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A NOTE ON DUTCH AUCTIONS WITH TIME CREDITS

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A German internet auction site creates a new auction procedure as an application for iPhone. It is a Dutch auction with time credit. First, the seller announces a time credit fee rate. Next, each bidder purchases time credit and pays corresponding participation cost. Lastly, a Dutch auction starts and those who buy positive time credit can submit bids. A bidder cannot stay in an auction longer than the amount of time credit he purchased in advance. We can solve the model explicitly for uniform distribution functions and show that the equilibrium time credit fee rate is positive. We cannot collect field data because it is not approved by Apple yet. Therefore, we ran an experiment and found that we could reject the null hypothesis of the seller's revenues being the same between a Dutch auction with zero and positive time credit fee rates. The seller 's revenues from Dutch auctions with time credit are close to the one from the optimal auction mechanism, but our data shows the rate of time credit does not affect the seller 's revenue as long as it is strictly positive, which contradicts to the theoretical prediction.

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A Note on Dutch Auctions with Time Credit*

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A German internet auction site creates a new auction procedure as an application for iPhone. It is a Dutch auction with time credit. First, the seller announces a time credit fee rate. Next, each bidder purchases time credit and pays corresponding participation cost. Lastly, a Dutch auction starts and those who buy positive time credit can submit bids. A bidder cannot stay in an auction longer than the amount of time credit he purchased in advance. We can solve the model explicitly for uniform distribution functions and show that the equilibrium time credit fee rate is positive. We cannot collect field data because it is not approved by Apple yet. Therefore, we ran an experiment and found that we could reject the null hypothesis of the seller's revenues being the same between a Dutch auction with zero and positive time credit fee rates. The seller's revenues from Dutch auctions with time credit are close to the one from the optimal auction mechanism, but our data shows the rate of time credit does not affect the seller's revenue as long as it is strictly positive, which contradicts to the theoretical prediction.

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1. Introduction

A German internet auction site company, SevenSnap plans to introduce a new type of auction procedure as an application for iPhone. It is a Dutch auction with time credit and has two different features from other popular auction procedures prevailed on internet. First, it is a Dutch auction instead of an English auction. Popular internet auction sites such as eBay and Yahoo! use variants of an English auction and a Dutch auction is rare in online auctions. Secondly, a bidder has to purchase "time credit" in advance of participating in a Dutch auction. Although Swoop already introduced pay-per-bid auction procedure, SevenSnap's time credit works quite differently from pay-per-bid procedure. In a pay-per-bid auction, a bidder has to pay fixed amount of fee every time he submits a new bid. SevenSnap's auction works as follows. First, SevenSnap announces how much a bidder has to pay per unit of time to stay in a Dutch auction, which we call time credit fee rate from now on. Next, a bidder observes the time credit fee rate and decides maximum amount of time to stay in an auction and pays for it, which is called "time credit" and the purchased amount of time credit is private information. Lastly, an actual auction procedure begins and those who buy positive time credit can participate in an auction. SevenSnap denote this as those who purchase a positive amount of time credit is allowed to "enter a Snap room." Therefore, time credit is sunk cost for a bidder in an auction stage, while it is not in a pay-per-bid auction. SevenSnap uses a Dutch auction in an auction stage and a price decreases over time until a bidder declares for the first time that he will purchase the item at the current price. Then, the auction is over and the item is sold to that bidder at the price the winner claims to purchase the item. It is worth noting that a bidder cannot stay in an auction longer than the amount of time credit he purchased in advance. Furthermore, he cannot buy additional amount of time credit once he enters into a Snap room. Nor, he cannot get refund if he loses the auction or he becomes the winner with remaining amount of time credit. He cannot carry over unused amount of time credit in the current auction to the future auction. To analyze the role of time credit, we construct the simplest model where the risk-neutral seller auctions off a single unit of indivisible item to n risk-neutral potential bidders by a Dutch auction with time credit. First, the seller announces a time credit fee rate. After observing the time credit fee rate, a bidder purchases any amount of time credit including zero which means he decides not to participate in an auction and the amount of purchased time credit is private information of a bidder. For simplicity, we assume that bidders' values of the item are i.i.d. draws from a differentiable probability distribution function. Many online selling companies add unique elements to traditional auction procedures such as buy-it-now option, an automatic bidding, an automatic extension of the closing time after a new bid, secret reserve price, feedback system after trade, and so on. Among them, we focus on the theoretical analysis of selling mechanisms with exogenous/endogenous participation costs. There are significant amount of previous works which examine auctions with endogenous entry and they are divided into two strands. A strand of literature assumes that a bidder learns his value of the item before making entry decision. Green and Laffont (1984) show the existence of a symmetric equilibrium with uniform distribution in a second price auction under the assumption that both the value of the item and participation cost are private information. Gal et al. (2007) extend their result to more generalized probability distribution functions, but still focus on a symmetric equilibrium. Cao and Tian (2008) consider a sealed-bid second-price auction with differentiated participation costs and show that a counter-cyclical equilibrium exists where a bidder with higher participation cost uses a lower cut-off value for entry when the probability distribution function is strictly convex and the differences of the participation costs are small. Due to tractability, there a few works examining first price auctions with participation costs. Menezes and Monteio (2000) analyze sealed-bid first and second price auctions with exogenous entry cost. A bidder knows his value of the item before making entry decision to an auction. They show that a bidder participates in an auction only when his value of the item is equal or greater than the cut-off value. They also show that the revenue equivalence between a sealed-bid first and a second price auctions holds. Cao and Tian (2010) consider a sealed-bid first-price auction and find that there exists only a symmetric equilibrium if underlying probability distribution function is inelastic, but an asymmetric equilibrium exists if underlying probability distribution function is elastic. Stageman (1996) studies ex-ante efficient auctions and shows that it is characterized by cut-off values and it can be asymmetric. He further shows that the symmetric equilibrium of a second price auction is always efficient whereas a first price auction might not have an efficient equilibrium. Celik and Yilankaya (2010) consider the same model as that of Stageman, but their focus is on characterizing the optimal auction. In line with the results obtained by Stageman, they show that the optimal auction is characterized by cut-off values and it can be asymmetric. Another strand of literature reverses the order of a bidder learning his value of the item and his making participation decision. Now, a bidder has to make participation decision before observing his value of the item. This assumption makes it possible to combine participation costs and information acquisition costs. McAfee and McMillan (1987) analyze auctions where a bidder has to make entry decision before observing his value of the item and show that the optimal number of bidders enters in a sealed-bid first-price auction. Engelbrecht-Wiggans (1993) restricts a bidder's entry decision to a pure strategy and characterizes the optimal auction. Levin and Smith (1994) allow a bidder to use a mixed strategy for an entry decision. Under the assumption that a bidder can observe the actual numbers of participants in an auction, they show that a bidder uses a mixed strategy for entry with strictly positive probability of entry and the revenue equivalence theorem holds. Our model has several differences from the literature. First, a bidder decides the amount of his participation cost through buying time credit. Therefore, the size of participation cost is not exogenously given as the literature assumes. Recently, many unique auction procedures are introduced mainly by on-line auction site companies and there are several papers which consider specific elements of auctions observed in the practical world. Gallice (2010) analyzes a price reveal auction which is a Dutch auction where the current price of the item is not observable to a bidder. A bidder has to incur cost every time to observe the current price. In addition, the price decreases only when some bidder decides to observe the current price and the price does not decrease at all if no one chooses to observe it. Under the assumption that the amount of the decrease of the price is smaller than the cost a bidder has to incur to observe the current price, he shows that no bidder participate into such kind of auctions. Augenblick (2009), Hinnosaar (2010), and Platt *et. al.* (2010) analyze a penny auction which is a variant of an English auction. In a penny auction, the price increases by predetermined amount (a penny) when someone submits a bid and restarts a public countdown. The winner is the one who submits the highest bid before the countdown expires. There are common factors and differences in a Dutch auction with time credit and penny auctions. Although they both introduce endogenous participation cost to auctions, SevenSnap's auction is quite different from penny auctions. First, of course, SevenSnap's auction is a variant of a Dutch auction and a penny auction is a variant of an English auction. More than that, in SevenSnap's auction, a bidder has to purchase time credit in advance of participating into a Dutch auction and cannot purchase additional amount of time credit once a Dutch auction starts. Therefore, money spent on time credit is sunk cost when a Dutch auction starts. On the contrary, in penny auctions, a bidder has to pay predetermined amount of fee every time he submits a bit. So, total amount of participation cost for a bidder is not sunk cost, but is determined in an English auction.

2. The Model

We construct the simplest possible model to examine the role of time credit on a bidder's equilibrium behavior. The risk-neutral seller tries to sell a single indivisible item through a Dutch auction with time credit. The seller's reserve price is zero. There are n risk-neutral potential bidders and we denote the set of potential bidders as N. Bidders' values of the items, v_is ($^\forall i \in N$) are i.i.d. draws from the differentiable probability distribution function $F(\cdot)$ who has density $f(\cdot)$ and whose support is $[0, \overline{v}]$. We impose PIV assumption where bidder i ($^\forall i \in N$) knows his realized value of the item v_i, but he only knows that $v_j, j \neq i$ is i.i.d. draws from $F(\cdot)$.
The timing of the game is as follows. The game consists of three periods, $s = 0,1,$ and 2. In period 0, the seller announces time credit fee rate c. Once she sets c, it is irrevocable and she has to commit to it for the rest of the game. In period 1, bidder i $\left(^\forall i \in N\right)$ observes c and decides how much time credit $t_i(v_i) \geq 0$ he purchases, and pays $t_i(v_i)c$ to the seller. Those bidders who choose $t_i(v_i) > 0$ enter the Dutch auction stage held in period 2. In the Dutch auction, the price starts from $\overline{v}$ and decreases at the constant rate until a bidder declares for the first time that he will purchase the item at the current price. Then, the auction is over and the item is sold to that bidder at that price. Note that bidder i cannot stay in an auction longer than $t_i(v_i)$.
If he is the first to announce his willingness of purchase of the item at time t where $t > t_i$ is the case, his announcement is ineffective. On the other hand, announcing his purchase decision at any $t \leq t_i$ is effective, but he can minimize his payment by choosing $t = t_i$ without affecting the outcome of the auction. From this, one thinks that the time credit constraint always binds and we can show later that this guess is correct. Literature examines auctions with exogenously given participation cost. On the contrary, a bidder endogenously chooses his participation cost in a Dutch auction with time credit.
Figure1 summarizes the timing of a Dutch auction with time credit.

Figure1. (timing of the game)

s=0 | s=1 | s=2

The seller announces c.

Bidder i purchases t_i and pays $t_i c$ to the seller.

The Dutch auction determines the winner and the payment.

We use the Bayesian perfect equilibrium as our equilibrium concept and apply backward induction. Note that the second period game after (some) bidders pay for time credits is no different from any Dutch auctions and hence the equivalent to the first-price sealed- bid auction; however, there might exist a cut-off value $\underline{v}$ and a bidder with $v_i < \underline{v}$ $({}^{\forall}i \in N)$ does not purchase positive amount of time credit and accordingly never participates in a Dutch auction. We show this is actually the case, A bidder with $v_i \geq \underline{v}$ $\left({}^{\forall}i \in N\right)$ purchases $t_i(v_i) \geq 0$. Especially, a bidder with $v_i = \underline{v}$ is indifferent between purchasing $t_i(v_i) > 0$ and participating in a Dutch auction and purchasing $t_i(v_i) = 0$ and not participating in a Dutch auction. It implies that the expected revenue of a bidder with $v_i = \underline{v}$ is zero at equilibrium. Further, note that without time credit, a bidder's equilibrium bidding function of a Dutch auction is the same as that of sealed-bid first-price auction. Therefore, we only need to show that there exists a cut-off value $\underline{v}$ and a bidder with $v_i < \underline{v}$ $({}^{\forall}i \in N)$never purchases positive amount of time credit and accordingly never participates in a sealed-bid first-price auction.

We prove $\underline{v} > 0$ by contradiction. Suppose $\underline{v} = 0$ which means everyone participates in a sealed-bid first-price auction. Then, the equilibrium bidding strategy of bidder i $({}^{\forall}i \in N)$is equivalent to that of a sealed-bid first-price auction and is obtained by solving the following problem.

$$Max_{\widetilde{v_i}} \; E\pi_i\,(v_i, \widetilde{v_i}) = \left(v_i - b(\widetilde{v_i})\right)F(\widetilde{v_i})^{n-1} \qquad \text{...}(1)$$

Therefore, the equilibrium bidding function $b(v_i)$ is characterized as

$$b(v_i) = \frac{\int_0^{v_i} w(n-1)F(w)^{n-2}f(w)dw}{F(v_i)^{n-1}} \qquad \text{...}(2)$$

However, for $v_i \cong 0$,

$$E\pi_i\,(v_i, v_i) = \left(v_i - \frac{\int_0^{v_i} w(n-1)F(w)^{n-2}f(w)dw}{F(v_i)^{n-1}}\right)F(v_i)^{n-1} - t_i c{<}0 \qquad \text{...}(3)$$

This is because the first term of (3) approaches to 0 as v_i approaches to 0, but the second term approaches to $\overline{v}c$ because $b(v_i)$ approaches to 0 as v_i approaches to 0 and bidder i's time credit t_i should approach to $\overline{v}$ to make $b(v_i) \cong 0$ effective. The result is summarized in the following lemma. Therefore, $\underline{v} = 0$ does not hold and we obtain the following lemma.

Lemma1. (an existence of $\underline{v} > 0$)

There exists a cut-off value $\underline{v} > 0$ where a bidder with $v_i < \underline{v}$ chooses $t_i(v_i) = 0$, a bidder with $v_i > \underline{v}$ chooses $t_i(v_i) > 0$, and a bidder with $v_i = \underline{v}$ is indifferent between

$t_i(v_i) = 0$ and $t_i(v_i) > 0$.

Next, we show by contradiction that the time credit constraint always binds in the sense that $t_i(v_i) = \bar{v} - b(v_i)$ holds. Suppose $t_i(v_i) = \bar{v} - b(v_i)$ does not hold. Then, $b(v_i) > \bar{v} - t_i$ holds because of the definition of time credit. Then, bidder i can save $\big(b(v_i) - \bar{v} - t_i(v_i)\big)c > 0$ by choosing $\widetilde{t_i}(v_i) = \bar{v} - b(v_i)$ instead of t_i without affecting the outcome of the auction.

Lemma2. ($t_i(v_i) = \bar{v} - b(v_i)$, the time credit constraint binds)
The time credit constraint always binds and $t_i = \bar{v} - b(v_i)$ holds.

We use backward induction to characterize the equilibrium and starts from solving a bidder's problem in period2, but lemma 2 combines the problem of determining the equilibrium bidding function in period 2 and the problem of determining the equilibrium time credit in period1. Accordingly, bidder i ($^{\forall}i \in N$) solves the following problem.

$$Max_{\widetilde{v_i}} \ \big(v_i - b(\widetilde{v_i})\big)F(\widetilde{v_i})^{n-1} - cb(\widetilde{v_i}) \qquad \text{...(4)}$$

F.O.C. of (4) w.r.t. $\widetilde{v_i}$ is

$$-b'(\widetilde{v_i})F(\widetilde{v_i})^{n-1} + (v - b(\widetilde{v_i}))(n-1)F(\widetilde{v_i})^{n-2}f(\widetilde{v_i}) + cb'(\widetilde{v_i}) = 0 \qquad \text{...(5)}$$

Since $\widetilde{v_i} = v_i$ at equilibrium, (5) becomes

$$-b'(v_i)F(v_i)^{n-1} + (v_i - b(v_i))(n-1)F(v_i)^{n-2}f(v_i) + cb'(v_i) = 0 \qquad \text{...(6)}$$

Rearranging (6) , we obtain

$$b'(v_i) + \big((n-1)F(v_i)^{n-2}f(v_i)b(v_i)\big)/\big((F(v_i)^{n-1} - c)\big)$$
$$= \big((n-1)F(v_i)^{n-2}f(v_i)v_i\big)/\big((F(v_i)^{n-1} - c)\big) \qquad \text{...(7)}$$

Solving (7) for $b(v_i)$ yields

$b(v_i) = \left(\int_{\underline{v}}^{v_i} w(n-1)F(w)^{n-2}f(w)dw + \tilde{c}\right)/(F(v_i)^{n-1} - c)$, where $\tilde{c}$ is a constant of integration.

An initial condition of $b(\underline{v}) = 0$ determines $\tilde{c} = 0$ in (8). Therefore, the final solution takes the form of

$$b(v_i) = \left(\int_{\underline{v}}^{v_i} w(n-1)F(w)^{n-2}f(w)dw\right)/(F(v_i)^{n-1} - c) \qquad \text{...(9)}$$

The result is summarized in the next proposition.

Proposition1. (the equilibrium bidding function $b(v_i)$ and time credit t_i)

The equilibrium bidding function $b(v_i)$ and the amount of time credit purchased at equilibrium $t_i(v_i)$ are characterized as follows.

$$b(v_i) = \left(\int_{\underline{v}}^{v_i} w(n-1)F(w)^{n-2}f(w)dw\right)/(F(v_i)^{n-1}-c)$$

and $$t_i(v_i) = \bar{v} - b(v_i)$$

Next, given $b(v_i)$ and $t_i(v_i)$ characterized in proposition1, we consider the problem in period 0 where the seller solves the following problem to set time credit fee rate c to maximize her expected revenue, $R(c)$.

$$R(c) = \sum_{k=1}^{n} {}_nC_k \int_{\underline{v}}^{\bar{v}} b(w)F(\underline{v})^{n-k}k\left(F(w)-F(\underline{v})\right)^{k-1} f(w)dw$$

$$+n\int_{\underline{v}}^{\bar{v}}\left(\bar{v}-b(w)\right)cf(w)dw \qquad \text{...(10)}$$

Since $\sum_{k=1}^{n} {}_nC_k\, F(\underline{v})^{n-k}k\left(F(w)-F(\underline{v})\right)^{k-1} = n\left(F(\underline{v})+F(w)-F(\underline{v})\right)^{n-1}$, (10) is rewritten as

$$R(c) = \int_{\underline{v}}^{\bar{v}} b(w)nF(w)^{n-1}f(w)dw + n\int_{\underline{v}}^{\bar{v}}\left(\bar{v}-b(w)\right)cf(w)dw \qquad \text{...(11)}$$

Plagging (9) into (11) yields

$$R(c) = \int_{\underline{v}}^{\bar{v}}\left(\frac{\left(\int_{\underline{v}}^{w} s(n-1)F(s)^{n-2}f(s)ds\right)}{(F(w)^{n-1}-c)}\right)nF(w)^{n-1}f(w)dw$$

$$+n\int_{\underline{v}}^{\bar{v}}\left(\bar{v}-\left(\frac{\left(\int_{\underline{v}}^{w} s(n-1)F(s)^{n-2}f(s)ds\right)}{(F(w)^{n-1}-c)}\right)\right)cf(w)dw \qquad \text{...(12)}$$

We can obtain (13) by arranging (12) further.

$$n\int_{\underline{v}}^{\bar{v}}\left(\frac{\left(\int_{\underline{v}}^{w} s(n-1)F(s)^{n-2}f(s)ds\right)}{(F(w)^{n-1}-c)}\right)(F(w)^{n-1}-c)f(w)dw + n\int_{\underline{v}}^{\bar{v}}\bar{v}cf(w)dw$$

$$= n\int_{\underline{v}}^{\bar{v}}\left(\int_{\underline{v}}^{w} s(n-1)F(s)^{n-2}f(s)ds\right)f(w)dw + n\int_{\underline{v}}^{\bar{v}}\bar{v}cf(w)dw$$

$$= n\int_{\underline{v}}^{\bar{v}}\left(\int_{\underline{v}}^{w} s(n-1)F(s)^{n-2}f(s)ds\right)f(w)dw + n\bar{v}c\left(1-F(\underline{v})\right) \qquad \text{...(13)}$$

Noting that $\underline{v}$ depends on c, F.O.C. of (13) w.r.t. c becomes as follows.

$$\frac{\partial R(c)}{\partial c} = n \begin{pmatrix} -\left(\int_{\underline{v}}^{v} s(n-1)F(s)^{n-2}f(s)ds\, f(\underline{v})\right)\underline{v}'(c) \\ -\left(\int_{\underline{v}}^{\overline{v}} \underline{v}(n-1)F(\underline{v})^{n-2}f(\underline{v})\, f(w)dw\right)\underline{v}'(c) \\ +\overline{v}\left(1-F(\underline{v})\right) \\ -\overline{v}cf(\underline{v})\underline{v}'(c) \end{pmatrix} = 0 \qquad \ldots(14)$$

Remember that a bidder with type $\underline{v}$ expects zero expected profit when he decides to enter the auction. Therefore, $F(\underline{v})^{n-1}\underline{v}=\overline{v}c$ holds. …(15)

Total differentiation of (12) becomes

$$\frac{d\overline{v}}{dc} = \frac{\overline{v}}{F(\underline{v})^{n-1}+(n-1)\underline{v}F(\underline{v})^{n-2}f(\underline{v})} \qquad \ldots(16)$$

Inserting (16) into (14) and solving it for c yields the equilibrium value of c if $\frac{\partial^2 R(C)}{\partial c^2} < 0$.

Since it is complicated to obtain an explicit solution for general case and we only use uniform distribution function whose support is [0,1] in our experiments, we focus on the case of $F(\cdot) = U[0,1]$ from now on.

Under the assumption of $F(\cdot) = U[0,1]$, we can write (15) as

$$\underline{v}^{n-1} \cdot \underline{v} = c \qquad \ldots(17)$$

Solving (17) for $\underline{v}$ yields

$$\underline{v} = c^{\frac{1}{n}} \qquad \ldots(18)$$

With help of (18), (13) is expressed as

$$R(c) = n\int_{c^{\frac{1}{n}}}^{1}\left(\frac{1}{w^{n-1}-c}\right)\left(\frac{n-1}{n}\right)(w^n-c)(w^{n-1}-c)dw + n\int_{c^{\frac{1}{n}}}^{1} 1\cdot cdw \qquad \ldots(19)$$

Rearranging (19),

$$n\int_{c^{\frac{1}{n}}}^{1}\left(\frac{1}{w^{n-1}-c}\right)\left(\frac{n-1}{n}\right)(w^n-c)(w^{n-1}-c)dw + n\int_{c^{\frac{1}{n}}}^{1} 1\cdot cdw$$

$$=\int_{c^{\frac{1}{n}}}^{1}(n-1)(w^n-c)dw + n\int_{c^{\frac{1}{n}}}^{1} 1\cdot cdw$$

$$=\int_{c^{\frac{1}{n}}}^{1}(n-1)w^n dw + \int_{c^{\frac{1}{n}}}^{1} cdw$$

$$=\frac{n-1}{n+1} + c - \frac{2n}{n+1}c^{\frac{n+1}{n}} \qquad \ldots(20)$$

Further, F.O.C. of (20) w.r.t. c takes the following form.

$$\frac{\partial R(c)}{\partial c} = 1 - \frac{2n}{n+1}\frac{n+1}{n}c^{\frac{1}{n}} = 0 \qquad \ldots(21)$$

Solving (21) for c yields

$$c = \left(\frac{1}{2}\right)^n \qquad \text{...(22)}$$

We can also show that S.O.C. is satisfied as follows.

$$\frac{\partial^2 R(c)}{\partial c^2} = -2\frac{1}{n}c^{\frac{1-n}{n}} < 0$$

Next, we would like to know the seller's expected revenue at equilibrium $R^*(c)$. We can calculate it by inserting (22) into (20).

$$R^*(c) = \frac{n-1}{n+1} + \left(\frac{1}{2}\right)^n - \frac{2n}{n+1}\left(\left(\frac{1}{2}\right)^n\right)^{\frac{n+1}{n}}$$

$$= \frac{n-1}{n+1} + \left(\frac{1}{2}\right)^n \left(1 - \frac{2n}{2(n+1)}\right)$$

$$= \frac{n-1}{n+1} + \left(\frac{1}{2}\right)^n \left(\frac{1}{n+1}\right)$$

$$= \left(\frac{1}{n+1}\right)\left((n-1) + \left(\frac{1}{2}\right)^n\right) \qquad \text{...(23)}$$

Proposition2. (the equilibrium bidding function $b(v_i)$, time credit $t_i(v_i)$, and time credit fee c)

Suppose $(\cdot) = U[0,1]$. Then, the equilibrium bidding function $b(v_i)$ and time credit $t_i(v_i)$ are characterized as follows.

The seller sets time credit fee, c, as $c = \left(\frac{1}{2}\right)^n$.

Given time credit fee c, a bidder whose value $v_i \geq \underline{v} = c^{\frac{1}{n}} = \frac{1}{2}$ purchases positive amount time credit, $t_i(v_i) = 1 - b_i(v_i)$,

and submits a bid, $b_i(v_i) = \frac{\int_{\underline{v}}^{v_i} s(n-1)s^{n-2}ds}{{v_i}^{n-1} - c} = \frac{\int_{\frac{1}{2}}^{v_i} s(n-1)s^{n-2}ds}{{v_i}^{n-1} - \left(\frac{1}{2}\right)^n}$.

Next, we would like to know the seller's expected revenue at equilibrium $R^*(c)$. We can calculate it by inserting (22) into (20).

$$R^*(c) = \frac{n-1}{n+1} + \left(\frac{1}{2}\right)^n - \frac{2n}{n+1}\left(\left(\frac{1}{2}\right)^n\right)^{\frac{n+1}{n}}$$

$$= \frac{n-1}{n+1} + \left(\frac{1}{2}\right)^n \left(1 - \frac{2n}{2(n+1)}\right)$$

$$= \frac{n-1}{n+1} + \left(\frac{1}{2}\right)^n \left(\frac{1}{n+1}\right)$$

$$=\left(\frac{1}{n+1}\right)\left((n-1)+\left(\frac{1}{2}\right)^n\right) \qquad \ldots(24)$$

Now, we conduct comparative statics. First, we examine whether the expected revenue of the seller is an increasing function of the number of potential bidders

$$\frac{\partial R^*(n)}{\partial n}=\frac{1}{(n+1)^2}\left(\left(1+\left(\frac{1}{2}\right)^n\left(log\frac{1}{2}\right)\right)(n+1)-\left((n-1)+\left(\frac{1}{2}\right)^n\right)\right)$$

$$=\frac{1}{(n+1)^2}\left(2+\left(\frac{1}{2}\right)^n\left((n+1)\left(log\frac{1}{2}\right)-1\right)\right)>0.$$

Therefore, the seller's expected revenue increases as the number of potential bidders increases.

Proposition3. ($\frac{\partial R^*(n)}{\partial n}>0$)

Suppose $F(\cdot)=U[0,1]$. Then, the seller's expected revenue is a strict increasing function of the number of potential bidders.

Further, we can show that the seller sets higher c as the numbers of potential bidders increases. To see it, we differentiate (19) w.r.t. n and obtains that $\frac{\partial c}{\partial n}=n\left(\frac{1}{2}\right)^{n-1}>0$.

Proposition4. ($\frac{\partial c}{\partial n}>0$)

Suppose $F(\cdot)=U[0,1]$. Then, the seller's equilibrium time credit fee is a strictly increasing function of the number of potential bidders.

3. Design of the Experiment

We examine the effect of time credit fee rate on a bidder's bidding behavior and on the expected revenue of the seller. For this purpose, we manipulate only one variable in a Dutch auction, namely, adding time credit fee to a Dutch auction. Basic design of our experiment follows that by Katok and Kwasnica (2008) although the purpose of their experiment is different from ours. They examine the role of the speed of the clock in the Dutch auction, but we examine the effect of time credit fee rate in a Dutch auction.

Now, let me explain the basic features of our experiment. First, we explain the design of a Dutch auction without time credit, which is equivalent to a Dutch auction where time credit fee rate is set to be zero. Then, we introduce strictly positive time credit fee rate to a Dutch auction. We follow the terms used in Katok and Kwasnica (2008). Three bidders compete against each other for a single indivisible item in a Dutch auction. Bidders' values of the item are i.i.d. draws from a uniform distribution function whose support is the integers of [0, 100]. In a Dutch auction, the price starts from 100 and goes down 5 experimental yen per 5 seconds. Each cohort consists of 9 participants and we randomly match 3 participants in every session and they play one session of four different Dutch auctions: a Dutch auction with zero time credit fee rate and three Dutch auctions with strictly positive time credit fee rates. There are 5sessions in each of four Dutch auctions with different time credit fee rates. New values for the asset are drawn for each of 5 sessions and the realized value of the asset is private information of each participant.

We ran experiments at Aoyamagakuin University. Participants are recruited in the class of industrial organization. Participants obtain 831 yen in advance as show-up fee and are received total profit from 20 sessions they participate at the rate of 1 experimental yen=1 yen. The instructions distributed in our experiment are available in the appendix.

4. Result

A Dutch auction ($\boldsymbol{c = 0}$***)***

Since we assume bidders are risk-neutral, the expected revenue of the seller is 50. The average of the seller's revenue in our experiment is 46.56experimental yen and we cannot reject the null hypothesis that it is equal to 50 which is the seller's expected revenue at the risk-neutral Nash equilibrium. This is different from Katok and Kwasnica (2008) which claims that bidders overbid in the experiment and they can reject the null hypothesis that it is equal to 50.

A Dutch auction with time credit

The average revenue of the seller is 54.42 when $c = 0.1$, 57.06 when $c = 0.5$, and 57.11 when $c = 1.0$. The average of the seller's revenues and the sample variances of four auctions with different time credit fee rates are summarized in Table2 below.

Table 2. (Comparison of c**=0,** c**=0.1,** c**=0.5, and** c**=1.0)**

		$\bar{X}_i$	s_i^2
Experiment1	c=0	46.56	422.02
Experiment2	c=0.1	54.42	349.79
Experiment3	c=0.5	57.06	517.27

Experiment4 c=1.0 57.11 859.92

$\bar{X}_i$ is the sample average of experiment $i (i = 1,2,3,4)$ and s_i^2 is the sample variance of experiment i $(i = 1,2,3,4)$

We would like to test the claim by SevenSnap: introducing a strictly positive time credit fee rate increases the sellers expected revenue. To fulfill this goal, we set the null hypothesis H_0: $\mu_j - \mu_1 = 0$ and H_1: $\mu_j - \mu_1 > 0$ $(j = 2,3,4)$. We use t-test for samples with unequal variance. For j = 2, $t_{12} = \frac{\overline{X_2} - \overline{X_1}}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} = \frac{54.42 - 46.56}{\sqrt{422.02/45 + 349.79/45}} = \frac{7.86}{7.94.14} \cong 1.90$ and the degree of freedom ν_{12} is the closest integer to

$$\frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\left(\left(s_1^2/n_1\right)^2 / (n_1 - 1) + \left(s_2^2/n_2\right)^2 / (n_2 - 1)\right)} = \frac{(422.02/45 + 349.79/45)^2}{((422.02/45)^2/44 + (349.79/45)^2/44)} \cong 87.20.$$

Therefore, ν_{12}=87. Since $t_{0.05}$=1.66 when the degree of freedom is 80[1], we can discard the null hypothesis of H_0: $\mu_2 - \mu_1 = 0$. Therefore, we can support the claim of SevenSnap which says that introducing a strictly positive time credit fee rate increases the sellers expected revenue.

We do the same exercise for Dutch auctions with higher time credit fee rates of c=0.5 and c=1.0. We also test the hypothesis whether increasing time credit fee affects the revenue of the seller or no. The result is summarized in Table3.

Table3. (Summary of hypothesis tests)

Hypothesis	t-value	degree of freedom	$t_{0.05}$	H_0
$H_0: \mu_2 - \mu_1 = 0$ and $H_1: \mu_2 - \mu_1 > 0$	t_{12} =1.90	$\nu_{12} = 87.20$	1.66	rejected
$H_0: \mu_3 - \mu_1 = 0$ and $H_1: \mu_3 - \mu_1 > 0$	t_{13}=2.30	$\nu_{13} = 87.10$	1.66	rejected
$H_0: \mu_4 - \mu_1 = 0$ and $H_1: \mu_4 - \mu_1 > 0$	t_{14}=1.98	$\nu_{14} = 78.80$	1.66	rejected
$H_0: \mu_3 - \mu_2 = 0$ and $H_1: \mu_3 - \mu_2 \neq 0$	t_{23}=0.60	$\nu_{23} = 84.89$	1.99	not rejected
$H_0: \mu_4 - \mu_2 = 0$ and $H_1: \mu_4 - \mu_2 \neq 0$	t_{24}=0.52	$\nu_{24} = 74.30$	1.99	not rejected
$H_0: \mu_4 - \mu_3 = 0$ and $H_1: \mu_4 - \mu_3 \neq 0$	t_{34}=0.0090	$\nu_{34} = 82.86$	1.99	not rejected

The seller's revenue increases if a strictly positive time credit fee rates are introduced,

[1] We refer to Department of Statistics, University of Tokyo Faculty of Arts (1991) which has t-distribution for the degree of freedom is 80 or 100. So, we use t-distribution with the degree of freedom being 80.

but the rate of time credit fee rate does not affect the seller's revenue. With or without time credit does matter for the seller's revenue. Interestingly enough, Katok and Kwasnica (2008) demonstrate that subjects overbid in a Dutch auction, but overbidding behavior is not observed in our experiments. In fact, we cannot reject the seller's revenue of 46.56 in our experiment with c=0 is equal to 50 as theory predicts. Furthermore, the seller's expected revenue from the optimal auction is calculated as $100\int_{1/2}^{1}(2x-1)3x^2 = 53.125$ for the uniform distribution whose support is $[0,100]$. We cannot reject the seller's revenues from three auctions with strictly positive time credit fee rates in our experiment are equal to the one in the optimal auction, namely, 53.125.

5. Conclusions and extensions

We theoretically prove that a time credit fee rate works as a reserve price and the Dutch auction with a strictly positive time credit fee rate can be optimal. Further, our experimental result supports the claim made by SevenSnap that introducing a strictly positive time credit fee rate increases the seller's expected revenue. Further, we cannot reject the null hypothesis that Dutch auctions with strictly positive time credit fee rates implement the optimal auction mechanism; however our data shows that the actual rate of time credit fee doesn't matter as long as it is strictly positive. All it does matter is whether there is a strictly positive time credit fee rate or not. There are several additional features in the actual SevenSnap's auction we omit in the main text. Firstly, the speed of the clock in a Snap room depends on the actual numbers of participants in a Dutch auction. This means that SevenSnap controls the speed of the clock in a Snap room. Katok and Kwasnica (2008) analyze the effect of the clock speed in a Dutch auction by laboratory experiments and show that the revenue of the seller is lower in a Dutch auction at fast clock than that in a sealed-bid first-price auction and is higher at slow clock than that in a sealed-bid first-price auction. SevenSnap's design is beyond their analysis because they assume the speed of the clock is exogenously given and does not change in an auction, but the numbers of eligible bidders can decrease in a SevenSnap's auction due to time credit because a bidder cannot stay in a Snap room longer than the time he purchased as time credit in advance of the actual auction. SevenSnap does not show whether they plan to announce the numbers of actual participants in a Snap room to bidders or not. A Dutch auction with endogenously determined speed of the clock is an interesting theoretical and experimental research topic especially when bidders are impatient.

Secondly, SevenSnap charges $1 for 1minute of stay and only four sets of time credit are available: 10minutes (=$10), 30minutes (=$30), 60minutes(=$60), and 240minutes(=$240). Although we assume that time credit is a continuous variable, we do not think discrete time credit cause significant difference to our main results. Thirdly, SevenSnap introduces a new item every 60 minutes. Therefore, their auction is not one-shot auction, but has a feature of repeated auction. When a bidder purchases time credit, he has to take into account the possibility that the same items are on auction in the near future. On the other hand, SevenSnap has to decide how frequently it sells the same item. Even for the simplest possible case where SevenSnap has two units of the same item, it can auction off two units at the same time, or one at a time and the choice of the sales procedure by SevenSnap influences a bidder's decision of the amount of time credit he purchases and his bid in the Dutch auction.

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Home page of SevenSnap, http://www.sevensnap.com/

Appendix.

This appendix contains the instructions used in our experiments.

Instructions

Introduction

This is an experiment in market decision-making. If you follow the instructions carefully and make good decisions, you can earn a considerable amount of CASH.

The experiment consists of 4 different auctions and 5 sessions in each. You receive show-up fee of 837 yen as participation fee. In addition, your earnings from all 5×4=20 sessions will be totaled and converted to yens at the rate of 1 experimental yen=1 yen. You will be paid this amount at the end of the experiment in private and in CASH.

You are not allowed to talk nor communicate with other participants during the experiment. If you disobey the rules, you may have to leave. In that case, you receive zero yen when you leave the room.

Auction Description.

In each auction, you and two other players compete against each other for an asset. There are 4 different auctions and each consists of 5 sessions.

(1) Dutch auction without time credit

- The price starts from 100 experimental yen and decreases 5 experimental yen per 5 seconds. The price never becomes strictly negative. (5seconds later: 95 experimental yen, 10 seconds later: 90 experimental yen, 15seconds later: 85 experimental yen,…, 20seconds later: 0 yen)
- Any of the bidders can stop the auction and purchases the asset at the price when s/he raises her/his hand. The first bidder to raise his/her hand wins the asset and pays the current price at the time, and the other two bidders earn zero experimental yen for that auction.
- For example, if you intend to buy the asset at 50 experimental yen, you should raise your hand after the price drops to 50 experimental yen, but before the price drops to 45 experimental yen.

(2) Dutch auction with time credit c=0.1

- The rule is the same as that in a Dutch auction without time credit EXCEPT you have to buy time credit before the auction starts.
- Time credit are the maximum amount of time you can stay in an auction and your purchasing decision of raising your hand becomes INEFFECTIVE after your time credit expires. You have to pay 0.1 experimental yen for every 5 seconds of time you stay in an auction.

Time to stay in an auction	Price	Your payment for time credit
5 seconds	95	0.1 experimental yen
10seconds	90	0.2experimental yen
15seconds	85	0.3 experimental yen
20seconds	80	0.4 experimental yen
25seconds	75	0.5 experimental yen
30seconds	70	0.6 experimental yen
.		.
.		.
.		.
100seconds	0	2.0 experimental yen

(3) Dutch auction with time credit c=0.5

The rule is the same as (2) EXCEPT one point. Now, you have to pay 0.5 experimental yen for every 5seconds of time you stay in an auction.

Time to stay in an auction	Price	Your payment for time credit
5seconds	95	0.5 experimental yen
10seconds	90	1.0 experimental yen
15seconds	85	1.5 experimental yen
20seconds	80	2.0 experimental yen
25seconds	75	2.5 experimental yen
30seconds	70	3.0 experimental yen
.		.
.		.
.		.
100seconds	0	10.0 experimental yen

(4) Dutch auction with time credit c=1.0

The rule is the same as (2) EXCEPT one point. Now, you have to pay 1.0 experimental yen for every 5seconds of time you stay in an auction.

Time to stay in an auction	Price	Your payment for time credit
5 seconds	95	1.0 experimental yen
10seconds	90	2.0 experimental yen
15seconds	85	3.0 experimental yen
20seconds	80	4.0 experimental yen
25seconds	75	5.0 experimental yen
30seconds	70	6.0 experimental yen
.		.
.		.
.		.
100seconds	0	20.0 experimental yen

Resale values and Earnings

If you purchase an asset, your earnings are equal to the difference between your resale value of the asset and the price you paid for the asset. You observe your resale value at the beginning of each auction.

In a Dutch auction without time credit,
YOUR EARNINGS=RESALE VALUE-PURCHASE PRICE

For example, if you pay 30 for the asset your resale value is 64, your earnings are
YOUR EARNINGS=64-30=34 experimental yen.

If you did not win the auction, your earning isZERO.

In a Dutch auction with time credit
YOUR EARNINGS=RESALE VALUE-PURCHASE PRICE-PAYMENT FOR TIME CREDIT

For example, if you pay 30 for the asset your resale value is 64, and you purchased time credit of 100seconds when time credit fee rate is 0.1yen for every 5seconds, your earnings are
YOUR EARNINGS=64-30-0.1×10=64-30-0.1×20=64-30-2=32 experimental yen.

If you did not win the auction, but purchased time credit of 100seconds when time credit fee is 0.1yen for every 5seconds, your earnings are
YOUR EARNINGS=-0.1×20=-2yen.

This example demonstrates that your earnings can be NEGAIVE in a Dutch auction with time credit.

Resale values differ among bidders and among sessions. For each bidder, the resale value of the asset in a session will be between 0 and 100 and it is an integer. Each number from 0 to 100 has an equal chance of being chosen. It is as if the numbers from were stamped on 101 balls, one number for each ball, and placed in an urn. A random draw from the urn determines the resale value of an asset for an individual in a session. After one draw, we put the ball back to the urn, shuffle the urn, and make another draw. Since there are 3 bidders in each auction, we repeat this process three times for each session.

At the end of each auction, all bidders will see the auction's outcome. If you won the auction, you will be informed of your earnings. If you do not win, you will be told that you did not acquire the asset, and your earnings for that auction is zero if it is a Dutch auction WITHOUT time credit and your earnings for that auction can be NEGATIVE if it is a Dutch auction with time credit because you have to make payment for time credit even you lose the auction.

Your earnings from all the previous auctions, along with your resale values, the winning prices, and the amount you paid, will be displayed on your sheet during each session.

Matching

You will not be matched with the same two participants for two consecutive auctions. You will not be told which of the other participants in the room you are matched with, and they will not be told that you matched with them. What happened in any auction has no effect on what happens in any other auction.

Ending the experiment

At the end of the experiment, your earnings from all 5×4=20 sessions will be totaled and converted to yens at the rate of 1 experimental yen=1 yen. You will be paid this amount at the end of the experiment in private and in CASH. The total payment will be displayed on your sheet at the end of the experiment. In addition, you will be paid 837yen at the beginning of the experiment for SHOW-UP FEE.

Now, please complete the quiz on the next page. If you have any questions, raise your hand and we will come to your seat and answer your questions. When everyone has completed the quiz, we will go over the answers and we start our experiment.

QUIZ

The purpose of this quiz is to see everyone understands the auction rules used in experiments. If you have any questions, raise your hand and we will come to your seat and answer your questions.

Question1.

Suppose it is a Dutch auction WITHOUT time credit and bidders' resale values of the asset on sale are as follows.

Bidder1's resale value is 85 experimental yen.
Bidder2's resale value is 80 experimental yen.
Bidder3's resale value is 63 experimental yen.

The price changes as follows.

Beginning of the auction:	price=100
After 5 seconds:	price=95
After 10 seconds:	price=90
After 15 seconds:	price=85
After 20 seconds:	price=80
After 25 seconds:	price=75

And after 50 seconds, bidder1 raises his/her hand and stops the auction.

1. Does bidder 1 win the auction? __________________
2. The earnings of bidder1 are __________________
3. Does bidder 2 win the auction? __________________
4. The earnings of bidder2 are __________________
5. Does bidder 3 win the auction? __________________
6. The earnings of bidder3 are __________________

Question2.

Suppose it is a Dutch auction WITH time credit. The costs of time credit are: you have to pay 0.1 experimental yen for every 5 seconds of your stay in an auction.

Bidders' resale values of the asset on sale are as follows.

Bidder1's resale value is 58 experimental yen.
Bidder2's resale value is 91 experimental yen.
Bidder3's resale value is 86 experimental yen.

Bidder1 purchased 40 seconds of time credit.
Bidder2 purchased 50 seconds of time credit.

Bidder3 purchased 60 seconds of time credit.

The price changes as follows.

Beginning of the auction:	price=100
After 5seconds:	price=95
After 10 seconds:	price=90
After 15 seconds:	price=85
After 20 seconds:	price=80
After 25 seconds:	price=75

And after 40 seconds, bidder2 raises his/her hand and stops the auction.

1. Does bidder 1 win the auction? __________________
2. Bidder1's payment for time credit __________________
3. The earnings of bidder1 are __________________
4. Does bidder 2 win the auction? __________________
5. Bidder2's payment for time credit __________________
6. The earnings of bidder2 are __________________
7. Does bidder 3 win the auction? __________________
8. Bidder3's payment for time credit __________________
9. The earnings of bidder3 are __________________