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Abstract

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POVERTY ORDERINGS WITH ORDINAL AND CATEGORICAL DATA.

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ABSTRACT

Most of the data available for measuring capabilities or dimensions of poverty is either ordinal or categorical. However, the majority of the indices introduced for the assessment of multidimensional poverty behave well only with cardinal variables. In a recent paper, Alkire and Foster (2008) propose a new methodology to measure multidimensional poverty that includes an identification method and a class of poverty measures. The identification step and the first of their measures, the *dimension adjusted headcount ratio,* are based on a counting approach and are well suited for use with ordinal and categorical data. The implementation of this methodology involves the choice of a minimum number of deprivations required in order to be identified as poor. This cut-off adds arbitrariness to poverty comparisons

In this paper we explore dominance conditions that guarantee unanimous poverty rankings in a counting framework. Our conditions are based on a simple graphical device that provides a tool for checking the robustness of poverty rankings to changes in the identification cut-off, and also for checking unanimous orderings in a wide set of multidimensional poverty indices that suit ordinal and categorical data.

Key words: Multidimensional Poverty Measurement, counting approach, dimension adjusted headcount ratio, ordinal data

JEL Classification: I30, I32, D63

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INTRODUCTION

In recent years there has been considerable agreement that poverty is a multidimensional phenomenon and great efforts have been made from both a theoretical and an empirical point of view, trying to assess multidimensional poverty.[2] However, since most of the data available to measure capabilities or dimensions of poverty is either ordinal or categorical, only indices that behave well with this sort of variable should be used in empirical applications.

The counting approach introduced by Atkinson (2003) takes into consideration the number of dimensions in which each person is deprived, and is an appropriate procedure that deals well with ordinal and categorical variables. Based on this framework there are two recent contributions.

On the one hand, Alkire and Foster (2008) propose a new framework for measuring multidimensional poverty that includes an identification procedure and a way of aggregation. The identification step extends the traditional union and intersection approaches and incorporates two cut-offs. The first has to do with the traditional identification of the poor within each dimension using a dimension-specific poverty line. In the second step, a counting approach is used to identify the poor people using a threshold of the number of dimensions in which a person should be deprived in order to be identified as multidimensional poor.[3]

As regards the aggregation step, they propose the Foster-Greer-Thobercke measures (Foster et al. (1984)) appropriately adjusted to the identification procedure. Specifically, the first of their measures, the *adjusted headcount ratio*, defined as the average of the number of deprivations suffered by the poor, is also based on a counting approach. In addition, it fulfils a number of desirable properties; among them the suitability for working with ordinal data and the dimensional monotonicity, which means that it will increase if a person already identified as poor becomes deprived in an additional dimension.

The second contribution based on a counting approach is Bossert et al. (2009) that characterizes a class of counting measures and generalizes Alkire and Foster indices.

In general, the choice of either the identification cut-offs, or the indices, adds arbitrariness to poverty comparisons, and different selections can lead to contradictory results.

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[2] See, among others, UNDP (1997), Chakravarty et al. (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Deutsch and Silber (2005), Alkire and Foster (2008), Chakravarty et al. (2008), Maasoumi and Lugo (2008), Diez et al. (2008), Bossert et al. (2009).

[3] Actually Alkire and Foster (2008) have explicitly formulated and analyzed this identification procedure although similar methods had already been used in the literature, for instance Marck and Lindsay (1985), Gordon et al. (2003).

For this reason it may be of interest to investigate dominance criteria to allow poverty comparisons to be robust to the different choices.[4] There exists a branch of the literature devoted to establishing dominance criteria which provide unanimous orderings when comparisons are made at a variety of poverty thresholds and measures. A comprehensive survey of dominance conditions in the poverty unidimensional field is provided by Zheng (2000). Taking this literature as a starting point, and more specifically the basic papers by Shorrocks (1983), Foster (1985) and Foster and Shorrocks (1988), this work explores ordering conditions of the adjusted headcount ratio for a range of identification cut-offs. We show that if the rankings provided by this index are unanimous over all the admissible identification thresholds, then this rank holds for any counting measure in a wide set of indices. The implementation of these conditions is based on what we call *dimension deprivation curves*, henceforth *DD* curves, a simple device which allows us to represent in the same picture the *headcount ratio*, the *adjusted headcount ratio* and the *average deprivation share* according to Alkire and Foster (2008)'s proposal. Since the Lorenz curve was introduced in the literature, a number of cumulative curves have been widely used to check unanimous orderings in the inequality, poverty, and polarization fields.[5] In this connection, the curves we propose become a very intuitive tool to make robust comparisons when a counting approach is used.

The paper is structured as follows. In the next section we present the notation and basic definitions. Section 2 introduces the DD curves, showing that these curves may be constructed in a similar way to the procedure used in the literature to derive the mentioned cumulative curves. Then, following Foster and Shorrocks (1988), in Section 3 we will show that the *DD* curves become a powerful tool for checking unanimous orderings according to a wide class of counting measures. They also avoid the choice of an arbitrary identification cut-off and offer a useful way to determine the bounds of the number of dimensions for which multidimensional comparisons are robust. As the dimensional headcount ratio behaves particularly well with ordinal and categorical data, the *DD* curves play an important role in making poverty comparisons when data is ordinal. The paper finishes with some concluding remarks.

[4] The robustness of poverty measures as regards the number of deprivations chosen to identify the poor has already been addressed by Batana (2008) and Subramanian (2009). The former proposes to follow the procedure introduced by Davidson and Duclos (2006) and already used by Batana and Duclos (2008) to check the robustness of adjusted headcount ratio. They suggest a statistical dominance test based on the empirical likelihood ratio. The latter author introduces a graphical device to check the robustness of the headcount ratio when different cut-offs are selected.

[5] Among them the TIP curves proposed by Jenkins and Lambert (1997), the polarization curve introduced by Foster and Wolfson (1992) and more recently the proposal of Shorrocks (2009) to derive unemployment indices.

1. NOTATION AND BASIC DEFINITIONS.

We consider a population of $n \geq 2$ individuals endowed with a bundle of $d \geq 2$ attributes considered as relevant to measure poverty. In a counting approach, poverty is measured taking into consideration the number of dimensions in which people are deprived. Thus, we assume that the dimensions are represented by binary variables where a value of 1 means that the individual is deprived in that attribute, and a value of 0 indicates that the individual is not deprived. In this framework it is implicitly assumed that comparing each person's achievements with the respective dimension poverty lines, determines whether the individual is deprived or not in each attribute. Thus, each individual's characteristics are represented by a *deprivation vector* $g_i \in \{0,1\}^d$, whose typical component *j* is defined by $g_{ij} = 1$ when individual *i* is deprived in attribute *j* and $g_{ij} = 0$ otherwise. For simplicity we assume that all the dimensions are equally weighted, although similar conclusions may be derived if different fixed weights are attached to the different dimensions.

Let's denote by $c_i \in \{0,1,..,d\} = C$ the number of dimensions in which person *i* is deprived, that is, $c_i = \sum_{1 \leq j \leq d} g_{ij}$. The vector $c = (c_1,...,c_n) \in C^n$ is referred to as the *vector of deprivation counts*. This vector plays an important role in the poverty measurement when ordinal data are involved. In fact this vector is invariant if the achievement levels and the poverty lines are transformed under the same monotonic transformations, and this is a crucial property when the achievements or capabilities are measured with ordinal variables. We will denote by $\overline{c}$ the permutation of *c* in which the number of deprived dimensions have been arranged in decreasing order, that is, $\overline{c}_i \geq \overline{c}_{i+1}$ for $i = 1,...,n$. Hence people are ranked from the most deprived to the least. Let $G = \bigcup_{n \geq 1} C^n$ be the set of all admissible vectors of deprivation counts.

We will say that the vector *c'* is obtained from the vector *c* by a *permutation* if $\overline{c}' = \overline{c}$; by a *replication* if $c' = (c,c,...,c)$; by an *increment* if $c'_i > c_i$ for some *i* and $c'_j = c_j$ for all $j \neq i$; and by a *deprived dimension (regressive) transfer* if $c'_i > c_i > c_j$, $c'_i + c'_j = c_i + c_j$; $c'_k = c_k$ for all $k \neq i, j$.

Following the framework proposed by Alkire and Foster (2008), a methodology for measuring multidimensional poverty consists of a method to identify the poor and an aggregative measure.

Two main methods have been used in the literature in the identification step, referred to as the 'union' and the 'intersection' approaches respectively. Whereas the union procedure identifies the poor as someone who is deprived in at least one dimension, the intersection definition requires a poor person to be deprived in all dimensions. These methods present well known drawbacks when the number of poverty dimensions is great. Whereas "almost nobody" is identified as poor with the intersection approach, "almost everybody" is poor with the union identification.

There is an intermediate procedure that proposes to identify a person as poor if they are deprived in at least *k* dimensions. According to this procedure, person *i* is identified as poor if $c_i \geq k$, i.e., the number of dimensions in which they are deprived is at least *k*; and person *i* is non-poor otherwise, that is, if $c_i < k$.Clearly, for $k = 1$, this method coincides with the union approach, whereas for $k = d$, it is equivalent to the intersection approach. Following Alkire and Foster (2008), we will use ρ_k to denote this procedure, which will be the identification method throughout this paper.

Let's denote by Q_k and q_k respectively, the set and number of poor identified using the dimension cut-off *k*. For each vector *c* of deprivation counts, we define the *censored vector of deprivation counts*, denoted by $c(k)$, as follows: $c_i(k) = c_i$ if $c_i \geq k$, and $c_i(k) = 0$ if $c_i < k$.

In what follows, a counting measure P is a non-constant function $P : G \times \{1, ..., d\} \to \mathbb{R}$, whose typical image $P_k(c)$ represents the level of poverty in a society where the poor are identified according to ρ_k. We assume that P fulfils the following four properties:

* *Poverty Focus (PF):* P_k remains unchanged if the number of deprived dimensions of a non-poor person decreases.

* *Dimensional Monotonicity (MON)*: $P_k(c) < P_k(c')$ if *c'* is obtained from *c* by an increment of a poor person.

* *Symmetry (SYM)*: $P_k(c) = P_k(c')$ if *c'* is obtained from *c* by a permutation.

* *Replication Invariance (RI)*: $P_k(c) = P_k(c')$ if *c'* is obtained from *c* by a replication.

Since poverty measurement is concerned with the deprivations of the poor people, the first two properties, postulated by Sen (1976) in the unidimensional setting, are considered as basic axioms for a poverty measure. Thus, PF requires that a poverty index should not depend on the non poor people's deprivations and MON demands that poverty should increase if the number of deprived dimensions suffered by a poor person increases. It may be worth noting that PF ensures that $P_k(c) = P_k(c(k))$.

SYM and RI are also standard requirements for a poverty measure. SYM establishes that no other characteristic apart from the number of dimensions in which a person is deprived matters in defining a counting poverty index. In turn, RI allows us to compare populations of different sizes.

According to Sen (1976), a poverty measure should be sensitive to inequality among the poor, then the counterpart of the Pigou-Dalton transfer principle for a counting measure may be introduced as follows:

* *Transfer Sensitivity* (TS): $P_k(c) < P_k(c')$ if *c'* is obtained from *c* by a deprived dimension transfer between two people that are poor before and after the transfer.

The class of counting poverty measures that fulfil these five axioms will be denoted by $\mathbf{P}$, that is:

$$\mathbf{P} = \left\{ P : G \times \{1,...,d\} \to \mathbb{R} \,/\, P \textit{ satisfies PF, MON, DDC, SYM, RI and TS} \right\}.$$

The first poverty counting measure introduced in the literature is the *multidimensional headcount ratio*, denoted by $H_k = q_k / n$, that is, the percentage of the population deprived in at least *k* dimensions. There are some advantages to this index, usually used to measure the incidence of poverty. One of them is that it can be used with ordinal and categorical data. There are also some shortcomings, since it is able to capture neither the intensity nor the inequality among the poor and violates MON, that is, it does not change if a person already identified as poor becomes deprived in an additional dimension in which the person was not poor previously.

The *adjusted headcount ratio*, M_k, introduced by Alkire and Foster (2008) is defined as the average of the number of deprivations suffered by the poor, that is, $M_k(c) = \frac{\sum_{1 \le i \le n} c_i(k)}{nd}$. This index overcomes the drawbacks of the headcount ratio since it satisfies MON. However M_k does not belong to class $\mathbf{P}$, since although it satisfies a weaker version of TS, it violates TS as proposed in this paper.

More information about poverty can be incorporated using the *average deprivation share across the poor* denoted by A_k, also introduced by Alkire and Foster (2008), which is defined as the mean among the poor, of the number of deprivations suffered by the poor, that is, $A_k = \frac{\sum_{1\le i\le n} c_i(k)}{q_k d}$. This index captures the intensity of poverty. Moreover, M_k can be computed as the product of the *multidimensional headcount ratio* and the *average deprivation share across the poor*.

Poverty rankings may be reversed depending on the identification threshold, or on the measure selected. Thus, in order to avoid contradictory results, dominance conditions ensure unanimous rankings for a set of identification cut-offs, or a class of poverty measures. Following the existing literature, given a poverty measure P we introduce a partial ordering denoted by $\precsim_P$ in the set of vectors of deprivation counts, as follows:[6]

$$c' \precsim_{\mathrm{P}} c \text{ if and only if } P_k(c') \ge P_k(c) \text{ for all } k:1,\ldots,d.$$

2. DEPRIVATION DIMENSION CURVES.

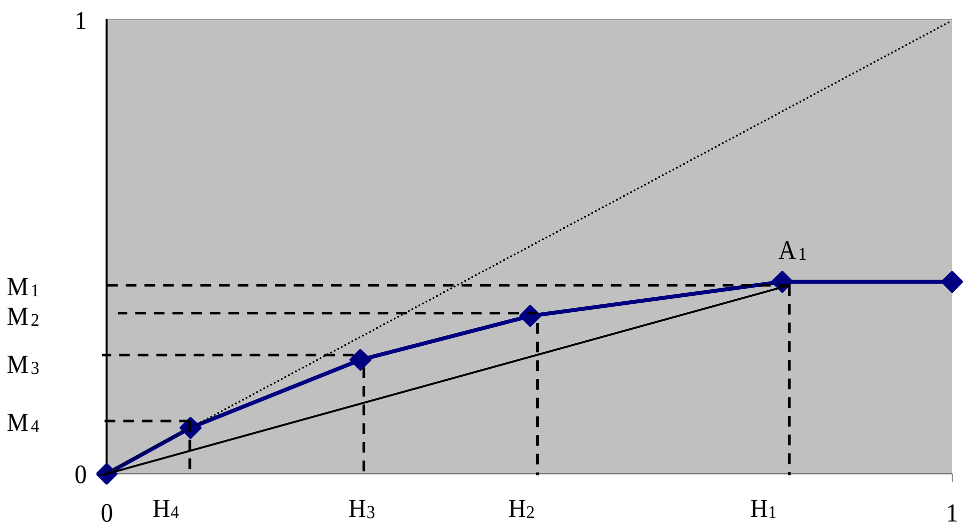
We can compute the previous indices in an example. Consider the vector of deprivation counts $c = (4,3,3,2,2,1,1,1,0,0)$ in a society of 10 individuals with 4 dimensions. The headcount ratio, the average deprivation share and the adjusted headcount ratio for all the possible identification cut-offs are displayed in the following table:

	k=4	*k=3*	*k=2*	*k=1*
H_k	*0.1*	*0.3*	*0.5*	*0.8*
M_k	*0.1*	*0.25*	*0.35*	*0.425*
A_k	*1*	*0.833*	*0.7*	*0.531*

[6] We follow Atkinson (1987) and adopt the weak definition of a partial ordering. Although not all the results derived in this paper hold for the other two levels (the semi-strict and the strict ones) similar conditions could be also obtained in these two cases.

We propose constructing the *DD curve*, for this vector *c*, plotting the headcount ratio against the adjusted headcount ratio, that is, pairs of points $\left(H_k, M_k\right)$. We also plot two extreme points $\left(0,0\right)$ as the start of the curve, and $\left(1, M_1\right)$, as the end of the curve. Then we join the dots, as showed in Figure 1:

Figure 1. Plotting the headcount ratio and the adjusted headcount ratio.



By definition, each time the slope of the curve changes the headcount ratio is displayed in the horizontal axis. By contrast, in the vertical axis, the adjusted headcount ratio is recovered. The slope of the line that connects (0,0) with $\left(H_k, M_k\right)$ coincides with the average deprivation share A_k.

In general, for any vector of deprivation counts *c*, ranked from the most deprived to the least, the *DD-curve* can equivalently be defined in the following way: for each integer $p = 0, ..., n-1$ the ordinate of the curve is computed as the cumulative of the sum of the total number of deprivations experienced by the first *p* people divided by the total number of deprivations that could possibly experienced by all people. At intermediate points the curve is determined by linear interpolation. Thus, the ordinates of the DD-curve are computed as follows:

$$DD(c;0)=0$$

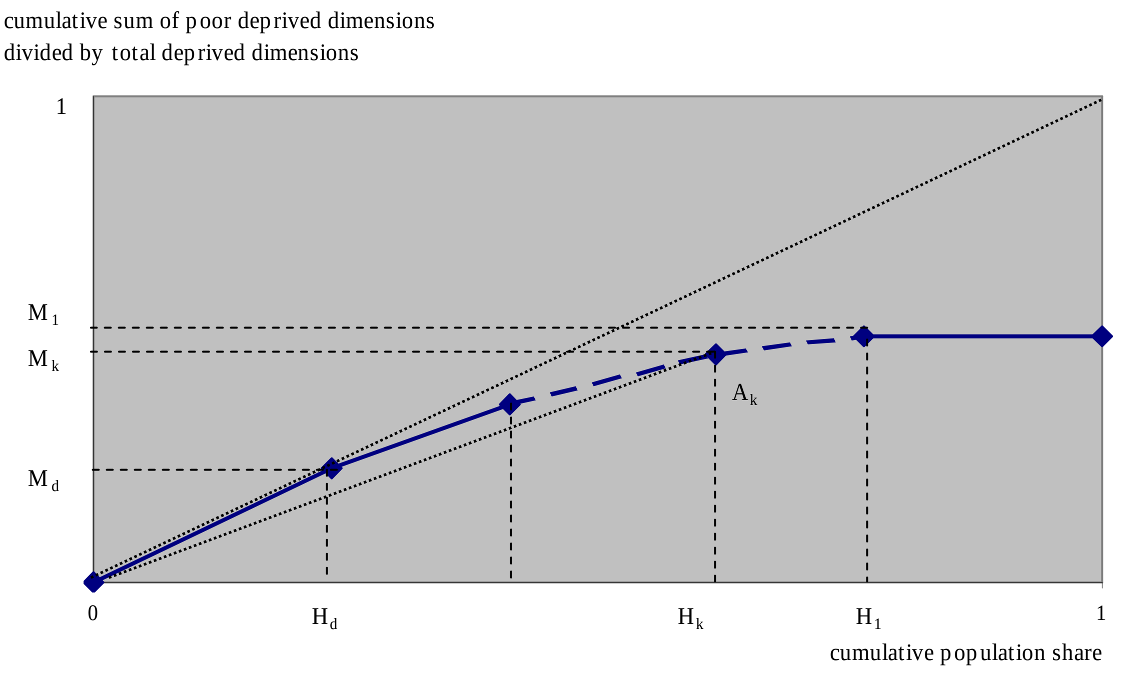
$$DD\left(c;\frac{p}{n}\right)=\frac{1}{nd}\sum_{1\le i\le p}\overline{c}_i\,,\; p=1,...,n$$

$$DD\left(c;\frac{p+\theta}{n}\right)=\frac{1}{nd}\sum_{1\le i\le p}\left(\overline{c}_i+\theta\overline{c}_{p+1}\right),\; p=0,...,n-1\,,\; \theta\in[0,1]$$

Some interesting properties of this curve may be mentioned. First of all, the ordinates of this curve are replication invariant, and are also invariant to permutation of *c*. The graph, as displayed in Figure 2, begins at the origin, and is a continuous non-decreasing concave function.

There are two bounding curves which correspond with the extreme situations of minimum and maximum deprivation. If nobody is deprived, the curve coincides with the horizontal axis. By contrast, if everybody is deprived in all dimensions, the curve becomes the diagonal line.

Figure 2. Deprivation Dimension Curves.



In general, the slope of the curve is to change *d* times, as many as the number of dimensions considered. Each point p/n at which the curvature of the curve changes, yields the percentage of people deprived in at least as many dimensions as person *p*. If we call this number *k*, we find that the adjusted headcount ratio, H_k, is recovered in these points. In contrast, the vertical axis displays, by definition, the dimension adjusted headcount ratio M_k. Thus, the first time the slope changes corresponds to the headcount ratio according to the intersection procedure, H_d. The last time, when the curve becomes horizontal, yields the headcount ratio as regards the union procedure, H_1. At this point the curve reaches its maximum value which corresponds to the ratio between the sum of the total number of deprivations experienced by all the people, and the total number of deprivations that could possibly be experienced, that is, M_1 according to Alkire and Foster (2008) designation.

The *average deprivation share across the poor*, A_k, is also represented in the graph by the slope of the ray from (0,0) to $\left(p, DD(p)\right)$.

3. Deprivation Dimension Dominance.

When the *DD* curves of two vectors *c* and *c'* do not intersect, they allow us to introduce a dominance criterion denoted by $\succsim_{DD}$, as follows:

$$c' \succsim_{DD} c \text{ if and only if } DD(c';p) \geq DD(c;p) \text{ for all } \mathrm{p} \in [0,1].$$

The following proposition is based on the results established by Marshall and Olkin (1979, propositions 4.A.2 and A.B.2) for vectors with the same number of components:

Proposition 1. *For any* $c, c' \in C^n$ *vectors of deprivation counts, the following statements are equivalent:*

i) *c' dominates c;*

ii) $M_k(c) \leq M_k(c')$ *for all* $k = 1, ..., d$ *;*

iii) $\sum_{1 \leq i \leq p} \overline{c}_i \leq \sum_{1 \leq i \leq p} \overline{c}_i^{'}$ *for all* $p = 1, ..., n$ *;*

iv) *c' may be obtained from c by a finite sequence of permutations, increments and/or deprived dimension transfers;*

v) $\sum_{1\le i\le p}\varphi(c_i)\le\sum_{1\le i\le p}\varphi(c_i^{'})$ *for all continuous, increasing and convex functions* $\varphi:[0,d]\longrightarrow\mathbb{R}$.

This proposition establishes that when the curve of a vector of deprivation counts lies above the curve of another with the same population size, or equivalently, when these two vectors can be ordered by the multidimensional headcount ratio, then one may be obtained from the other by a sequence of increments and/or permutations. Consequently, any poverty measure belonging to class **P** will rank these two vectors in exactly the same way. In addition, as the deprivation curves are invariant under replication, and the same holds for any measure $P\in\mathbf{P}$, the result also holds for vectors with different population sizes.

Then, this result reveals that although the dimension adjusted headcount ratio violates TS, if two vectors of deprivation counts can be unanimously ranked by M_k at all cut-offs, then all poverty counting measures satisfying TS will rank societies in the same way.

The reverse of this proposition is also true. In particular, consider the class of counting measures:

$$P(c,k)=\frac{1}{n}\sum_{1\le i\le n}\psi(c_i(k))$$

with $\psi:[0,d]\longrightarrow\mathbb{R}$ a continuous increasing and strictly convex function. It is quite simple to show that $P(c,k)$ belongs to class **P**. Given any continuous increasing and convex function $\varphi:[0,d]\longrightarrow\mathbb{R}$ and $\varepsilon>0$, then the measures $P_\varepsilon(c,k)=\frac{1}{n}\sum_{1\le i\le n}(\varepsilon\psi+\varphi)(c_i(k))$ also belong to class **P**. Consequently, given two vectors *c* and *c'* with $P_\varepsilon(c,k)\le P_\varepsilon(c',k)$, when $\varepsilon\to 0$ we get statement v) in Proposition 1 and have the following result:

Proposition 2. *For any* $c,c'\in G$ *vectors of deprivation counts:*

$$DD(c';p)\ge DD(c;p)\ \textit{for all}\ \mathrm{p}\in[0,1]$$

if and only if $P_k(c')\ge P_k(c)$ *for all* $P\in\mathbf{P}$ *and for all identification cut-off* $k\in\{1,...,d\}$.

When the *DD* curves of two vectors, *c* and *c'*, intersect, it is always possible to restrict the set of identification cut-offs in order to establish similar dominance conditions. In fact, there exists a threshold $k^* \in \{1,...,d\}$ such that $M_k(c) \leq M_k(c')$ *for all* $k = 1,...,k^*$. Taking into consideration the respective censored vectors $c(k^*)$ and $c'(k^*)$, we may obtain the following proposition:

Proposition 3. *For any* $c, c' \in G$ *vectors of deprivation counts:*

$$DD(c'(k^*); p) \geq DD(c(k^*); p) \textit{ for all } p \in [0,1]$$

if and only if $P_k(c') \geq P_k(c)$ *for all* $P \in \mathbf{P}$ *and for all identification cut-off* $k \in \{1,...,k^*\}$.

This proposition shows that, even when the DD curves intersect, they allow us to obtain robust conclusions in a wide set of counting measures restricting the set of identification cut-offs.

CONCLUDING REMARKS.

A counting approach based on the number of deprivations suffered by the poor is quite an appropriate framework to measure multidimensional poverty with ordinal or categorical data.

The choice of a cut-off to identify the poor, and a poverty measure to aggregate the data are two sources of arbitrariness and different selections may lead to contradictory conclusions. In this paper we derive dominance conditions in order to obtain unanimous rankings in a wide set of counting measures, and a set of identification cut-offs. The implementation of these conditions is based on the *DD* curves, a simple and intuitive device.

The dominance conditions proposed in this paper correspond to what in the literature are known as second degree dominance conditions. It is not difficult to derive the counterpart of the first degree dominance requirements.

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