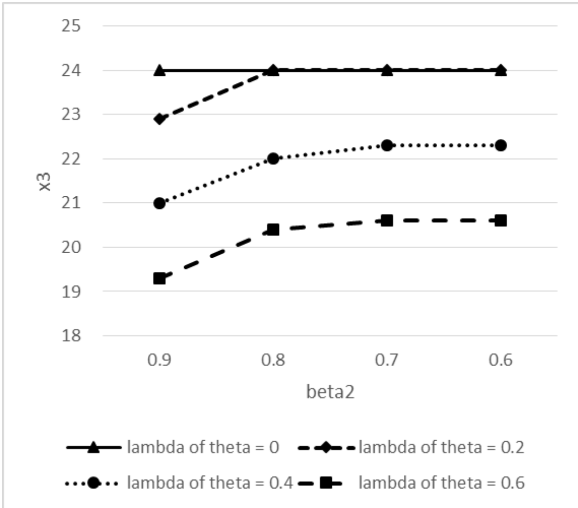
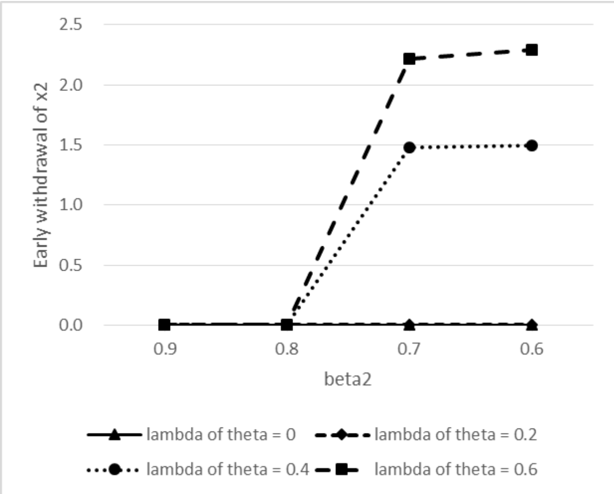


Figure 32

Expected early withdrawal of semi-illiquid savings in period 2 by households with β_2 and with moderate mental accounting ($\beta_3 = 0.8$) under the optimal penalty rate for the aggregate society

(a) Expected early withdrawal under future-consumption-control



(b) Expected early withdrawal under all-consumption-control

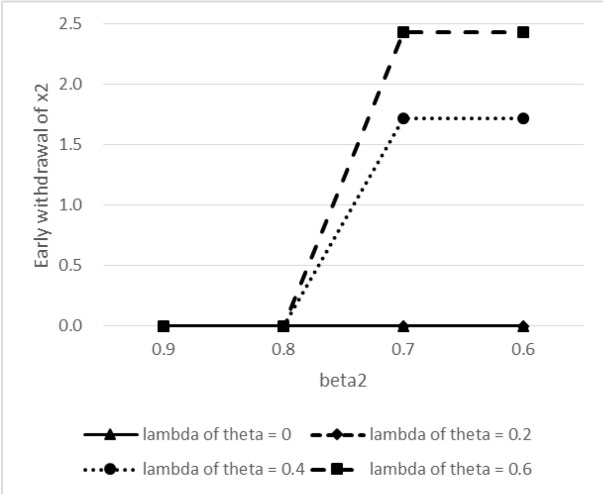
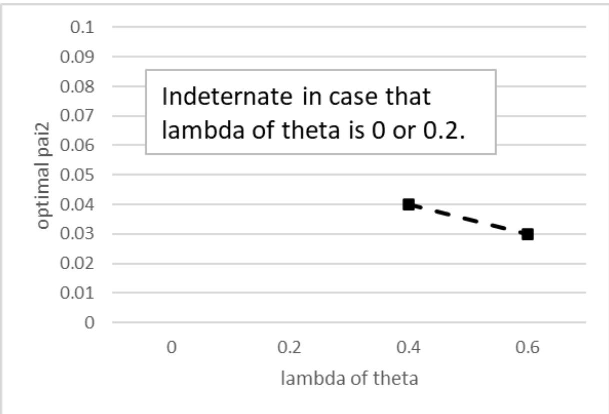


Figure 33

Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) for the aggregate society with moderate mental accounting ($\beta_3 = 0.8$)

(a) Optimal penalty rate under future-consumption-control



(b) Optimal penalty rate under all-consumption-control

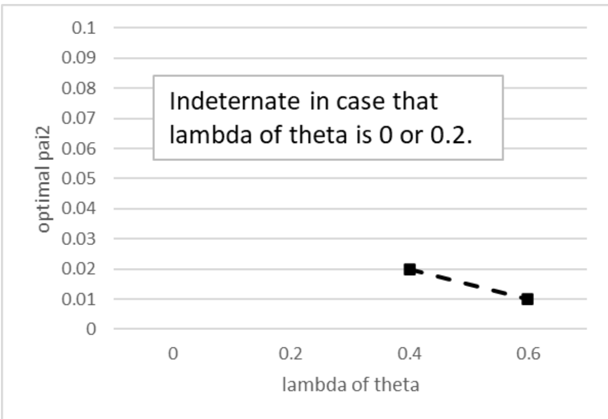


Figure 34

Welfare improvements of households with β_2 and with moderate mental accounting ($\beta_3 = 0.8$) on semi-illiquid savings under the optimal penalty rate

(a) Welfare improvement under future-consumption-control ($\lambda_\theta = 0.6$)

(b) Welfare improvement under all-consumption-control ($\lambda_\theta = 0.6$)

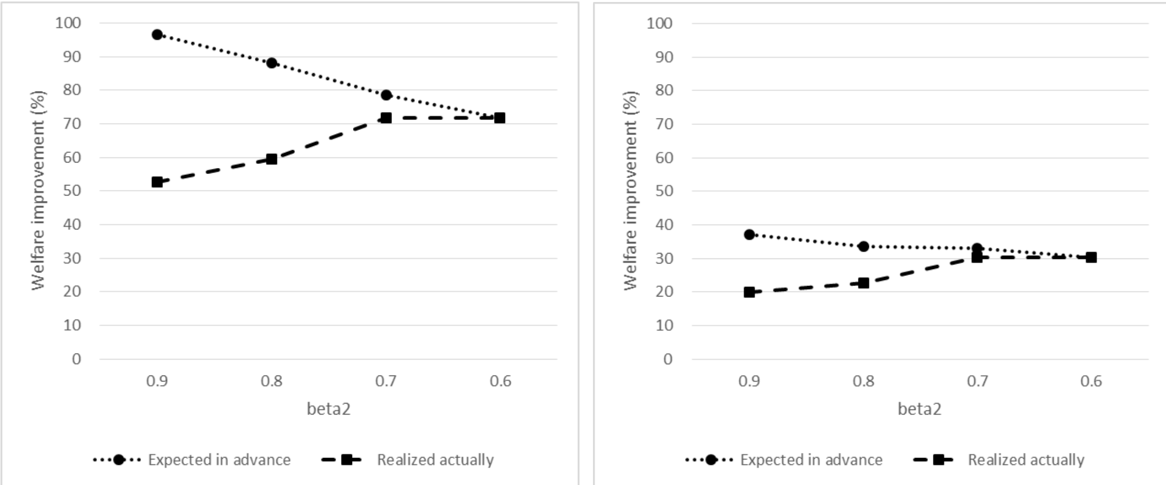


Table 6

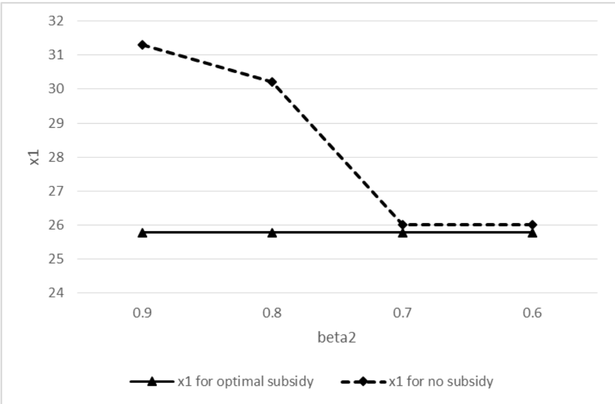
Optimal penalty rate on early withdrawal from semi-illiquid savings (x_2) and optimal subsidy rate for the aggregate society ($\lambda_\theta = 0.6$) with moderate mental accounting ($\beta_3 = 0.8$)

Type of optimization	Optimized policy variables	
	Penalty rate on early withdrawal	Subsidy rate on illiquid and/or semi-illiquid savings
Future-consumption-control	0.05	0.04
No subsidy case	0.03	---
All-consumption-control	0.21	0.20
No subsidy case	0.01	---

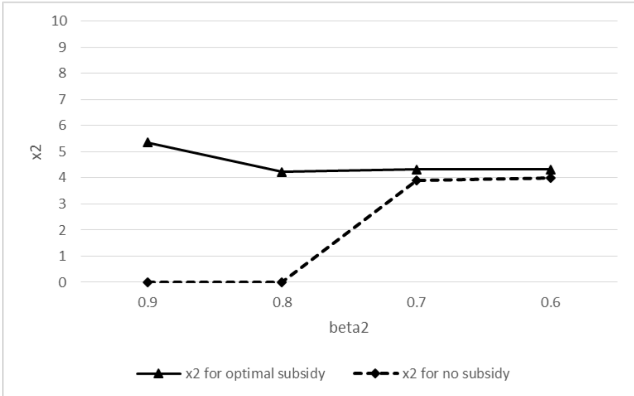
Figure 35

Savings in period 1 by households with β_2 and with moderate mental accounting (β_3 = 0.8) on semi-illiquid savings (λ_θ = 0.6): with the optimal subsidy and penalty rates

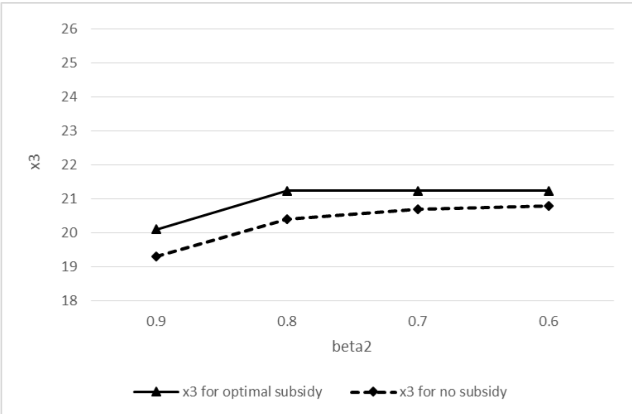
(a) x_1 under future-consumption-control



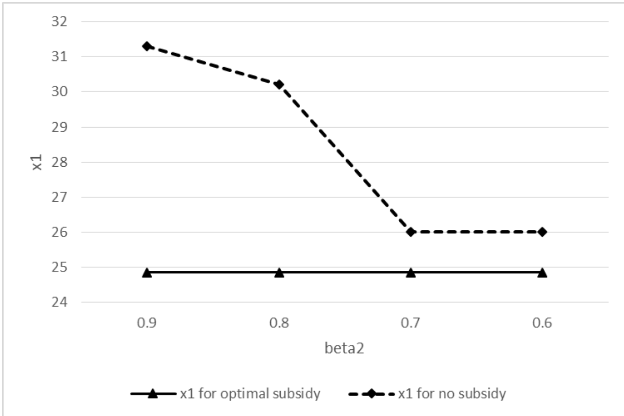
(b) x_2 under future-consumption-control



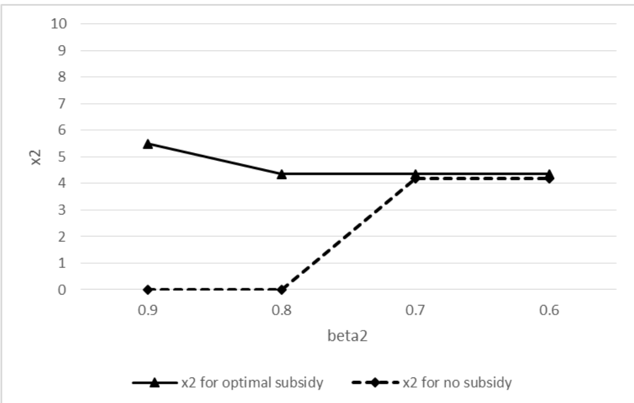
(c) x_3 under future-consumption-control



(d) x_1 under all-consumption-control



(e) x_2 under all-consumption-control



(f) x_3 under all-consumption-control

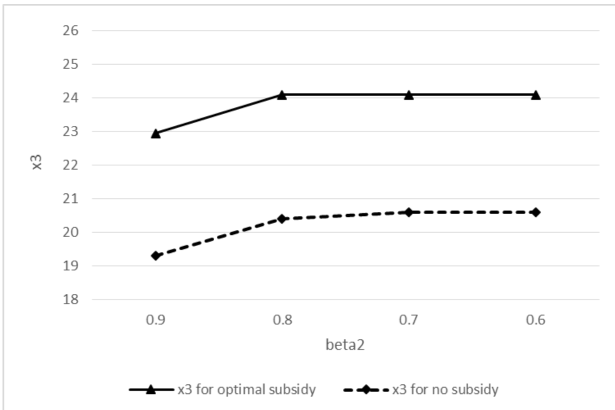
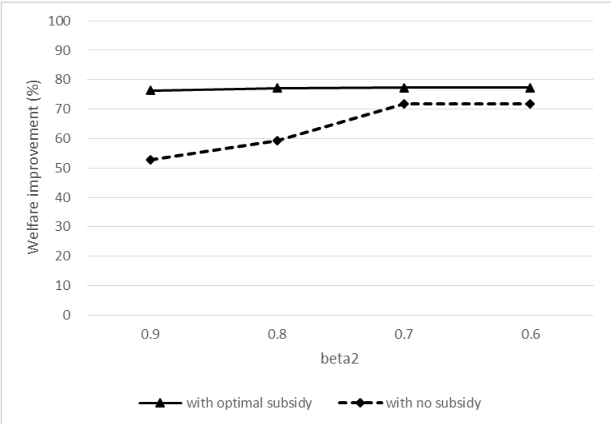


Figure 36

Welfare improvements of households with β_2 and with moderate mental accounting ($\beta_3 = 0.8$) on semi-illiquid savings under the optimal subsidy and penalty rates

(a) Welfare improvement under future-consumption-control ($\lambda_\theta = 0.6$)



(b) Welfare improvement under all-consumption-control ($\lambda_\theta = 0.6$)