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CREDIT-EASING POLICY IN AN INCOMPLETE MARKETS ECONOMY

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Abstract

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Credit-easing Policy in an Incomplete Markets Economy*

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1 Introduction

It is widely agreed that high financial leverage, namely high ratios of assets to underlying capital, was a critical factor in generating and magnifying the recent global financial crisis. In addition, the recent financial crisis has also highlighted the critical role of unconventional monetary policies, interpreted as 'expanding central bank credit intermediation to offset a disruption of private financial intermediation' (Gertler and Kardi, 2010). The role of financial leveraging in propagating business cycle shocks is discussed intensively in the literature.[1] On the other hand, relatively few papers explore the factors that cause excessive leveraging and the resulting financial crises.[2]

In the context of asset pricing literature, Aiyagari and Gertler (1999, hereafter AG) constructs a very simple framework for analyzing the relationship between the high financial leverage and the volatility of equity prices. In AG, there are households and financial intermediaries. The households receive the disutility from the trading equity shares. Intermediaries can borrow from the households in order to invest equity shares. That is, the intermediaries can take short positions. However, the leverage constraints limit the use of leverage. When the leverage constraints bind, the intermediaries have to unload the shares to the households. If it is very costly for the household to quickly absorb shares, the price must drop sharply. Although it is very intuitive, the main shortcoming of their model is that financial intermediaries have no incentive to maintain a leveraged position over time.

In this paper, we extend the model of AG to understand why financial intermediaries continue to make leveraged investments despite the risk of having to do distress selling later. Then, we conduct numerical exercises to examine the effects of a type of unconventional monetary policy on the consumption and wealth distributions of households. In particular, we focus on the credit-easing (hereafter, CE) policy, which involves the central bank buying risky assets such as equities and selling risk-free bonds.

We introduce leverage-constrained financial intermediaries, incomplete markets, and financial frictions into a three-period pure exchange economy with ex ante homogeneous risk-averse households. Because of uninsured idiosyncratic income shocks and market incompleteness, there is ex post heterogeneity in households' consumption and wealth distributions. As is well known, market incompleteness implies the existence of arbitrage opportunities.[3] To pursue arbitrage profits, the intermediaries finance positions in risky securities by borrowing from households. When shocks push asset prices to a level low enough to constrain leverage, the intermediaries are forced to liquidate assets. This drives asset prices below what they would have been in a frictionless market. We analytically derive the conditions under which the intermediaries raise their leverage positions ex ante and are forced to liquidate assets to satisfy their leveraged constraints ex post. We also numerically demonstrate that such price falls actually emerge in a competitive equilibrium.

Our model allows us to analyze how CE policy affects the asset prices and welfare of ex ante identical, but ex post heterogeneous, households. Because CE policy raises the prices of risky assets but lowers the prices of risk-free bonds, effects of CE

[1] For example, Gertler and Kardi (2011) and Kiyotaki and Moore (2011) explore the role of unconventional monetary policy. Gertler, Gilchrist, and Natalucci (2007), Mendoza (2008), and Devereux and Yetman (2011) explore such policies in the international context.

[2] Lorenzoni (2007) develops a three-period model where a simple externality gives rise to excess credit and the ensuing fire sales. Kato and Tsuruga (2011) extends the model of Diamond and Rajan (2009) to understand the interaction between financial intermediaries' leverage and the probability of a bank run.

[3] See Duffie (2001) and Cochrane (2005).

policy on household's welfare vary due to the portfolio positions. In other words, CE policy may have substantial distributional effects. In fact, we demonstrate that CE policy generates conflicts of interest between high- and low-income households, although it may raise average welfare. Because it mitigates falls in equity prices during recessions, CE policy improves the welfare of low-income households, who have to sell shares to raise their current consumption. However, it may lower the welfare of high-income households, who want to buy shares to raise their future consumption.

2 Model

There are three periods: $t = 0, 1$ and 2; three types of agents: households, financial intermediaries, and a government; and two types of assets: risky equities and risk-free bonds. Households are risk averse, receive endowments in each period, and trade assets. We assume that agents cannot trade state-contingent securities. In other words, asset markets are incomplete. Thus, households are ex ante identical but ex post heterogeneous because of uninsured idiosyncratic shocks to endowments.

While the household may hold risky shares directly in its portfolio, it cannot trade them cost free. The financial intermediaries that are owned by the households and valued in a competitive market have comparative advantage in pursuing arbitrage profits. They can exchange shares and risk-free bonds free of charges. They finance their positions in shares using the capital provided by intermediaries and by borrowing from households. This borrowing takes the form of short-term risk-free bonds, which the households can trade cost free. In other words, the intermediaries take the leveraged investment. There are two important frictions. First, leverage constraints limit the use of leverage to some multiple of their capital. Second, the only way the intermediaries can build their capital is by retaining earnings from trading profits. We assume that directly issuing new equity is prohibitively expensive. These assumptions, originally constructed by AG, generate a potential link between the value of the intermediaries' existing capital and their gross holding of risky assets.

2.1 Households

Aggregate states, j, realize a normal state, n, with probability of $1 - \phi$, but a bad state, b, with probability of ϕ. In state $j = b$, there are aggregate and idiosyncratic shocks to endowments. In particular, idiosyncratic states i realize a high-endowment state, h, with probability of $1 - \psi$, but a low-endowment state, l, with probability of ψ. Suppose that households cannot trade securities contingent on idiosyncratic states in period $t = 0$ and borrow in period $t = 1$.

We assume that there are no idiosyncratic shocks in state $j = n$. In this economy, there is a single Lucas tree, which produces dividends, d_t, in each period. For analytical purpose, households cannot trade the Lucas tree cost free in period 1, while they can in period 0.

Let $\{c_0, c_{ti}^j\}$, $\{e_0, e_{ti}^j\}$, $\{s_0, s_{ti}^j\}$, and $\{f_0, f_{ti}^j\}$ be the households' consumption, endowments, shares of the Lucas tree, and holdings of riskless bonds in periods 0 and $t = 1,\ 2$ and states $j = \{n,\ b\}$ and $i = \{h,\ l\}$. Let $\{p_0, p_1^j\}$ and $\{q_0, q_1^j\}$ be prices of the Lucas tree and the riskless bonds. $\{d_0, d_t^j\}$ denote the dividends received from the Lucas tree, $\{D_0, D_t^j\}$ denote the dividends received from the intermediaries, and $\{T_0,\ T_t^j\}$ denote lump-sum taxes.

Households receive utility from consumption and disutility in the form of costs of buying or selling shares. Households maximize the following expected utility:

$$u(c_0)+\beta E_0\Big\{u(c^j_{1i})-\tilde{a}\frac{(s^j_{2i}-s_1)^2}{2}+\beta u(c^j_{2i})\Big\}, \tag{1}$$

subject to the following budget and borrowing constraints:

$$\begin{aligned} c_0+p_0s_1+q_0f_1+T_0 &= (p_0+d_0)s_0+f_0+D_0+e_0, &(2)\\ c^j_{1i}+p^j_1s^j_{2i}+q^j_1f^j_{2i}+T^j_1 &= (p^j_1+d^j_1)s_1+f_1+D^j_1+e^j_1, &(3)\\ c^j_{2i}+T^j_2 &= d^j_2s^j_{2i}+f^j_{2i}+D^j_2+e^j_{2i}, &(4)\\ f^j_{2i} &\geq \bar{f}. &(5)\end{aligned}$$

Periodic utility is the power function $u(c)=\frac{c^{1-\gamma}-1}{1-\gamma}$. Equation (5) is borrowing constraints. For simplicity, we assume $e^b_{1h}>e^b_{1l}$, $e^b_{2h}<e^b_{1h}$, and $e^b_{2l}>e^b_{1l}$ so that households with binding borrowing constraints are type l.

Note that in equation (1), the disutility of effort arising from transactions in shares is quadratic in size of households' equity trades.[4] In this paper, we assume that the parameter $\tilde{a}=a_+$ when households buy shares, while $\tilde{a}=a_+$ when the households sell shares, $\tilde{a}=a_-$. That is, we assume that the adjustment costs can be asymmetric. In section 3.2, we demonstrate that the asymmetry generates an important arbitrage opportunity in this three-period framework.

2.2 The Financial Intermediaries

Because the financial intermediaries are owned by the households, the financial intermediaries' objective is to maximize the expected discounted value of dividend payouts to households. While the intermediaries may exchange securities cost free, there are some restriction on their ability to construct portfolios. Let M^j_t be the stochastic discount factors (hereafter, SDF) of the intermediaries and $\{s^*_0, s^{*j}_{ti}, f^*_0, f^{*j}_{ti}\}$ be the intermediaries' respective holdings of shares and risk-free bonds in periods 0 and $t=1,\ 2$ and states $j=\{n,\ b\}$.

The market value of the intermediaries in period 0 is:

$$D_0+E_0\Big\{M^j_1\Big(D^j_1+M^j_2D^j_2\Big)\Big\},$$

where the dividend payouts at t, D^j_t, are the difference between the net assets the intermediaries have at the beginning of period t and the net assets they have at the end of t. The budget constraints are:

$$\begin{aligned} &D_0=(p_0+d_0)s^*_0+f^*_0-p_0s^*_1-q_0f^*_1, &(6)\\ &D^j_1=(p^j_1+d^j_1)s^{*j}_1+f^{*j}_1-p^j_1s^{*j}_2-q^j_1f^{*j}_2, &(7)\\ &D^j_2=d^j_2s^{*j}_2+f^{*j}_2. &(8)\end{aligned}$$

That is, using the beginning of period capital, $(p_t+d_t)s^*_t+f^*_t$, the intermediaries choose their financial positions, s^*_{t+1} and f^*_{t+1}. The residuals, D_t, are dividend

[4]The quadratic adjustment costs are originally introduced by AG. "Think of the household as having a fixed amount of time to either trade securities or enjoy leisure (e.g., watch football or play with children). Since the household is not a specialist, large transactions absorb a large amount of time (due to, say, double checking, rearranging funds, etc.). With diminishing marginal utility of leisure this could lead to convex costs of trading, which may be approximated by a quadratic loss function." (AG p. 9)

payouts to the households. The leverage constraints are as follows:

$$\frac{1}{\kappa}\{(p_0 + d_0)s_0^* + f_0^* - D_0\} \geq p_0 s_1^*, \tag{9}$$

$$\frac{1}{\kappa}\{(p_1^j + d_1^j)s_1^{*j} + f_1^{*j} - D_1^j\} \geq p_1^j s_2^{*j}. \tag{10}$$

The leverage constraints imply that the intermediaries can purchases the equity shares to the amount of $\frac{1}{\kappa}$ times their own capital, $(p_t + d_t)s_t^* + f_t^* - D_t$, where $\kappa \in \{0, 1\}$. In addition, because directly issuing new equity is prohibitively expensive, dividend payouts cannot be negative:

$$D_0 \geq 0,\ D_1^j \geq 0,\ \textit{and}\ D_2^j \geq 0. \tag{11}$$

We call inequalities (11) dividend constraints.

2.3 The Government

The government levies lump-sum taxes on households and conducts CE policy in short-term risk-free bonds and shares, subject to the following budget constraints:

$$T_0 + f_0^g + (p_0 + d_0)s_0^g \geq p_0 s_1^g + q_0 f_1^g,$$
$$T_1 + f_1^g + (p_1^j + d_1^j)s_1^g \geq p_1^j s_2^{gj} + q_1 f_2^{gj},$$
$$T_2 + f_2^g + d_2^j s_2^{gj} \geq 0,$$

where $\{f_0^b, f_t^{gj}\}$ and $\{s_0^g, s_t^{gj}\}$ denote government's beginning-of-period holdings of risk-free bonds and shares in periods 0 and $t = 1,\ 2$ and states $j = \{n,\ b\}$. Risk-free bonds that the government issues are no different from those issued by intermediaries. We assume there is no government consumption or spending.

2.4 Market-clearing Conditions

The total number of shares is normalized to unity. There are no external bonds. Therefore, market-clearing conditions for goods, shares, and risk-free bonds are as follows:

$$c_0 = e_0 + d_0 = y_0\ \textit{and}\ (1-\psi)c_{th}^j + \psi c_{tl}^j = (1-\psi)e_{th}^j + \psi e_{tl}^j + d_t^j = y_t^j,$$
$$s_0 + s_0^* + s_0^g = 1,\ \textit{and}\ (1-\psi)s_{th}^j + \psi s_{tl}^j + s_t^{*j} + s_t^{gj} = 1,$$
$$f_0 + f_0^* + f_0^g = 0,\ \textit{and}\ (1-\psi)f_{th}^j + \psi f_{tl}^j + f_t^{*j} + f_t^{gj} = 0,$$

for $t = 1, 2$ and $j = \{n, b\}$. $\{y_0, y_t^j\}$ denote aggregate endowments.

3 Equilibrium

3.1 Households

While households are subject to borrowing constraints, they do not face any costs of adjusting holdings of risk-free bonds. The unconstrained households' intertemporal marginal rates of substitution (hereafter, IMRS) determine risk-free prices in a standard fashion:

$$q_0 = (1-\phi)\beta_1^n + \phi\beta_1^b, \tag{12}$$
$$q_1^n = \beta_2^n, \tag{13}$$
$$q_1^b = \beta\frac{u'(c_{2i}^b)}{u'(c_{1i}^b)} + \frac{\xi_i}{\beta\phi u'(c_{1i}^b)}, \tag{14}$$

where $\beta_1^n \equiv \beta\frac{u'(c_1^n)}{u'(c_0)}$, $\beta_2^n \equiv \beta\frac{u'(c_2^n)}{u'(c_1^n)}$, $\beta_1^b \equiv \beta\frac{(1-\psi)u'(c_{1h}^b)+\psi u'(c_{1l}^b)}{u'(c_0)}$, and ξ denote the Lagrangian multipliers of borrowing constraints, which take the value 0 if constraints do not bind. Note that because our assumption ensures that households with high income have no incentive for borrowing, equation (14) implies that the IMRS of high-income households, who have the strongest willingness to save, determines the risk-free prices in state $j = b$.

Using first-order conditions with respect to shares, we can derive the following conditions:

$$p_0 = (1-\phi)\beta_1^n\{d_1^n + \beta_2^n d_2^n\} + \phi\beta_1^b\{d_1^b + \hat{\beta}_2^b d_2^b\}, \tag{15}$$
$$s_2^n - s_1 = \left\{\frac{\beta_2^n d_2^n}{p_1^n} - 1\right\}\frac{p_1^n u'(c_1^n)}{\tilde{a}}, \tag{16}$$
$$s_{2i}^b - s_1 = \left\{\beta\frac{u'(c_{2i}^b)}{u'(c_{1i}^b)}\frac{d_2^b}{p_1^b} - 1\right\}\frac{p_1^b u'(c_{1i}^b)}{\tilde{a}}, \tag{17}$$

where

$$\hat{\beta}_2^b \equiv \beta\frac{(1-\psi)u'(c_{2h}^b) + \psi u'(c_{2l}^b)}{(1-\psi)u'(c_{1h}^b) + \psi u'(c_{1l}^b)}.$$

In period 1, the high-income households, $i = h$, buy shares, while the low-income households, $i = l$, would like to sell their shares in order to raise their consumption. Thus, the adjustment cost parameters are $\tilde{a} = a_+$ for the high-income households and are assigned $\tilde{a} = a_-$. From market-clearing condition and the first order conditions, (17), for the high and low endowment households, the following conditions imply that the sum of the high and low endowment households' equity shares transactions is zero:

$$p_1^j = \bar{p}_1^j \equiv \beta_2^j d_2^j,$$

where

$$\beta_2^b \equiv \beta\frac{(1-\psi)a_- u'(c_{2h}^b) + \psi a_+ u'(c_{2l}^b)}{(1-\psi)a_- u'(c_{1h}^b) + \psi a_+ u'(c_{1b}^b)}.$$

The following definitions imply that the net of households and government transactions equal zero:

$$p_1^n \;=\; \tilde{p}_1^n \equiv \beta_2^n d_2^n + \frac{\tilde{a}g^n}{u'(c_1^n)}, \tag{18}$$
$$p_1^b \;=\; \tilde{p}_1^b \equiv \beta_2^b d_2^b + \frac{g^b}{\alpha_1}, \tag{19}$$

where $\alpha_1 \equiv \frac{(1-\psi)a_- u'(c_{1h}^b)+\psi a_+ u'(c_{1l}^b)}{a_+ a_-}$ and $g^j \equiv s_2^{jg} - s_1^{jg}$. In this paper, we denote the above equity prices as the fundamental equity prices.

3.2 The Financial Intermediaries

Financial intermediaries maximize their market value subject to budget constraints, dividend constraints, and leverage constraints. Introducing the budget constraints

(6), (7), and (8) into the leverage constraints, (9) and (10), and the dividend constraints, (11), we can derive the following constraints:

$$f_1^* \geq -(1-\kappa)\frac{p_0}{q_0}s_1^*, \tag{20}$$

$$f_1^* \leq -\frac{p_0}{q_0}s_1^* + \frac{(p_0+d_0)s_0^* + f_0^*}{q_0}, \tag{21}$$

$$f_2^{*j} \geq -(1-\kappa)\frac{p_1^j}{q_1^j}s_2^*, \tag{22}$$

$$f_2^{*j} \leq -\frac{p_1^j}{q_1^j}s_2^{*j} + \frac{(p_1^j + d_1^j)s_1^* + f_1^*}{q_1^j}, \tag{23}$$

$$f_2^{*j} \geq -d_2^j s_2^j. \tag{24}$$

Because of $d_2^j \geq \frac{p_1^j}{q_1^j} \geq (1-\kappa)\frac{p_1^j}{q_1^j}$, dividend constraints in period 2 never bind. Thus, we can ignore the dividend constraints in period 2 (24).

SDF and arbitrage opportunities The intermediaries employ households' IMRS as the SDF, M_t^j for $t = 1, 2$, and $j = \{n, b\}$. In particular, $M_t^n = \beta_t^n$ and $M_1^b = \beta_1^b$. On the other hand, because M_2^b can take the value of β_2^b, $\hat{\beta}_2^b$, or q_1^b, the SDF in period 2 and state b are not uniquely determined. Therefore, there are some arbitrage opportunities for the intermediaries.

q_1^b is the price of the risk-free bonds in period 1 and state b. On the other hand, β_2^d is the fundamental price–dividend ratio of shares in period 1 and state b. Because the risk-free prices are determined by the highest IMRS, the fundamental price–dividend ratio of shares is relatively cheaper than the price of the risk-free bonds in period $t = 1$ and state $j = b$; that is, $q_1^b = \beta\frac{u'(c_{2h}^b)}{u'(c_{1h}^b)} > \beta_2^b$. This implies that selling the risk-free bonds and buying shares make arbitrage profits without any costs. This is 'the interassets arbitrage.'

The period 0 fundamental values of dividends in period 2 are $\beta_1^b\hat{\beta}_2^b$. On the other hand, the period 0 value of the fundamental price–dividend ratio in period 1 and state b is $\beta_1^b\beta_2^b$. $\hat{\beta}_2^b$ is the IMRS between periods 1 and 2 of the ex ante homogeneous households. β_2^b is the price of the equities that equalize the households' supply and demand for shares. Symmetric adjustment cost, $a_+ = a_-$, implies $\hat{\beta}_2^b = \beta_2^b$. However, if $a_- > a_+$, $\beta_2^b > \hat{\beta}_2^b$. Because high-income households are those who have high IMRS and strong incentives to save and can buy shares with relatively cheaper adjustment costs in period 1, the fundamental prices of shares are relatively higher in period 1 than in period 0. Therefore, buying stocks in period 0 and selling in period 1 creates arbitrage profits. This is 'the intertemporal arbitrage.'

Determinations of $\{s_2^{*j}, f_2^{*j}\}$

Lemma 1 *In state n, $p_1^n = q_1^n d_2^n$ implies that both dividend and leverage constraints are slack while $p_1^n < q_1^n d_2^n$ implies that both constraints are binding. On the other hand, in state b, both constraints are always binding.*

Proof. See Appendix.

Figure 1 displays the relationship among the constraints, (22), (23), and (24). In state b, because of the interasset arbitrage opportunities, intermediaries gain profits from their long position in shares and their short position in risk-free bonds. Therefore, the intermediaries borrow from the households and invest equity shares as possible as they can. As a results, the intermediary choose the portfolio where both the dividend and leveraged constraints bind. This is the point of intersection of "Dividend constraints in period 1" and "Leverage constraints" in Figure 1.

Determinations of $\{s_1^*, f_1^*\}$ We call (20) and (21) the period 0 leverage constraint and the period 0 dividend constraint. Both constraints define the portfolio that the intermediaries can choose in period 0.

If leverage constraints do not bind under $\tilde{p}_1^j$ in period 1 and state j, the intermediaries need not change their portfolio: $s_2^{*j} = s_1^*$ and $f_2^{*j} = -(1-\kappa)d_2^j s_1^*$. The following inequality holds:

$$\begin{aligned} & (\tilde{p}_1^j + d_1^j)s_1^* + f_1^* - \kappa \tilde{p}_1^j s_1^* \geq 0, \\ \iff \quad & f_1^* \geq -\{d_1^j + (1-\kappa)\tilde{p}_1^j\}s_1^*. \end{aligned} \tag{25}$$

On the other hand, if leverage constraints are binding under $\tilde{p}_1^j$, the following inequality holds:

$$f_1^* < -\{d_1^j + (1-\kappa)\tilde{p}_1^j\}s_1^*.$$

In this case, intermediaries have to sell their equities in order to satisfy their leverage constraints in period 1 and state j. If the intermediaries choose a portfolio such that (26) holds in period 0, they have to deleverage in period 1 and state b. We call inequality (25) the period 1 leverage constraints.

Assumption 1 *To ensure that the leverage constraints can bind in state b, we assume the following inequality:*

$$(1-\kappa)\frac{p_0}{q_0} \geq \{d_1^b + (1-\kappa)\tilde{p}_1^b\}. \tag{26}$$

Lemma 2 *If inequality (26) holds, the leverage constraints bind only in state b. If inequality (26) does not hold, the leverage constraints bind only in state n.*

Proof. See Appendix.

Figure 2 displays the above constraints.

Lemma 3 *The intermediary have no incentive to pay dividends to households in period 0.*

Proof. See Appendix.

From Lemma 1, the intermediaries raise their capital in order to gain arbitrage profits in period 1 and state b. Thus, the intermediaries have no willingness to pay dividends in period 0. The lemma 3 indicates that the optimal portfolio exists in the "Dividend constraints in period 0" in the Figure 2.

We can derive the following proposition.

Proposition 1 *Individual optimality of full leveraged investment in period 0.* Leveraged investment raises financial intermediaries' market value, if and only if the following conditions hold:

$$\frac{\Delta^B(d_1^b + p_1^b) - (d_1^b + \hat{\beta}_2^b d_2^b)}{p_0} > \frac{\Delta^B - 1}{q_0}, \tag{27}$$

where Δ^B denotes the market value of leveraged investment in period 1:

$$\Delta^B \equiv M_2^b \Big[\frac{1}{\kappa}\frac{d_2^b}{p_1^b} - (\frac{1}{\kappa} - 1)\frac{1}{q_1^b}\Big].$$

Proof. See Appendix.

Because intermediaries gain arbitrage profits through taking short positions in risk-free bonds and long positions in shares in period 1 and state b, they seek to raise their own capital in period 0. However, there are two ways of raising intermediaries' capital in period 0: holding risk-free bonds or shares. The right-hand side of inequality (27) shows the returns on leveraged investments in period 1 financed by the risk-free bonds in period 0. The left-hand side shows the returns on those financed by shares in period 0. If the left-hand side is higher than the right-hand side, intermediaries prefer investing in shares in period 0. Because of the intertemporal arbitrage opportunities, $\beta_2^b > \hat{\beta}_2^b$, investing in equities in period 0 is more profitable than holding risk-free bonds.

Sources of arbitrage opportunities We now consider the roles of asset trading constraints in generating arbitrage opportunities. First of all, if markets are complete and state contingent securities are traded, the IMRS are equalized among households in all periods. Thus, there are no arbitrage opportunities. This implies that market incompleteness is crucial in generating arbitrage opportunities. Second, if there are no borrowing constraints and adjustment costs, consumption growth is equalized between periods 1 and 2, but not in periods 0 and 1 due to market incompleteness. In this case, there are no arbitrage opportunities because of $\frac{u'(c_{2h})}{u'(c_{1h})} = \frac{u(c_{2l})}{u'(c_{1l})}$ and $\beta_2^b = q_1^b$, $\beta_2^b = \hat{\beta}_2^b$. Therefore, borrowing constraints and adjustment costs also play an important role in generating arbitrage opportunities. Finally, what is the role of heterogeneity in transaction costs? Homogeneous transaction costs imply that the SDF that prices the shares in period 1 is $\beta_2^b = \hat{\beta}_2^b$. That is, the intertemporal arbitrage opportunity does not exist, while interasset arbitrage exists.

3.3 Credit-easing Policy

We consider the policy whereby the government or the central bank buys risky assets and sells short-term risk-free bonds in period 1. This is the so-called 'credit-easing' policy, which aims to normalize the dysfunctional markets. We model CE policy with the following government budget constraints:

$$0 = p_1^b s_2^{gb} + q_1^b f_2^{gb}, \quad T_2^{bg} + f_2^{bg} + d_2^b s_2^{bg} = 0.$$

As described by the above equations, the central bank or the government issues the risk-free bonds, which are no different from those that the intermediaries issue, and purchases equities in period 1. In period 2, the government transfers the profits to the households. That is, a government engages in arbitrage trading through open market operations between risky and safe assets in period 1 and transfers the profits to the households in period 2.

3.4 Equilibrium

Equity prices If the leverage constraints in period 1 do not bind, the equity price equals its fundamental value, $\tilde{p}_1^b$. On the other hand, if the period 1 leverage constraint binds, the intermediaries have to unload the shares to the households. If it is very costly for the household to quickly absorb shares, the price must drop sharply. As explained in AG, in such a deleveraging process, the market-clearing equity prices are determined as follows:[5]

$$p_1^j = \frac{1}{2}\Big[\tilde{p}_1^b + \frac{1-\kappa}{\kappa}\frac{s_1^*}{\alpha_1} + \Big\{\Big(\tilde{p}_1^b + \frac{1-\kappa}{\kappa}\frac{s_1^*}{\alpha_1}\Big)^2 + 4\frac{d_1^b s_1^* + f_1^*}{\alpha_1}\Big\}^{\frac{1}{2}}\Big], \tag{28}$$

where $\tilde{p}_1^b \geq p_1^b$. If the following inequality holds, there exists unique equilibrium equity prices:

$$f_1^* \geq \frac{s_1^{*2}}{\alpha_1} - (\tilde{p}_1^b + d_1^b)s_1^*.$$

The financial intermediaries' portfolio Using inequality (27) and the equilibrium equity prices (28), we can determine the equilibrium portfolio of the intermediaries. If inequality (27) evaluated at the fundamental price, $p_1^b = \tilde{p}_1^b$, does not hold, the leveraged investment lowers the market value of the intermediaries. Then, intermediaries do not hold shares.

On the other hand, if inequality (27) evaluated at the fundamental price, $p_1^b = \tilde{p}_1^b$, holds, the intermediaries prefer leveraged investments. If both the period 1 leverage constraint and the period 0 dividend constraint bind, the intermediaries' portfolio can be written as:

$$s_1^{*bN} \equiv \frac{(p_0 + d_0)s_0^* + f_0^*}{p_0 - q_0\{d_1^b + (1-\kappa)\tilde{p}_1^b\}},$$
$$f_1^{*bN} \equiv -\frac{\{d_1^b + (1-\kappa)\tilde{p}_1^b\}\{(p_0 + d_0)s_0^* + f_0^*\}}{p_0 - q_0\{d_1^b + (1-\kappa)\tilde{p}_1^b\}}.$$

$\{s_1^{*bN}, f_1^{*bN}\}$ is the point of intersection of "Dividend constraint in period0" and "Leverage constraints in period 1 ($j = b$)" in Figure 2. If both the period 0 leverage and dividend constraints bind, the intermediaries' portfolio can be written as:

$$s_1^{*bB} \equiv \frac{1}{\kappa}\frac{(p_0 + d_0)s_0^* + f_0^*}{p_0},$$
$$f_1^{*bB} \equiv -\frac{1-\kappa}{\kappa}\frac{(p_0 + d_0)s_0^* + f_0^*}{q_0}.$$

$\{s_1^{*bB}, f_1^{*bB}\}$ is the point of intersection of "Dividend constraint in period0" and "Leverage constraints in period 0" in Figure 2. An equilibrium portfolio, $\{s_1^*, f_1^*\}$, exists in the region of $s_1^* \in [s_1^{*bN}, s_1^{*bB}]$ and $f_1^* \in [f_1^{*bN}, f_1^{*bB}]$.

If conditions (27) and (28) evaluated at $\{s_1^{*bB}, f_1^{*bB}\}$ hold, the full leverage portfolio is the equilibrium portfolio. If conditions (27) and (28) evaluated at $\{s_1^{*bB}, f_1^{*bB}\}$ do not hold, a portfolio and an equity price determined by (28) that equalize inequality (27) are the equilibrium portfolio and equity price.

Definition of competitive equilibrium Competitive equilibrium is defined as the set of consumption allocation, $\{c_0, c_{ti}^j\}$, the portfolio of households, the financial intermediaries, and the government, $\{s_0, s_{ti}^j, s_0^*, s_t^{*j}, s_0^g, s_t^{gj}\}$, $\{f_0, f_{ti}^j, f_0^*, f_t^{*j}, f_0^g, f_t^{gj}\}$, and the prices of assets $\{p_0, p_t^j\}$ and $\{q_0, q_t^j\}$, for $t = 1, 2$, $i = \{h, l\}$, and $j = \{n, b\}$, that satisfy the first-order conditions of households, solve the intermediaries' optimization problem, and are consistent with the market-clearing conditions.

[5] Details are described in the Appendix.

4 Computations

We now conduct numerical computations. Our simulation explores whether an equilibrium with binding leverage constraints exists and how CE policy affects households' consumption and wealth distribution. Our goal is not to try to match data, but to provide qualitative implications of the model.

4.1 Computational Algorithm

In state n, consumption equals endowments, $c_1^n = y_1^n$ and $c_2^n = y_2^n$. On the other hand, in state b, consumption is heterogeneous. We define consumption shares as follows: $\theta_1 \equiv \frac{(1-\psi)c_{1h}^b}{y_1^b}$ and $\theta_2 \equiv \frac{(1-\psi)c_{2h}^b}{y_2^b}$. Consumption can be written as $c_{th} = \theta_t \frac{y_t^b}{1-\psi}$. Using consumption shares, we can compute equilibrium as follows:

1. Given the subjective time discount factor, β, the coefficient of relative risk aversion, γ, and endowments, $\{e_0,\ d_0, e_{t,i}^j, d_t^j\}$ for $t = 1, 2$, $j = n, b$, and $i = h, l$, we estimate the consumption shares θ_1 and θ_2. Note that consumption shares satisfy the goods market-clearing conditions.
2. Using estimated consumption shares and the households' first-order conditions, we compute SDFs and asset prices, $\{p_0, p_1^n, \tilde{p}_1^b\}$ and $\{q_0, q_t^j\}$.
3. Using (27) and (28), we compute the optimal portfolio of intermediaries and equilibrium equity prices, p_1^b.
4. Using the households' first-order conditions, we compute households' holdings of equities and risk-free bonds.
5. Check periods 1 and 2 budget constraints of households. If budget constraints do not clear, go to step 1.

4.2 Calibration Parameters

Table 1 presents the basic calibration parameters. We specify the time discount factor, β, 0.9, the degree of relative risk aversion, γ, 5.0, the probability of aggregate bad state, ϕ, 0.5, and the low idiosyncratic state, ψ, 0.5. Labor endowments are 9.0, but idiosyncratic shocks drive the endowment of period 1 to 0.675. Dividends from Lucas trees are 1.0, but when bad states emerge, it becomes 0.01 in period 1, but it recovers to 1.0 in period 2.

For computational tractability, we follow AG's specification of the adjustment cost of equity transactions: $a_+ = ap_1^b u'(c_{1h})$, $a_- = ap_1^b u'(c_{1l})$, and $a = 1.0$. In this case, the SDF and equity prices can be written as $\beta_2^b = (1-\psi)\beta\frac{u'(c_{2h}^b)}{u'(c_{1h}^b)} + \psi\beta\frac{u'(c_{2l}^b)}{u'(c_{1l}^b)}$ and $p_1^b = \frac{\kappa\beta_2^b d_2^b + a(d_1^b s_1^* + f_1^*)}{(1-ag_1^b)\kappa - a(1-\kappa)s_1^*}$. In this case, $\frac{\beta_2^j d_2^j}{a(s_1^* - g)+1} > -\frac{d_1^* s_1^* + f_1^*}{s_1^*}$ ensures the uniqueness of equilibrium equity prices.

4.3 Results

4.3.1 The Effects of Credit-easing Policy

Panels A and B of Figure 3 demonstrate how a CE policy affects high- and low-income households and their average welfare level, as measured by certainty equivalent consumption. In particular, Panel A plots percentage changes of the certainty equivalent consumption level from the level where there is no government intervention to the level where the government buys 20% of total shares. On the other hand, Panel B plots the percentage declines in equity prices from the fundamental value. As Figure 3 shows, CE policies raise average- and low-income households' welfare and mitigate price drops. However, those lower the welfare of high-income households.

Tables 2, 3, 4, 5, and 6 show the details in portfolio transactions when the bad state $j = b$ occurs. Tables 2 and 3 describe the portfolio transaction behavior of households and the intermediaries in the realization of state b, where $\kappa = 0.75$ and there are no government interventions. For example, the intermediaries buy 11.8% of shares and issue safe assets of 0.016, which are held as precautionary savings by households. In period 1, when the bad state occurs, the equity prices become 0.279 and the realized values of shares and dividends are 0.255. Low-income households sell shares to the high-income households in order to raise their current consumption, which amounts to $0.115 = 0.255 - 0.140$. The sum of sales of shares and the precautionary savings, $0.131 = 0.115 + 0.016$, equals the difference between income and consumption, $0.132 = 6.882 - 6.750$. High-income households raise their equity positions from 0.882 to 1.330.

On the other hand, Tables 4, 5, and 6 report the portfolio transactions when the government purchases 20% of shares, that is, the case of CE policy. In this case, the intermediaries buy 11.8% of equities and issue safe assets of 0.017, which are held as precautionary savings by households. In period 1, when the bad state occurs, the equity prices become 0.345 and the realized value of equity share and dividends is 0.313. Due to the CE policy, the falls in equity prices in the bad state are mitigated. Low-income households sell shares to the high-income households in order to raise their consumption, which amounts to $0.168 = 0.313 - 0.145$. The sum of sales in shares and precautionary savings, $0.185 = 0.168 + 0.017$, is equal to the difference between the income and consumption, $0.186 = 6.936 - 6.750$. High-income households raise their equity positions from 0.882 to 0.987.

As described above, because the government buys shares and sells risk-free bonds, CE policy raises the price of shares and lowers the price of risk-free bonds. In other words, returns on shares, which have relatively high returns, decrease and returns on risk-free bonds, which have relatively low returns, increase. The low-income households favor CE policy because a rise in the price of shares at the beginning of period 1 raises the insurance value of their shares. However, the high-income households dislike the policy because CE policy worsens the yields on their leveraged investment opportunities.

This result shows that CE policy generates conflicts of interest between high- and low-income households, although it may raise average welfare. Because it mitigates falls in equity prices during recessions, CE policy improves the welfare of low-income households, who have to sell equities to raise their current consumption. However, it may lower the welfare of high-income households, who want to buy equities to raise their future consumption.

5 Conclusion

In this paper, we have extended the model constructed by AG into a three-period general equilibrium model with incomplete markets and some financial frictions. Although the financial intermediaries have no incentive to construct leverage positions in equilibrium in the original AG framework, we demonstrate that the financial intermediaries have an incentive to maintain leverage positions under specific financial constraints.

The main shortcoming of the model is that our assumptions about financial constraints are exogenous. In particular, there is no consensus about the adjustment cost and its asymmetry of households' equity trading. However, in the infinite horizon environment, we may not need such asymmetric adjustment costs. The asymmetry in adjustment costs is necessary for the intertemporal arbitrage opportunity but not for the interasset arbitrage. The interasset arbitrage may generate intermediaries' incentive to construct leverage positions in a stationary equilibrium in the infinite horizon setups. Therefore, it seems worthwhile to extend the model to the Bewly (1983), Hugget (1993), and Aiyagari (1994) type of infinite horizon incomplete market economies.

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Appendix

Proof of Lemma 1

From the intermediaries' optimization problem, we can derive the following first-order conditions:

$$M_1^j(M_2^j d_2^j - p_1^j) - \{\lambda_1^j - \mu_1^j(1-\kappa)\}p_1^j = 0, \tag{29}$$
$$M_1^j(M_2^j - q_1^j) - (\lambda_1^j - \mu_1^j)q_1^j = 0, \tag{30}$$
$$-p_0 + (1-\phi)M_1^b(d_1^n + p_1^n) + \phi M_1^b(d_1^b + p_1^b) = 0$$
$$\{1 + \lambda_0 - \mu_0(1-\kappa)\}p_0$$
$$= (1-\phi)(\lambda_1^n + M_1^b)(d_1^n + p_1^n) + \phi(\lambda_1^b + M_1^b)(d_1^b + p_1^b), \tag{31}$$
$$\{1 + \lambda_0 - \mu_0\}q_0 = (1-\phi)(\lambda_1^n + M_1^b) + \phi(\lambda_1^b + M_1^b), \tag{32}$$

where λ_t^j denotes Lagrangian multipliers on the dividend constraints and μ_t^j denotes multipliers on the leverage constraints. We can analyze s_2^{*j} and f_2^{*j} using first-order conditions, (29) and (30), leverage constraints in period 1, (22), and dividend constraints in period 1, (23).

In state n, because $M_2^n = q_1^n$, equation (30) implies $\lambda_1^n = \mu_1^n$ and equation (30) implies $M_1^n(M_2^n d_2^n - p_1^n) = \{\lambda_1^n - \mu_1^n(1-\kappa)\}p_1^n > 0$. Then, while $p_1^n = q_1^n d_2^n$ implies $\lambda_1^n = \mu_1^n = 0$, $p_1^n < q_1^n d_2^n$ implies $\lambda_1^n = \mu_1^n > 0$. That is, if equity prices, p_1^n, are lower than the level of the fundamental prices, $q_1^n d_2^n$, the intermediaries gain profits from leverage trading. Then, both dividend and leverage constraints are binding.

In state b, the above arguments can be applied in the case of $M_2^b = q_1^b$. However, as discussed in AG and 3.4 in this paper, because $p_1^b < q_1^b d_2^b$ in equilibrium, both constraints are always binding. On the other hand, if $M_2^b < q_1^b$, for example, $M_2^b = \beta_2^b$, (30) implies $(\mu_1^b - \lambda_1^b) = M_1^b(q_1^b - M_2^b) > 0$, that is, $\mu_1^b > \lambda_1^b$ and $\lambda_1^b = \mu_1^b = 0$ are not feasible. Then, if $p_1^b = M_2^b d_2^b$, $\{\lambda_1^b - \mu_1^b(1-\kappa)\}p_1^b = 0$. In this case, $\lambda_1^b > 0$ and $\mu_1^b > 0$, that is, both constraints bind. Otherwise, if $p_1^b < M_2^b d_2^b$, $(\lambda_1^j - \mu_1^j(1-\kappa)) > 0$. In this case, $\lambda_1^b > 0$ and $\mu_1^b > 0$. That is, both dividend and leverage constraints are binding.

From this discussion, in state n, $p_1^n = q_1^n d_2^n$ implies that constraints are slack, while $p_1^n < q_1^n d_2^n$ implies that constraints are binding. On the other hand, in state b, both constraints are always binding.

Proof of Lemma 2

Suppose that $\beta_2^b \equiv \hat{\beta}_2^b + \delta$ $(\delta > 0)$. We can demonstrate that $(1-\kappa)\frac{p_0}{q_0} > d_1^b + (1-\kappa)\tilde{p}_1^b \cap (1-\kappa)\frac{p_0}{q_0} > d_1^n + (1-\kappa)\tilde{p}_1^n$ contradicts. If both inequalities hold, the following inequality has to hold: $\phi\beta_1^b(1-\kappa)\frac{p_0}{q_0} > \phi\beta_1^b\big\{d_1^b + (1-\kappa)(\beta_2^b d_2^b + \tilde{g}^b)\big\} \cap (1-\phi)\beta_1^n(1-\kappa)\frac{p_0}{q_0} > (1-\phi)\beta_1^n\big\{d_1^b + (1-\kappa)(\beta_2^n d_2^n + \tilde{g}^n)\big\}$. However, sums of the inequalities imply that $0 > (1-\phi)\beta_1^n\big(\kappa d_1^n + (1-\kappa)\tilde{g}^n\big) + \phi\beta_1^b\big(\kappa d_1^b + (1-\kappa)\tilde{g}^b\big) + (1-\kappa)\phi\beta_1^b\delta d_2^b$. Because the right-hand side is positive, the above inequality is the contradiction.

Note that if we do not assume inequality (26), $(1-\kappa)\frac{p_0}{q_0} < d_1^b + (1-\kappa)\tilde{p}_1^b \cap (1-\kappa)\frac{p_0}{q_0} < d_1^n + (1-\kappa)\tilde{p}_1^n$ does not contradict.

Proof of Lemma 3

From Lemma 1 and Assumption 1, the leverage constraints cannot bind in state n, $p_1^n = \beta_2^n d_2^n$, $\lambda_1^n = \mu_1^n = 0$, $\lambda_1^b > 0$, and $\mu_1^b > 0$. Equation (32) can be reformulated as

$q_0(\lambda_0 - \mu_0) = \phi\lambda_1^b > 0$. Because $\mu_0 \geq 0$, $\lambda_0 > 0$, the dividend constraint is always binding in period 0.

Proof of Proposition 1

The period 0 objective function of the intermediaries is written as follows:

$$\begin{aligned}\hat{V}_{0b}^* \equiv\ & (p_0 + d_0)s_0^* + f_0^* - p_0 s_1^* - q_0 f_1^* \\ & +(1-\phi)\beta_1^n\Big\{(d_1^n + \beta_2^n d_2^n)s_1^* + f_1^*\Big\} \\ & +\phi\beta_1^b M_2^b\Big\{\frac{d_2^b}{p_1^b} - (1-\kappa)\frac{1}{q_1^b}\Big\}\frac{1}{\kappa}\Big\{(p_1^b + d_1^b)s_1^* + f_1^*\Big\}.\end{aligned}$$

Suppose that Lemma 3 holds. If so, the period 0 dividend constraints bind. We examine whether the intermediaries should raise their leveraged positions or not. Using:

$$\begin{aligned}\frac{\partial \hat{V}_{0,b}^*}{\partial f_1^*} &= \phi\beta_1^b\Big[M_2^b\Big\{\frac{1}{\kappa}\frac{d_2^b}{p_1^b} - \Big(\frac{1}{\kappa} - 1\Big)\frac{1}{q_1^b}\Big\} - 1\Big], \\ \frac{\partial \hat{V}_{0,b}^*}{\partial s_1^*} &= \phi\beta_1^b\Big[\frac{M_2^b}{\kappa}\Big\{\frac{1}{\beta_2^b} - (1-\kappa)\frac{1}{q_1^b}\Big\}(d_1^b + p_1^b) - (d_1^b + \hat{\beta}_2^b d_2^b)\Big],\end{aligned}$$

$$\frac{\partial \hat{V}_{0,b}^*}{\partial s_1^*} - \frac{p_0}{q_0}\frac{\partial \hat{V}_{0,b}^*}{\partial f_1^*} = \phi\beta_1^b\Big[M_2^b\Big\{\frac{1}{\kappa}\frac{d_2^b}{p_1^b} - \Big(\frac{1}{\kappa} - 1\Big)\frac{1}{q_1^b}\Big\}\big(d_1^b + p_1^b - \frac{p_0}{q_0}\big) - \big(d_1^b + \beta_2^b d_2^b - \frac{p_0}{q_0}\big)\Big].$$

After some manipulations, we acquire equation (27).

If $\frac{\partial \hat{V}_{0,b}^*}{\partial s_1^*} - \frac{p_0}{q_0}\frac{\partial \hat{V}_{0,b}^*}{\partial f_1^*} > 0$, the intermediaries raise their leverage positions even though they have to deleverage ex post.

Determination of Equity Prices under the Binding Leverage Constraints

Following AG, we explain the determination of equity prices under leverage binding. If leverage constraints are binding under the fundamental equity price, $\tilde{p}_1$, equation (10) implies that equity prices, p_1^j, and the equity share of the intermediaries, s_2^{*j}, are determined by the following equation:

$$s_2^{*j,D} = \frac{1}{\kappa p_i^j}\big\{(p_1^j + d_1^j)s_1^* + f_1^*\big\}. \tag{33}$$

This is the equity demand function of the financial intermediaries. This can be written as $p_1^j = -\frac{d_1^j s_1^* + f_1^*}{s_1^* - \kappa s_2^{*j}}$. Note that $d_1^j s_1^* + f_1 < 0$. For the leverage constraint to bind, the intermediaries must be unable to cover their debt obligation simply with dividend earnings. Thus, s_2^{*j} varies positively with p_1^j. In addition, the intermediaries reduce their shares, $s_2^{*j} \leq s_1^{*1}$, while $p_1^j = \tilde{p}_1^j$ and $s_2^{*j} = s_1^{*1}$ hold in the case where the leverage constraint does not bind.

From households' first-order conditions, (16) and (17), and the market-clearing conditions, the equity supply function of the intermediaries can be written as follows:

$$s_2^{*j,S} = s_1^* - g^j - \Big(\alpha_2 d_2^b - \alpha_1 p_1^b\Big), \tag{34}$$

where $\alpha_2 \equiv \beta \frac{(1-\psi)a_- u'(c^b_{2h}) + \psi a_+ u'(c^b_{2l})}{a_+ a_-}$, $g^j \equiv (s^{gj}_2 - s^g_1)$. This can be written as $p^b_1 = \frac{s^{*b,S}_2}{\alpha_1} + \tilde{p}^b_1 - \frac{s^*_1}{\alpha_1}$. Equity supply curves are upward sloping and take the value of $s^{*j}_2 = s^*_1$ when $p^j_1 = \tilde{p}^j_1$.

If inequality $\tilde{p}^b_1 - \frac{s^*_1}{\alpha_1} > -\frac{d^*_1 s^*_1 + f^*_1}{s^*_1}$ holds, equilibrium equity prices exist uniquely (see Figure 3). Equilibrium equity prices are given by:

$$p^j_1 = \frac{1}{2}\Big[\tilde{p}^b_1 + \frac{1-\kappa}{\kappa}\frac{s^*_1}{\alpha_1} + \Big\{\Big(\tilde{p}^b_1 + \frac{1-\kappa}{\kappa}\frac{s^*_1}{\alpha_1}\Big)^2 + 4\frac{d^b_1 s^*_1 + f^*_1}{\alpha_1}\Big\}^{\frac{1}{2}}\Big]. \tag{35}$$

Table 1: The basic calibration parameters

time discount factor	β	0.9	endowments	e_0	9.00	dividends	d_0	1.00
relative risk aversion	γ	5.0		e_1^n	9.00		d_1^n	1.00
equity adjustment cost	a	1.0		e_2^n	9.00		d_2^n	1.00
aggregate prob.	ϕ	0.5		e_1^{bh}	9.00		d_1^b	0.01
idiosyncratic prob.	ψ	0.5		e_2^{bh}	9.00		d_2^b	1.00
leverage ratio	κ	0.75		e_1^{bl}	6.75			
				e_2^{bl}	9.00			

Table 2: Portfolio transaction behavior in the realization of state $j = b$; (κ=0.75, no government intervention)

	e_t	$(p_t + d_t)s_t$	f_t	D_t	c_t	s_{t+1}	$p_t s_{t+1}$	f_{t+1}	$q_t f_{t+1}$	T_t
				high income household						
time 0	9.000	2.175	0.000	0.000	10.000	0.882	1.137	0.016	0.038	0.000
time 1	9.000	0.255	0.016	0.000	8.888	1.330	0.372	0.029	0.012	0.000
time 2	9.000	1.330	0.029	0.070	10.429					
				low income household						
time 0	9.000	2.175	0.000	0.000	10.000	0.882	1.137	0.016	0.038	0.000
time 1	6.750	0.255	0.016	0.000	6.882	0.501	0.140	0.000	0.000	0.000
time 2	9.000	0.501	0.000	0.070	9.571	0.000				

Note: e_t denotes endowments, $(p_t + d_t)s_t$ denotes realized tree value, f_t denote safe assets receipt, D_t denotes the intermediaries' dividend payments, c_t denotes consumption, $p_t s_{t+1}$ denotes invested trees value, f_{t+1} denotes invested safe asset, $q_t f_{t+1}$ denotes invested safe asset values, and T_t denotes Tax payments.

Table 3: The intermediaries portfolio transaction behavior in the realization of state $j = b$; (κ=0.75, no government intervention)

	D_t	$(p_t + d_t)s_t^*$	f_t^*	s_{t+1}^*	$p_t s_{t+1}^*$	f_{t+1}^*	$q_t f_{t+1}^*$	p_t	q_t
time 0	0.000	0.114	0.000	0.118	0.153	-0.016	-0.038	1.290	2.313
time 1	0.000	0.034	-0.016	0.085	0.024	-0.015	-0.006	0.279	0.405
time 2	0.070	0.085	-0.015						

Note: D_t denotes the intermediaries' dividend payments, $(p_t + d_t)s_t^*$ denotes realized tree value, f_t^* denote safe assets receipt, s_t^* denotes invested trees, $p_t s_{t+1}^*$ denotes invested trees value, f_{t+1}^* denotes invested safe asset, and $q_t f_{t+1}^*$ denotes invested safe asset values.

Table 4: Household portfolio transaction behavior in the realization of state $j = b$; (κ=0.75, $s_2^{gb} = 0.2$)

	e_t	$(p_t + d_t)s_t$	f_t	D_t	c_t	s_{t+1}	$p_t s_{t+1}$	f_{t+1}	$q_t f_{t+1}$	T_t
					high income household					
time 0	9.000	2.178	0.000	0.000	10.000	0.882	1.140	0.017	0.038	0.000
time 1	9.000	0.313	0.017	0.000	8.834	0.987	0.341	0.406	0.155	0.000
time 2	9.000	0.987	0.406	0.075	10.487					-0.019
					low income household					
time 0	9.000	2.178	0.000	0.000	10.000	0.882	1.140	0.017	0.038	0.000
time 1	6.750	0.313	0.017	0.000	6.936	0.419	0.145	0.000	0.000	0.000
time 2	9.000	0.419	0.000	0.075	9.513					-0.019

Note: e_t denotes endowments, $(p_t + d_t)s_t$ denotes realized tree value, f_t denote safe assets receipt, D_t denotes the intermediaries' dividend payments, c_t denotes consumption, $p_t s_{t+1}$ denotes invested trees value, f_{t+1} denotes invested safe asset, $q_t f_{t+1}$ denotes invested safe asset values, and T_t denotes Tax payments.

Table 5: The intermediaries portfolio transaction behavior in the realization of state $j = b$; (κ=0.75, $s_2^{gb} = 0.2$)

	D_t	$(p_t + d_t)s_t^*$	f_t^*	s_{t+1}^*	$p_t s_{t+1}^*$	f_{t+1}^*	$q_t f_{t+1}^*$	p_t	q_t
time 0	0.000	0.115	0.000	0.118	0.153	-0.017	-0.038	1.293	2.270
time 1	0.000	0.042	-0.017	0.097	0.034	-0.022	-0.008	0.345	0.382
time 2	0.075	0.097	-0.022						

Note: D_t denotes the intermediaries' dividend payments, $(p_t + d_t)s_t^*$ denotes realized tree value, f_t^* denote safe assets receipt, s_t^* denotes invested trees, $p_t s_{t+1}^*$ denotes invested trees value, f_{t+1}^* denotes invested safe asset, and $q_t f_{t+1}^*$ denotes invested safe asset values.

Table 6: The government portfolio transaction behavior in the realization of state $j = b$; (κ=0.75, $s_2^{gb} = 0.2$)

	T_t	$(p_t + d_t)s_t^g$	f_t^g	s_{t+1}^g	$p_t s_{t+1}^g$	f_{t+1}^g	$q_t f_{t+1}^g$
time 1	0.000	0.000	0.000	0.200	0.069	-0.181	-0.069
time 2	-0.019	0.200	-0.181				

Note: D_t denotes the intermediaries' dividend payments, $(p_t + d_t)s_t^i$ denotes realized tree value, f_t^i denote safe assets receipt, s_t^i denotes invested trees, $p_t s_{t+1}^i$ denotes invested trees value, f_{t+1}^i denotes invested safe asset, and $q_t f_{t+1}^i$ denotes invested safe asset values.

Figure 1: Feasible Portfolio Region in Period 1

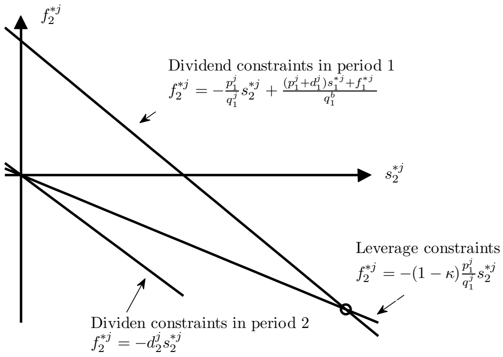

Figure 2: Determination of Equilibrium Equity Prices in Period 1

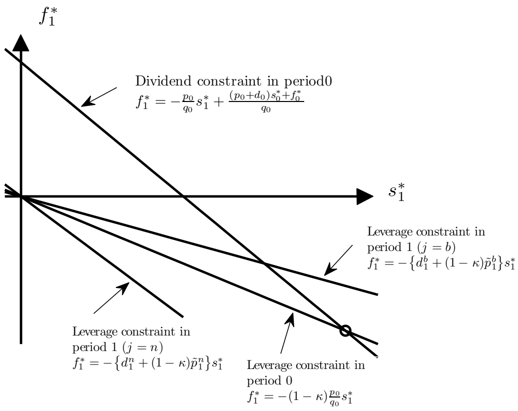

Figure 3: The Effects of Credit-easing Policy on the Welfare (Measured by the Certainty Equivalent Consumption) and Equity Price (%)

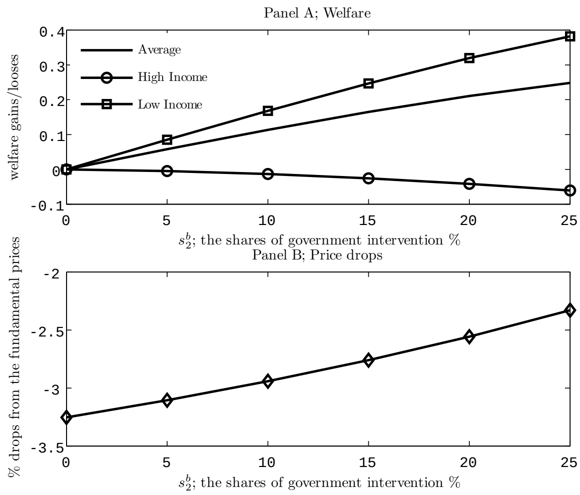

Figure 4: Feasible Portfolio and Leverage Biding Portfolio in Period 0