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Abstract

This article examines the optimality of the Freidman rule in an overlapping generations model with spatial separation, wherein asymmetric liquidity shocks are observed. Suboptimality of the Freidman rule is shown. Furthermore, when the number of locations is sufficiently large, there is no room for monetary policy to improve social welfare.

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Asymmetric Liquidity Shocks and Optimality of the Freidman Rule*

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Abstract: This article examines the optimality of the Freidman rule in an overlapping generations model with spatial separation, wherein asymmetric liquidity shocks are observed. Suboptimality of the Freidman rule is shown. Furthermore, when the number of locations is sufficiently large, there is no room for monetary policy to improve social welfare.

Keywords: Money; Spatial separation; Friedman rule; Overlapping generations model.

JEL Classification Numbers: E30; E31; E40; E41.

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1 Introduction

Since the Lehman Crisis, many economists have been studied financial crises. Most of these studies have been interested in the financial-intermediate role of banks and have been tried to explain the relationship between bank runs and financial crises. Especially, Champ, Smith, and Williamson (1996), Smith (2002), and Haslag and Martin (2007) have incorporated the essence of Diamond and Dybvig (1983) into an overlapping generations (OLG) model and explored implications of monetary policies on the financial crises.[1] In their model, they showed suboptimality of the Friedman rule.

In their model, there are two islands between which there is no communication (spatial friction). In each island, there is a single bank as a coalition of agents. Furthermore, liquidity shocks are modeled by random relocation of agents. To analyze a symmetric situation, the size of liquidity shocks are common between two islands. Therefore, each bank faces common liquidity shocks in the existing models. However, liquidity shocks are usually asymmetric among banks. In this article, we modify the existing model to allow asymmetric liquidity shocks among banks and reexamine optimality of the Friedman rule.

This article presents an OLG model with random relocations among more-than-two islands, numbered from 0 to $N \geq 1$. The fraction π of agents of the 0-th island are assumed to randomly selected and equally distributed to other islands. At the same time, the fraction π/N of agents of the n-th island are randomly selected and move to the 0-th island. This framework, then, allows asymmetric liquidity shocks among islands.

In the model, we first show the suboptimality of the Friedman rule. More precisely, the optimal monetary policy is, if any, greater than one. This is a natural extension of the existing result obtained per Smith (2002) and Haslag and Martin (2007). Furthermore, it is shown that, when the number of islands diverges, the economy converges to the autarkic situation and there is no room for monetary policy to improve social welfare. This is because the welfare loss by the liquidity shocks becomes relatively small when the number of islands increases.

The organization of this paper is as follows: Section 2 describes the model considered in this article. Our model is an extension of that studied per Haslag and Martin (2007). Section 3 examines the behavior of each bank in the monetary steady state. Section 4 presents main results of this article. Proofs are provided in Section 5.

[1] See also Schreft and Smith (2002) and Matsuoka (2011) for an OLG model with spatial frictions.

2 The Environment

We consider an overlapping generations model with spatial separation, wherein there exist more-than-two locations (islands). The model is closely related to Haslag and Martin (2007).[2] The interested reader can be also referred to their works for related issues.

Time is discrete and runs from $-\infty$ to $+\infty$. There are $1+N$ islands, where N is an integer more than or equal to one. Also, there is a single good. At each date, a new young generation consisting of a continuum of ex-ante identical agents with unit mass appears on each island. Each agent lives for two-periods. There is no population growth.

Money is issued by the central bank. We denote by M_t the per-capita money stock of period t. This is assumed to be common to all islands. The stock of money follows the equation $M_t = \sigma M_{t-1}$, where $\sigma > 0$ is a known constant. Each agent born in period t is endowed with ω units of the good and lump-sum money transfer $Z_t := M_t - M_{t-1}$ when young and nothing when old. Each young agent cares only about old age consumption c and has lifetime utility $u(c) = c^{1-\rho}/(1-\rho)$, where $0 < \rho < 1$.[3]

To create a transaction role for money, we assume limited communication among spatially distinct islands. At each date, trade is assumed to occur only among agents who inhabit the same location. Then, some fraction π of young agents in the 0-th island is selected at random to move to the other locations. We assume that the movers from the 0-th island are randomly but equally allocated to the other islands $1, \ldots, N$. At the same time, some fraction π/N of young agents in the n-th island, $n = 1, \ldots, N$, is selected at random to move to the 0-th island. Let $\pi_0 = \pi$ and $\pi_n = \pi/N$ for each $n = 1, \ldots, N$. An example of this relocation mechanism is depicted in Figure 1, which treats the case that $N = 2$.

Finally, there is a storage technology whereby one unit stored at date t generates $x > 1$ units of the good at date $t+1$. The gross return of the storage technology, x, is a known constant.

3 Bank Behavior in a Monetary Steady State

At each date t, young agents deposit all of their after-tax/transfer endowment with a bank. A bank in each island $n = 1, \ldots, N$ then enters into a local spot market, wherein the price level is p_t^n, and allocates its portfolio. For each $n = 1, \ldots, N$, we denote by s_t^n the amount of storage

[2] When the number of locations is two, our model degenerates into that studied per Haslag and Martin (2007).
[3] This is a standard assumption in the existing literature. This is because, if $\rho \geq 1$, the model produces the counterintuitive result that bank reserves increase when inflation increase.

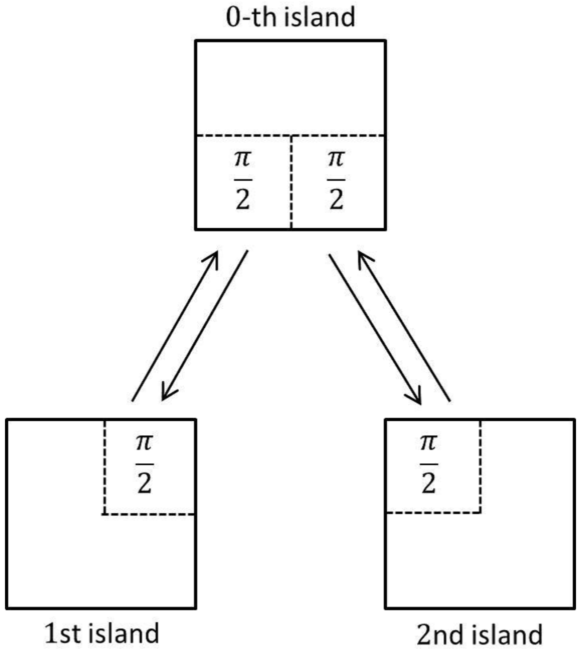

Figure 1: Relocation Mechanism: The Case of $N = 2$

investment and by m_t^n the real value of cash reserves of the bank at n-th island. Then, the balance sheet constraint of each bank at n-th island is

$$s_t^n + m_t^n \leq \omega + \tau_t^n, \tag{1}$$

where $\tau_t^n = [M_t - M_{t-1}]/p_t^n = [(\sigma - 1)/\sigma]M_t/p_t^n = [(\sigma - 1)/\sigma]m_t^n$. The bank also chooses a schedule of returns on deposits contingent upon the realization of the location numbering. Let $d_t^{m,n}$ denote the gross real return offered to agents who are relocated between t and $t+1$ and let d_t^n denote the corresponding rate for agents who are not relocated, when the location is the n-th island for $n = 0, \ldots, N$.

Each bank at n-th island ($n = 1, \ldots, N$) must have sufficient liquidity to meet the needs of movers. This is captured by the expression:

$$\pi_n d_t^{m,n}(\omega + \tau_t) \leq \sigma_t^{-1} m_t^n, \tag{2}$$

where $\sigma_t := p_{t+1}/p_t$. A similar condition for nonmovers, who consume all the proceeds from the storage technology, is given by

$$(1 - \pi_n) d_t^n(\omega + \tau_t^n) \leq x s_t^n. \tag{3}$$

The bank at n-th island now chooses $d_t^{m,n}$, d_t^n, m_t^n, and s_t^n to maximize the expected welfare function

$$\frac{(\omega+\tau_t^n)^{1-\rho}}{1-\rho}\left[\pi_n\left(d_t^{m,n}\right)^{1-\rho}+(1-\pi_n)\left(d_t^n\right)^{1-\rho}\right]$$

subject to constraints (1–3). We denote by γ_t^n the reserve-to-deposit ratio of period t bank at n-th island, i.e., $\gamma_t^n := m_t^n/(\omega+\tau_t^n)$. Because constraints hold with equality at a solution, the banks' objectives are now to choose γ_t^n to maximize

$$\frac{(\omega+\tau_t^n)^{1-\rho}}{1-\rho}\left[\pi_n^\rho\left(\frac{\gamma_t^n}{\sigma_t}\right)^{1-\rho}+(1-\pi_n)^\rho\left(x(1-\gamma_t^n)\right)^{1-\rho}\right].$$

The first-order condition of an interior solution is then given by

$$0=\frac{(\omega+\tau_t^n)^{1-\rho}}{1-\rho}\left[\pi_n^\rho\frac{1}{\sigma_t}\left(\frac{\gamma_t^n}{\sigma_t}\right)^{-\rho}-\left(\frac{1-\pi_n}{1-\gamma_t^n}\right)^\rho x^{1-\rho}\right],$$

which is equivalent to

$$\gamma_t^n=\left[1+\frac{1-\pi_n}{\pi_n}(\sigma_t x)^{\frac{1-\rho}{\rho}}\right]^{-1}. \tag{4}$$

Throughout the rest of this paper, we focus on stationary monetary equilibrium (monetary steady-state). Here, an equilibrium occurs when demand and supply for money coincide. By the assumption of the steady-state, the time subscript can be dropped in what follows when it is convenient. Furthermore, in a monetary steady-state, we can observe that the inflation rate, σ_t, is equal to the money growth rate, σ. Then, we can now obtain the following proposition.

Proposition 1 *In a monetary steady-state, (a) the optimal reserve-to-deposit ratio, $\gamma_n(\sigma)$, of the bank at n-th island, $n=0,\ldots,N$, is given by*

$$\gamma_n(\sigma)=\begin{cases}\left[1+\frac{1-\pi}{\pi}(\sigma x)^{\frac{1-\rho}{\rho}}\right]^{-1} & \text{if} \quad n=0,\\ \left[1+\frac{N-\pi}{\pi}(\sigma x)^{\frac{1-\rho}{\rho}}\right]^{-1} & \text{otherwise,}\end{cases}$$

(b) in the limit, as $\sigma\to 1/x$, $\gamma_0(\sigma)\to\pi$, $\gamma_n(\sigma)\to\pi/N$ for each $n=1,\ldots,N$, and the mover's consumption converges to the nonmover's, x, and (c) $\gamma_n'(\sigma)\leq 0$ for each $\sigma\geq 1/x$ and each $n=0,\ldots,N$.

By their characterizations, $\gamma_n(\sigma)\in(0,1)$ for $n=0,\ldots,N$. Also remark that $\gamma_0(\sigma)$ is independent of the number of islands, $1+N$, whereas $\gamma_n(\sigma)$, $n\geq 1$, depends on N and $\gamma_n(\sigma)\to 0$ as $N\to\infty$.

4 The Optimum Quantity of Money

We now examine the optimality of the Friedman rule. Consistent with Friedman's dictum (1969), the Friedman rule corresponds to the limit as $\sigma \to 1/x < 1$. In this case, the rate of return of money is equal to the rate of return of storage.

Here, we assume that the central bank chooses $\sigma \geq 1/x$ in order to maximize the equally-weighted sum of objective functions of banks at all island, i.e.,

$$\sum_{n=0}^{N} \frac{1}{1+N} \frac{(\omega + \tau^n)^{1-\rho}}{1-\rho} \left[\pi_n^\rho \left(\frac{\gamma_n(\sigma)}{\sigma} \right)^{1-\rho} + (1-\pi_n)^\rho \left(x(1-\gamma_n(\sigma)) \right)^{1-\rho} \right],$$

subject to $\tau^n = [(\sigma - 1)/\sigma] m^n$ for each $n = 1, \ldots, N$. Let $\Omega_0(\sigma) := \omega/[1 - (1-\sigma)\gamma_0(\sigma)/\sigma]$, $\Omega(\sigma) := \omega/[1 - (1-\sigma)\gamma_1(\sigma)/\sigma]$, $\Gamma_0(\sigma) := \pi^\rho \left(\gamma_0(\sigma)/\sigma\right)^{1-\rho} + (1-\pi)^\rho \left(x(1-\gamma_0(\sigma))\right)^{1-\rho}$, and $\Gamma(\sigma) := (\pi/N)^\rho \left(\gamma_1(\sigma)/\sigma\right)^{1-\rho} + (1-\pi/N)^\rho \left(x(1-\gamma_1(\sigma))\right)^{1-\rho}$. Then, the objective function of the central bank can be rewritten as

$$W(\sigma) := \frac{1}{1+N} \frac{[\Omega_0(\sigma)]^{1-\rho}}{1-\rho} \Gamma_0(\sigma) + \frac{N}{1+N} \frac{[\Omega(\sigma)]^{1-\rho}}{1-\rho} \Gamma(\sigma).$$

We can now obtain our main result on the suboptimality of the Friedman rule.

Proposition 2 *The Friedman rule does not maximize social welfare and the maximizing rate of growth of the money supply is, if any, strictly greater than* $1/x$.

Finally, we should mention a role of asymmetric liquidity shocks. By the fact that $\gamma_n(\sigma)$, $n = 1, \ldots, N$, converges to 0 as $N \to \infty$, we can say that the economy (except for 0-th island) converges to the autarkic situation when the number of islands is sufficiently large. This is because the size of liquidity shocks on n-th island, π/N, decreases when N increases; i.e., the bank in n-th island can ignore the liquidity shocks when the number of islands is sufficiently large. Then, while money still circulates in 0-th island, we can show that the social welfare is independent of monetary policy.

Proposition 3 *As* $N \to \infty$, *the economy (except for* 0*-th island) converges to the autarkic situation and there is no room for monetary policy to improve social welfare.*

5 Proofs

Proof of Proposition 1. The statement (a) follows immediately from Eq.(4). Convergences of $\gamma_n(\sigma)$ in the statement (b) is straightforward because $\sigma x \to 1$ as $\sigma \to 1/x$. Convergences of consumptions follows from Eqs. (2) and (3). Finally,

$$\gamma_n'(\sigma) = \frac{1-\rho}{\rho}\frac{\gamma_n(\sigma)}{\sigma}(\gamma_n(\sigma)-1) \leq 0$$

because $\gamma_n(\sigma) \leq 1$. This completes the proof of Proposition 1. Q.E.D.

Proof of Proposition 2. For each $\sigma \geq 1/x$,

$$\begin{aligned}\Gamma_0'(\sigma) &= \pi^\rho(1-\rho)\left[\frac{\gamma_0'(\sigma)}{\sigma}\left(\frac{\gamma(\sigma)}{\sigma}\right)^{-\sigma} - \frac{1}{\sigma}\left(\frac{\gamma(\sigma)}{\sigma}\right)^{1-\rho}\right] - (1-\rho)\gamma_0'(\sigma)\left(\frac{1-\pi}{1-\gamma_0(\sigma)}\right)^\rho x^{1-\rho} \\ &= -\pi^\rho(1-\rho)\frac{1}{\sigma}\left(\frac{\gamma_0(\sigma)}{\sigma}\right)^{1-\sigma} < 0,\end{aligned}$$

since $\pi^\rho(\gamma_0(\sigma)/\sigma)^{-\rho}/\sigma - [(1-\pi)/(1-\gamma_0(\sigma))]^\rho x^{1-\rho} = 0$. Similarly, $\Gamma'(\sigma) < 0$.

By differentiating W, we can obtain that

$$\begin{aligned}W'(\sigma) &= \frac{1}{1+N}\left[[\Omega_0(\sigma)]^{-\rho}\Omega_0'(\sigma)\Gamma_0(\sigma) + \frac{[\Omega_0(\sigma)]^{1-\rho}}{1-\rho}\Gamma_0'(\sigma)\right] \\ &\quad + \frac{N}{1+N}\left[[\Omega(\sigma)]^{-\rho}\Omega'(\sigma)\Gamma(\sigma) + \frac{[\Omega(\sigma)]^{1-\rho}}{1-\rho}\Gamma'(\sigma)\right].\end{aligned}$$

Therefore, $W'(\sigma) > 0$ if

$$\Omega_0'(\sigma)\frac{\sigma}{\Omega_0(\sigma)} > -\frac{1}{1-\rho}\Gamma_0'(\sigma)\frac{\sigma}{\Gamma_0(\sigma)} \quad \text{and} \quad \Omega'(\sigma)\frac{\sigma}{\Omega(\sigma)} > -\frac{1}{1-\rho}\Gamma'(\sigma)\frac{\sigma}{\Gamma(\sigma)}.$$

Furthermore, we can obtain that

$$\begin{aligned}\Omega_0'(\sigma)\frac{\sigma}{\Omega_0(\sigma)} &= \frac{\gamma_0(\sigma)\left[1+\frac{1-\rho}{\rho}(\sigma-1)(\gamma_0(\sigma)-1)\right]}{\sigma-\gamma_0(\sigma)(\sigma-1)}, \\ \Omega'(\sigma)\frac{\sigma}{\Omega(\sigma)} &= \frac{\gamma(\sigma)\left[1+\frac{1-\rho}{\rho}(\sigma-1)(\gamma(\sigma)-1)\right]}{\sigma-\gamma(\sigma)(\sigma-1)}, \\ -\frac{1}{1-\rho}\Gamma_0'(\sigma)\frac{\sigma}{\Gamma_0(\sigma)} &= \gamma_0(\sigma) \\ -\frac{1}{1-\rho}\Gamma'(\sigma)\frac{\sigma}{\Gamma(\sigma)} &= \gamma(\sigma).\end{aligned}$$

Now, it follows that $W'(\sigma) > 0$ if

$$\frac{1+\frac{1-\rho}{\rho}(\sigma-1)(\gamma_0(\sigma)-1)}{\sigma-\gamma_0(\sigma)(\sigma-1)} > 1 \quad \text{and} \quad \frac{1+\frac{1-\rho}{\rho}(\sigma-1)(\gamma(\sigma)-1)}{\sigma-\gamma(\sigma)(\sigma-1)} > 1,$$

Both of the last two inequalities are equivalent to $\sigma < 1$. Therefore, $W'(\sigma) > 0$ for $\sigma < 1$, which implies the statement of Proposition 2. Q.E.D.

Proof of Proposition 3. As remarked in Section 3, $\gamma_n(\sigma) \to 0$ as $N \to \infty$ for each $n = 1, \ldots, N$. This implies that the economy, except for 0-th island, converges to the autarkic situation. Furthermore, one can easily observe that

$$W(\sigma) \to \frac{(x\omega)^{1-\rho}}{1-\rho}$$

as $N \to \infty$. Because social welfare is independent of the money growth rate, there is no room for monetary policy to improve social welfare. Q.E.D.

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