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An Equilibrium Foundation of the Soros Chart

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Abstract

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1. Introduction

Understanding of bumpy unpredictable movements in nominal exchange rates with solid economic reasoning is always a challenging business. Since Meese and Rogoff's (1983) seminal exercise, a random walk has been recognized as a primary property of flexible nominal exchange rates in post-Bretton Woods samples of major advanced economies.[1] The fact that nominal exchange rates are most described by a naïve random walk statistical model has negated past attempts of academic researchers to enjoy equilibrium models of nominal exchange rates and of policy makers to extract macroeconomic policy implications. Nominal exchange rates resemble a beast that resists a casual explanation stubbornly.

A random walk is also a major characteristic of the Japanese yen/U.S. dollar nominal exchange rate, at least, after the Plaza Accord in 1985. In fact, the serial correlation of the currency return of the Japanese yen against the U.S. dollar is estimated to be statistically low and economically negligible. Moreover, the Japanese yen/U.S. dollar rate seems to be disconnected with any real economic variables such as output and consumption. Neither common trend nor common cycle does it share with both the output and consumption differentials between the two major exchange rate floaters.

Nevertheless, there are two outstanding statistical properties of the Japanese yen/U.S. dollar exchange rate to be noted for profoundly figuring out nominal exchange rate fluctuations. As the first property, the Soros chart is well-known anecdotal evidence that the Japanese yen/U.S. dollar exchange rate is traced by the two countries' relative size of money supply.[2] Figures 1(a) and (b) are two versions of the Soros chart. The former plots the logarithm of the Japanese yen/U.S. dollar rate (the solid black line) and the differential in the log of the monetary base between Japan and the United States (the dashed blue line). This version of the Soros chart appears unsuccessful. In particular, after 2001 when the Bank of Japan (BOJ) initiated the first quantitative easing (QE) policy, the monetary base differential moves far apart from the Japanese yen/U.S. dollar exchange rate. This failure of the first Soros chart stays obvious even after the Lehman shock with subsequent QE policies conducted by the Federal Reserve System (Fed).

The reason behind the failure of the first version of the Soros chart clearly stems from the massive accumulation of the excess reserves at the BOJ and the Fed through the unconventional monetary policies after 2001. Figure 1(b) depicts the logarithm of the Japanese yen/U.S. dollar exchange rate (the solid black line) and the Japan/U.S. differential in the logarithm of the monetary

[1] Engel (2014) provides the most recent survey on past studies on nominal exchange rates.

[2] The Soros chart is named after George Soros who pointed out this anecdotal evidence behind the Japanese yen/U.S. dollar nominal exchange rate. See, for example, an article in the Nikkei Shinbun news paper on April 6, 2013 entitled "Kuroda Kanwa: Enyasu kouka wo tsuyoku ishiki (Monetary easing by Mr.Kuroda: Strong intention to the Japanese yen depreciation)"

base augmented by subtracting the excess reserve from each country's monetary base (the dashed blue line). Observe that the augmented Soros chart traces the low-frequency slow-moving component of the Japanese yen/U.S. dollar exchange rate surprisingly well. It, therefore, is empirically plausible that the augmented monetary base differential shares a common stochastic trend with the Japanese yen/U.S. dollar exchange rate.

The second outstanding property is the historically tight linkage between the two countries' interest rate differential and the low-frequency component of the currency return (i.e., the depreciation rate) of the Japanese yen against the U.S. dollar. Figure 2 displays the differential of the three-month Treasury Bill rates between the two countries (the black line) on the left axis and the currency return of the Japanese yen against the U.S. dollar (the green line) on the right axis, where each time series is demeaned by its own unconditional mean. Notice that the interest rate differential comoves tightly with the slow-moving component of the currency return, at least, prior to the Lehman shock when the Fed started the zero interest rate policy. This fact suggests that the conventional uncovered interest parity (UIP) condition is likely to capture an important low-frequency property of the Japanese yen/U.S. dollar foreign exchange market in the post-Plaza Accord sample.

In this paper, we develop a simple two-country incomplete-market model that can describe the above major characteristics of the post-Plaza Accord sample of the Japanese yen/U.S. dollar exchange rate. The sample moments our model targets include (i) the two versions of the Soros chart, i.e., the adjusted monetary base differential forms the stochastic trend of the Japanese yen/U.S. dollar rate, (ii) the near random-walk behavior of the Japanese yen/U.S. dollar rate with a negligible serial correlation of the currency return, (iii) the disconnection of the Japanese yen/U.S. dollar rate with real output and consumption differentials, and (iv) the historically tight linkage between the interest rate differential and the currency return. Recently, Kano (2014) theoretically establishes the equilibrium random walk property of nominal exchange rates within a canonical two-country incomplete-market model for the post-Bretton Woods sample of Canada and the United States.[3] In this paper, we extend Kano's (2014) exercise by explicitly modeling a money creation process in the reserve market to describe the two versions of the Soros chart simultaneously. Exploiting the post-Plaza Accord sample of Japan and the United States, we then estimate the proposed two-country model through a Bayesian restricted unobserved component approach. To our best knowledge, this paper is the first attempt to figure out the Soros chart within an equilibrium open-economy model with solid microfoundations.

[3] The important predecessors of this paper are Engel and West (2005), Nason and Rogers (2008), and Kano (2014). Engel and West (2005) establish the equilibrium random walk property in a partial equilibrium asset approach of nominal exchange rates when economic fundamentals are I(1) and the discount factor approaches one. Nason and Rogers (2008) show that the equilibrium random walk property holds in a two-country incomplete market model. Kano (2014) confirms Nason and Rogers's (2008) claim even when the two-country model of Nason and Rogers is closed suitably to find a balanced growth path with a stationary net foreign asset distribution.

Section 2 introduces our two-country model. Section 3 establishes the equilibrium random walk property of the nominal exchange rate. Section 4 describes the Bayesian unobserved component approach of this paper and reports the empirical results. Section 5 concludes.

2. A two-country incomplete-market model for the Soros chart

2.1. The model

In this paper, we extend the canonical incomplete market model with two countries, which is investigated in Kano (2014), for understanding the Soros chart. Consider the home h and foreign f countries. Each country is endowed with a representative household whose objective is the lifetime money-in-utility

$$\sum_{j=0}^{\infty} \beta^j E_t \left\{ \ln C_{i,t+j} + \Gamma_{i,t+j} \ln \left(\frac{M^d_{i,t+j}}{P_{i,t+j}} \right) \right\}, \quad 0 < \beta < 1, \quad \text{for } i = h, f,$$

where $C_{i,t}$, $M^d_{i,t}$, and $P_{i,t}$ represent the ith country's consumption, money demand, and price index, respectively. The money-in-utility function is subject to a preference shock $\Gamma_{i,t}$. The representative households in the home and foreign countries maximize their lifetime utility functions subject to the home budget constraint

$$B^h_{h,t}+S_tB^f_{h,t}+P_{h,t}C_{h,t}+M^d_{h,t} = (1+r^h_{h,t-1})B^h_{h,t-1}+S_t(1+r^f_{h,t-1})B^f_{h,t-1}+M^d_{h,t-1}+P_{h,t}Y_{h,t}+T_{h,t}, \quad (1)$$

and its foreign counterpart

$$\frac{B^h_{f,t}}{S_t} + B^f_{f,t} + P_{f,t}C_{f,t} + M^d_{f,t} = (1+r^h_{f,t-1})\frac{B^h_{f,t-1}}{S_t} + (1+r^f_{f,t-1})B^f_{f,t-1} + M^d_{f,t-1} + P_{f,t}Y_{f,t} + T_{f,t}, \quad (2)$$

respectively, where $B^l_{i,t}$, $r^l_{i,t}$, $Y_{i,t}$, $T_{i,t}$, and S_t denote the ith country's holdings of the lth country's nominal bonds at the end of time t, the ith county's returns on the lth country's bonds, the ith country's output level, the ith country's government transfers, and the level of the bilateral nominal exchange rate, respectively. Each country's output $Y_{i,t}$ is given as an exogenous endowment following a stochastic process $Y_{i,t} = y_{i,t}A_{i,t}$, where $y_{i,t}$ is the transitory component and $A_{i,t}$ is the permanent component. Below, we interpret the permanent component $A_{i,t}$ as the TFP in the underlying production technology.

The first-order necessary conditions (FONCs) of the home country's household are given by the budget constraint (1), the Euler equation

$$\frac{1}{P_{h,t}C_{h,t}} = \beta(1 + r^h_{h,t})E_t \left(\frac{1}{P_{h,t+1}C_{h,t+1}} \right), \quad (3)$$

the utility-based uncovered parity condition (UIP)

$$(1+r^h_{h,t})E_t\left(\frac{1}{P_{h,t+1}C_{h,t+1}}\right)=\frac{(1+r^f_{h,t})}{S_t}E_t\left(\frac{S_{t+1}}{P_{h,t+1}C_{h,t+1}}\right), \tag{4}$$

and the money demand function

$$\frac{M^d_{h,t}}{P_{h,t}}=\Gamma_{h,t}\left(\frac{1+r^h_{h,t}}{r^h_{h,t}}\right)C_{h,t}. \tag{5}$$

The foreign country's FONC counterparts are the budget constraint (2), the Euler equation

$$\frac{1}{P_{f,t}C_{f,t}}=\beta(1+r^f_{f,t})E_t\left(\frac{1}{P_{f,t+1}C_{f,t+1}}\right), \tag{6}$$

the utility-based uncovered parity condition (UIP)

$$(1+r^h_{f,t})E_t\left(\frac{1}{S_{t+1}P_{f,t+1}C_{f,t+1}}\right)=\frac{(1+r^f_{f,t})}{S_t}E_t\left(\frac{1}{P_{f,t+1}C_{f,t+1}}\right), \tag{7}$$

and the money demand function

$$\frac{M^d_{f,t}}{P_{f,t}}=\Gamma_{f,t}\left(\frac{1+r^f_{f,t}}{r^f_{f,t}}\right)C_{f,t}. \tag{8}$$

The most important extension of this model from Kano's (2014) is found in the paper's explicit modeling of a money creation process, which specifies the linkage among money supply $M_{i,t}$, monetary base $H_{i,t}$, and excess reserve $ER_{i,t}$ in country $i=h,f$. The monetary base consists of cash in circulation $V_{i,t}$, required reserve $RR_{i,t}$, and excess reserve $ER_{i,t}$ held by private banks in the accounts at the central bank of country i:

$$H_{i,t}=V_{i,t}+RR_{i,t}+ER_{i,t}, \quad \text{for } i=h,f. \tag{9}$$

The money supply is defined as the sum of the cash in circulation and demand deposit at private banks denoted by $D_{i,t}$:

$$M_{i,t}=V_{i,t}+D_{i,t}, \quad \text{for } i=h,f. \tag{10}$$

Let $v_{i,t}\in(0,1)$ denote the ratio of the cash to the deposit, $V_{i,t}/D_{i,t}$. Similarly, let $rr_{i,t}\in(0,1)$ denote the required reserve rate, $RR_{i,t}/D_{i,t}$. From eqs (9) and (10), we can derive the following money creation process

$$M_{i,t}=\Psi_{i,t}(H_{i,t}-ER_{i,t})=\Psi_{i,t}(1-er_{i,t})H_{i,t}, \tag{11}$$

where $\Psi_{i,t}=(1+v_{i,t})/(rr_{i,t}+v_{i,t})>1$ is the money multiplier and $er_{i,t}$ is the ratio of the excess reserve to the monetary base, $ER_{i,t}/H_{i,t}$. In this paper, we assume that both the money multiplier and the excess reserve ratio follow exogenous stochastic processes that we specify below more in details.[4]

The central bank of country i controls for the monetary base. We decompose the monetary base into permanent and transitory components $H^{\tau}_{i,t}$ and $h_{i,t}$: $H_{i,t} \equiv h_{i,t} H^{\tau}_{i,t}$. Then, from eq.(11), the money supply also contains a permanent component:

$$M_{i,t} = h_{i,t} \Psi_{i,t}(1 - er_{i,t}) H^{\tau}_{i,t} = h_{i,t} M^{\tau}_{i,t}.$$

where $M^{\tau}_{i,t}$ is the permanent component of the money supply, $\Psi_{i,t}(1 - er_{i,t}) H^{\tau}_{i,t}$.[5] Each country's government transfers the seigniorage collected through the money creation process (11) to the household as a lump-sum. Hence, the government's budget constraint is

$$M_{i,t} - M_{i,t-1} = T_{i,t}, \quad \text{for } i = h, f.$$

To close the model within an incomplete international financial market, we allow for a debt-elastic risk premium in the interest rates faced only by the home country:

$$r^l_{h,t} = r^l_{w,t}[1 + \psi\{\exp(-B^l_{h,t}\Gamma_{l,t}/M^{\tau}_{l,t} + \bar{d}) - 1\}], \quad \bar{d} \leq 0, \quad \psi > 0, \quad \text{for} \quad l = h, f \tag{12}$$

where $r^l_{w,t}$ is the equilibrium world interest rate of the lth country's bond. Notice that the home country needs to pay a risk premium over the world interest rate level $r^i_{w,t}$ when the transitory components of the home country's net foreign debt positions $B^i_{h,t}\Gamma_{l,t}/M^{\tau}_{l,t} < 0$ is beyond its threshold level $\bar{d} < 0$. The risk premium is given as an externality: The household does not take into account the effect of the debt position on the risk premium when maximizing the lifetime utility function. On the other hand, we do not attach a risk premium to the foreign country's interest rates: $r^l_{f,t} = r^l_{w,t}$ for $l = h, f$.[6]

[4]This assumption of the exogenous money multiplier and excess reserve, of course, is counterfactual and it would be better to endogenize the determination of them for a better understanding of the reserve markets of the two countries. This assumption prevents from deriving a realistic monetary policy implication on the exchange rate from the model. Nevertheless, we think that this extreme assumption is innocuous to the primary purpose of this paper, which is not to describe the monetary policy frameworks and their transmission mechanisms of the two countries precisely, but to replicate the empirical fact that the adjusted monetary base differential forms the stochastic trend of the near random-walk Japanese yen/U.S. dollar exchange rate within a two-country model as parsimonious as possible. For this purpose, taking a strategy of explaining the money multiplier and the excess reserves by independent exogenous stochastic processes, we rather focus on the endogenous determination of the near random-walk exchange rate. We appreciate Kozo Ueda's correctly pointing out this limitation of the paper.

[5]Therefore, the permanent component of the money supply depends on the money multiplier that also relies on the household's portfolio choice between the cash and the demand deposit.

[6]Since the elasticity of the risk premium toward the debt position, ψ, is set to a very small number, this asymmetric treatment of the debt elastic risk premium between the home and foreign countries does not affect the equilibrium outcome much.

The purchasing power parity (PPP) is assumed to hold only up to a persistent PPP deviation shock $\ln q_t$:

$$S_t P_{f,t} = P_{h,t} q_t.$$

The market-clearing conditions of the two countries' bond markets are

$$B^h_{h,t} + B^h_{f,t} = 0 \quad \text{and} \quad B^f_{h,t} + B^f_{f,t} = 0,$$

i.e., along an equilibrium path, the world net supply of nominal bonds is zero on a period-by-period basis.

As claimed by Kano (2014), to find a balanced growth path in the two-country incomplete market model, the permanent TFPs of the two countries need to be cointegrated in the long run. Fo this purpose, we assume that the logarithm of the total factor productivity (TFP) of each country is I(1) and the cross-country TFP differential, $\ln a_t \equiv \ln A_{h,t}/A_{f,t}$, is I(0). This assumption requires that two country's TFPs must be cointegrated. Hence, we specify the TFP processes as the following error correction models (ECMs)

$$\begin{aligned} \Delta \ln A_{h,t} &= \ln \gamma_A - \frac{\lambda}{2}(\ln A_{h,t-1} - \ln A_{f,t-1}) + \epsilon^h_{A,t}, \\ \Delta \ln A_{f,t} &= \ln \gamma_A + \frac{\lambda}{2}(\ln A_{h,t-1} - \ln A_{f,t-1}) + \epsilon^f_{A,t}, \end{aligned} \tag{13}$$

where $\gamma_A > 1$ is the common drift term and $\lambda \in [0, 1)$ is the adjustment speed of the error correction mechanism. $\epsilon^h_{A,t}$ and $\epsilon^f_{A,t}$, respectively, are the i.i.d. shocks to the home and foreign TFPs. ECMs (13) imply that the cross-country TFP differential is I(0) because

$$\ln a_t = (1 - \lambda) \ln a_{t-1} + \epsilon^h_{A,t} - \epsilon^f_{A,t}.$$

Importantly, if the adjustment speed λ is sufficiently close to zero, the cross-country TFP differential can be realized near I(1), as maintained by Nason and Rogers (2008).

We assume the logarithm of the permanent component of the monetary base of each country, $\ln H^\tau_{i,t}$, to be I(1). Moreover, we allow for a two-period ahead news shock to the permanent component of the monetary base, ξ_t, to identify anticipated permanent changes in the monetary policy.[7] We then specify each country's monetary base growth rate $\Delta \ln H^\tau_{i,t} \equiv \gamma^i_{H,t}$ as the following stochastic process:

$$\gamma^i_{H,t} = (1 - \rho_H) \ln \gamma_H + \rho_H \gamma^i_{H,t-1} + \xi^i_{t-2} + \epsilon^i_{H,t}, \quad 0 < \rho_H < 1, \quad \text{for } i = h, f.$$

where $\ln \gamma_H$ is the mean of the monetary base growth rate common to the two countries. The news shock ξ^i_t is assumed to an i.i.d. shock $\epsilon^i_{\xi,t}$. Importantly, this specification implies that the cross-country differential in the permanent component of the monetary base between the two countries, $\ln H_t \equiv \ln H^\tau_{h,t}/H^\tau_{f,t}$ is I(1).

[7]That is, news shock ξ_t is a shock to the future permanent component of the monetary base $\ln H^\tau_{i,t+2}$, which is realized at period t.

The fraction of the non-excess reserve component in the total monetary base, $\ln(1-er_{i,t})$, is also assumed to be I(1) and so is the corresponding cross country differential, $\ln(1-er_t) \equiv \ln(1-er_{h,t})/(1-er_{f,t})$.[8] Therefore, we specify the growth rate of each country's non-excess reserve component of the total monetary base $\Delta \ln(1-er_{i,t}) \equiv \gamma^i_{er,t}$ to be independent AR(1) process:

$$\gamma^i_{er,t} = \rho_{er}\gamma^i_{er,t-1} + \eta^i_{t-2} + \epsilon^i_{er,t}, \quad 0 < \rho_{er} < 1, \quad \text{for } i = h, f.$$

where η^i_t is a two-period ahead news shock to the non-excess reserve component. News shock η_t is assumed to an i.i.d. white noise $\epsilon_{\eta,t}$. We characterize the stochastic processes of the preference shocks and the money multipliers, $\Gamma_{i,t}$ and $\Psi_{i,t}$ for $i = h, f$, jointly as a single I(1) permanent stochastic process. Let define a new variable $\Phi_{i,t}$ by $\Gamma_{i,t}/\Psi_{i,t}$ for $i = h, f$. Then the growth rate of the variable $\Delta \ln \Phi_{i,t} \equiv \gamma^i_{\Phi,t}$ is

$$\gamma^i_{\Phi,t} = \rho_{\Phi}\gamma^i_{\Phi,t-1} + \epsilon^i_{\Phi,t}, \quad 0 < \rho_{\Phi} < 1, \quad \text{for } i = h, f.$$

We call the variable $\Phi_{i,t}$ the money demand shock throughout the paper below.[9]

The stochastic processes of the other structural shocks are assumed to be stationary. The logarithm of the transitory output component for each country, $\ln y_{i,t}$, is specified as the following AR(1) process:

$$\ln y_{i,t} = (1-\rho_y)\ln y_i + \rho_y \ln y_{i,t-1} + \epsilon^i_{y,t}, \quad 0 < \rho_y < 1, \quad \text{for } i = h, f.$$

Similarly, the stochastic process of the logarithm of the transitory monetary base component for each country, $\ln h_{i,t}$, is specified as the following AR(1) process:

$$\ln h_{i,t} = (1-\rho_h)\ln h_i + \rho_h \ln h_{i,t-1} + \epsilon^i_{h,t}, \quad 0 < \rho_h < 1, \quad \text{for } i = h, f.$$

The PPP shock q_t follows an AR(1) process.

$$\ln q_t = \rho_q \ln q_{t-1} + \epsilon_{q,t}, \quad 0 < \rho_q < 1.$$

Throughout this paper, we assume that all structural shocks are distributed independently.

2.2. The log-linear approximation of the stochastically de-trended system

[8] Because, by construction, $\ln(1-er_{i,t})$ is a bounded sequence, its specification by an I(1) process is, in fact, statistically irrelevant. It is not obvious and quite difficult to find a suitable stationary process to fit to the data of the excess reserves in Japan and the U.S. Hence, in this paper, we tract the data of the excess reserves in the two countries by independent stochastic trends.

[9] Notice that our definition of the money demand shock contains shocks to the money demand functions and the money multipliers. The money multiplier depends on the cash-deposit ratio, which should be determined by the portfolio deception of the households between cash and demand deposit. We include any time-series variations in the portfolio decision into the money demand shocks.

Define stochastically de-trended variables as $c_{i,t} \equiv C_{i,t}/A_{i,t}$, $p_{i,t} \equiv P_{i,t}A_{i,t}\Gamma_{i,t}/M^{\tau}_{i,t}$, $b^{l}_{i,t} \equiv B^{l}_{i,t}\Gamma_{l,t}/M^{\tau}_{l,t}$, $\gamma^{i}_{A,t} \equiv A_{i,t}/A_{i,t-1}$, $\gamma^{i}_{M,t} \equiv M^{\tau}_{i,t}/M^{\tau}_{i,t-1}$, $\gamma^{i}_{\Gamma,t} = \Gamma_{i,t}/\Gamma_{i,t-1}$, and $s_t \equiv S_t M^{\tau}_{f,t}\Gamma_{h,t}/(M^{\tau}_{h,t}\Gamma_{f,t})$. In Appendix A, we summarize the stochastically de-trended FONCs of the home and foreign countries. The resulting ten equations determine the ten endogenous variables $c_{h,t}$, $c_{f,t}$, $p_{h,t}$, s_t, $b^{h}_{h,t}$, $b^{f}_{h,t}$, $r^{h}_{h,t}$, $r^{f}_{h,t}$, $r^{h}_{w,t}$, and $r^{f}_{w,t}$, given nine exogenous variables $\gamma^{h}_{H,t}$, $\gamma^{f}_{H,t}$, $\gamma^{h}_{\Gamma,t}$, $\gamma^{f}_{\Gamma,t}$, a_t, $h_{h,t}$, $h_{f,t}$, $y_{h,t}$, and $y_{f,t}$.

Let $\hat{x}$ denote a percentage deviation of any variable x_t from its deterministic steady state value x^*, $\hat{x} \equiv \ln x_t - \ln x^*$. Below, the steady state value of the nominal market discount factor is denoted by $\kappa \equiv 1/(1+r^*) = \beta/\gamma_H$. Also, let $\tilde{x}$ denote a deviation of x from its deterministic steady state, $\tilde{x} = x - x^*$.[10] The log-linear approximation of the stochastically de-trended home budget constraint is

$$p^*_h(c^*_h - y_h)\hat{p}_{h,t} + p^*_h c^*_h \hat{c}_{h,t} - p^*_h y_h \hat{y}_{h,t} + \tilde{b}^{h}_{h,t} + \bar{d}(1-\beta^{-1})s^*\hat{s}_t + s^*\tilde{b}^{f}_{h,t}$$
$$= \beta^{-1}\bar{d}[(1+\hat{r}^{h}_{h,t-1}) - \hat{\gamma}^{h}_{M,t} + \hat{\gamma}^{h}_{\Gamma,t-1}] + s^*\beta^{-1}\bar{d}[(1+\hat{r}^{f}_{h,t-1}) - \hat{\gamma}^{f}_{M,t} + \hat{\gamma}^{f}_{\Gamma,t}] + \beta^{-1}\tilde{b}^{h}_{h,t-1} + s^*\beta^{-1}\tilde{b}^{f}_{h,t-1}; \tag{14}$$

that of the home Euler equation is

$$\hat{p}_{h,t} + \hat{c}_{h,t} + (1+\hat{r}^{h}_{h,t}) = E_t(\hat{p}_{h,t+1} + \hat{c}_{h,t+1} + \hat{\gamma}^{h}_{M,t+1} - \hat{\gamma}^{h}_{\Gamma,t+1}); \tag{15}$$

that of the home UIP condition is

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}^{h}_{h,t}) - (1+\hat{r}^{f}_{h,t}) + E_t(\hat{\gamma}^{h}_{\Gamma,t+1} - \hat{\gamma}^{f}_{\Gamma,t+1} - \hat{\gamma}^{h}_{M,t+1} + \hat{\gamma}^{f}_{M,t+1}); \tag{16}$$

and that of the home money demand function is

$$\hat{p}_{h,t} + \hat{c}_{h,t} - \hat{m}_{h,t} = \frac{1}{r^*}(1+\hat{r}^{h}_{h,t}). \tag{17}$$

The foreign country's counterparts are the log-linear approximation of the stochastically de-trended foreign budget constraint

$$p^*_h(c^*_f - y_f)(\hat{p}_{h,t} + \hat{q}_t - \hat{a}_t - \hat{s}_t) + p^*_h c^*_f \hat{c}_{f,t} - p^*_h y_f \hat{y}_{f,t} - \tilde{b}^{h}_{h,t} - \bar{d}(1-\beta^{-1})s^*\hat{s}_t - s^*\tilde{b}^{f}_{h,t}$$
$$= -\beta^{-1}\bar{d}[(1+\hat{r}^{h}_{w,t-1}) - \hat{\gamma}^{h}_{M,t} + \hat{\gamma}^{h}_{\Gamma,t}] - s^*\beta^{-1}\bar{d}[(1+\hat{r}^{f}_{w,t-1}) - \hat{\gamma}^{f}_{M,t} + \hat{\gamma}^{f}_{\Gamma,t}] - \beta^{-1}\tilde{b}^{h}_{h,t-1} - s^*\beta^{-1}\tilde{b}^{f}_{h,t-1}; \tag{18}$$

that of the foreign Euler equation

$$\hat{a}_t + \hat{s}_t - \hat{p}_{h,t} - \hat{c}_{f,t} - \hat{q}_t - (1+\hat{r}^{f}_{w,t}) = E_t(\hat{a}_{t+1} + \hat{s}_{t+1} - \hat{p}_{h,t+1} - \hat{c}_{f,t+1} - \hat{q}_{t+1} - \hat{\gamma}^{f}_{M,t+1} + \hat{\gamma}^{f}_{\Gamma,t+1}); \tag{19}$$

that of the foreign UIP condition

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}^{h}_{w,t}) - (1+\hat{r}^{f}_{w,t}) + E_t(\hat{\gamma}^{h}_{\Gamma,t+1} - \hat{\gamma}^{f}_{\Gamma,t+1} - \hat{\gamma}^{h}_{M,t+1} + \hat{\gamma}^{f}_{M,t+1}); \tag{20}$$

[10] In particular, for an interest rate r_t, $(1+\hat{r}_t) = (r_t - r^*)/(1+r^*)$.

and that of the home money demand function

$$\hat{a}_t + \hat{s}_t + \hat{m}_{f,t} - \hat{p}_{h,t} - \hat{c}_{f,t} - \hat{q}_t = -\frac{1}{r^*}(1 + \hat{r}^f_{w,t}). \tag{21}$$

The log-linear approximations of the home country's interest rates are

$$(1 + \hat{r}^h_{h,t}) = (1 + \hat{r}^h_{w,t}) - \psi(1-\kappa)\tilde{b}^h_{h,t}, \quad \text{and} \quad (1 + \hat{r}^f_{h,t}) = (1 + \hat{r}^f_{w,t}) - \psi(1-\kappa)\tilde{b}^f_{h,t}. \tag{22}$$

Notice that the home interest rates (22) redefine the home UIP condition (16) as

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1 + \hat{r}^h_{w,t}) - (1 + \hat{r}^f_{w,t}) - \psi(1-\kappa)(\tilde{b}^h_{h,t} - \tilde{b}^f_{h,t}) + E_t(\hat{\gamma}^h_{\Gamma,t+1} - \hat{\gamma}^f_{\Gamma,t+1} - \hat{\gamma}^h_{M,t+1} + \hat{\gamma}^f_{M,t+1}).$$

Comparing the above home UIP condition with the foreign UIP condition (20) implies that the home and foreign bonds are perfectly substitutable along the equilibrium path. Hence, the equilibrium condition $\tilde{b}_t \equiv \tilde{b}^h_{h,t} = \tilde{b}^f_{h,t}$ holds.[11]

3. Equilibrium random-walk property

We will now show that the equilibrium random-walk property of the exchange rate holds in this two-country model. To prove this proposition, we first characterize a unique analytical solution of the exchange rate as expected present discounted values of future economic fundamentals. Below we call this analytical solution of the exchange rate the present value model (PVM) of the exchange rate. Let c_t and h_t denote the de-trended consumption ratio and the transitory monetary base ratio between the two countries, $c_t \equiv c_{h,t}/c_{f,t}$ and $h_t \equiv h_{h,t}/h_{f,t}$, respectively. Furthermore, let M^τ_t denote the ratio of the permanent money supplies of the home and foreign countries $M^\tau_{h,t}/M^\tau_{f,t}$; let M_t denote the ratio of the money supplies of the home to the foreign countries $M_{h,t}/M_{f,t} \equiv h_t M^\tau_t$; let C_t denote the ratio of the consumptions of the home and foreign countries $C_{h,t}/C_{f,t}$; and let Γ_t denote the ratio of the preference shocks of the home and foreign countries $\Gamma_t = \Gamma_{h,t}/\Gamma_{f,t}$. The home and foreign money demand functions, (17) and (21), and the home interest rates (22) yield the following interest rate differential:

$$(1 + \hat{r}^h_{w,t}) - (1 + \hat{r}^f_{w,t}) = r^*(\hat{a}_t + \hat{s}_t + \hat{c}_t - \hat{h}_t - \hat{q}_t) + \psi(1-\kappa)\tilde{b}_t. \tag{23}$$

Substituting the interest rate differential (23) into the foreign UIP condition (20) leads to the expectational difference equation of the de-trended exchange rate $\hat{s}_t$:

$$\hat{s}_t = \kappa E_t\hat{s}_{t+1} - (1-\kappa)(\hat{a}_t + \hat{c}_t) + (1-\kappa)(\hat{h}_t + \hat{q}_t) - \kappa E_t(\hat{\gamma}^h_{\Gamma,t+1} - \hat{\gamma}^f_{\Gamma,t+1} - \hat{\gamma}^h_{M,t+1} + \hat{\gamma}^f_{M,t+1}) - \psi\kappa(1-\kappa)\tilde{b}_t.$$

[11] Appendix B characterizes the equilibrium transitory dynamics of the model for a simplified case including two symmetric countries.

After unwinding stochastic trends, the above expectational difference equation can be rewritten as

$$\ln S_t = \kappa E_t \ln S_{t+1} + (1-\kappa)(\ln M_t - \ln \Gamma_t) - (1-\kappa)\ln C_t + (1-\kappa)\ln q_t - \psi\kappa(1-\kappa)\tilde{b}_t.$$

Solving this expectational difference equation by forward iterations under a suitable transversality condition provides the PVM of this model:

$$\ln S_t = (1-\kappa)\sum_{j=0}^{\infty}\kappa^j E_t\left(\ln M_{t+j} - \ln \Gamma_{t+j} - \ln C_{t+j} - \psi\kappa\tilde{b}_{t+j} + \ln q_{t+j}\right). \tag{24}$$

If the fundamental $\ln M_t - \ln \Gamma_t - \ln C_t$ is I(1), so is the exchange rate.

The PVM (24) implies an error-correction representation of the currency return $\Delta \ln S_t$. Appendix B shows that after rearranging the PVM (24) in several steps, the currency return is

$$\Delta \ln S_t = \frac{1-\kappa}{\kappa}\left(\ln S_{t-1} - \ln M_{t-1} + \ln \Gamma_{t-1} + \ln C_{t-1} - \ln q_{t-1}\right) + \psi(1-\kappa)\tilde{b}_{t-1} + u_{s,t}, \tag{25}$$

where $u_{s,t}$ is the i.i.d., rational expectations error

$$u_{s,t} = (1-\kappa)\sum_{j=0}^{\infty}\kappa^j(E_t - E_{t-1})(\ln M_{t+j} - \ln \Gamma_t - \ln C_{t+j} - \psi\kappa\tilde{b}_{t+j} + \ln q_{t+j}).$$

Recall that the PVM (24) is constructed as an equilibrium condition from a part of the model's FONCs. The general equilibrium property of the model, however, imposes another restriction on the expected present discounted values of the future economic fundamentals in the right hand side of the PVM (24). Note that combining the log-linearized Euler equations of the home and foreign countries, (15) and (19), with those of the home country's interest rates (22), yields the first-order expectational difference equation of $\ln S_t + \ln \Gamma_t - \ln M_t + \ln C_t - \ln q_t$:

$$\begin{aligned}\ln S_t + \ln \Gamma_t - \ln M_t + \ln C_t - \ln q_t &= \kappa E_t(\ln S_{t+1} + \ln \Gamma_{t+1} - \ln M_{t+1} + \ln C_{t+1} - \ln q_{t+1}) \\ &\quad + \kappa\rho_H\hat{\gamma}_{H,t} + \kappa\xi_{t-1} + \kappa\rho_{er}\hat{\gamma}_{er,t} + \kappa\eta_{t-1} - \kappa\rho_\Phi\hat{\gamma}_{\Phi,t} + \kappa(\rho_h - 1)\ln h_t,\end{aligned}$$

where $\hat{\gamma}_{H,t}$, $\hat{\gamma}_{er,t}$, and $\hat{\gamma}_{\Phi,t}$ denote the two-country differentials of the growth rates of the permanent component of monetary base $\hat{\gamma}^h_{H,t} - \hat{\gamma}^f_{H,t}$, the excess reserve $\hat{\gamma}^h_{er,t} - \hat{\gamma}^f_{er,t}$, and the money demand shock $\hat{\gamma}^h_{\Phi,t} - \hat{\gamma}^f_{\Phi,t}$, respectively. Because κ is less than one, the difference equation above has the unique forward solution

$$\begin{aligned}\ln S_t = \ln M_t - \ln \Gamma_t - \ln C_t + \ln q_t &+ \frac{\kappa\rho_H}{1-\kappa\rho_H}\hat{\gamma}_{H,t} + \frac{\kappa}{1-\kappa\rho_H}\xi_{t-1} + \frac{\kappa^2}{1-\kappa\rho_H}\xi_t \\ &+ \frac{\kappa\rho_{er}}{1-\kappa\rho_{er}}\hat{\gamma}_{er,t} + \frac{\kappa}{1-\kappa\rho_{er}}\eta_{t-1} + \frac{\kappa^2}{1-\kappa\rho_{er}}\eta_t - \frac{\kappa\rho_\Phi}{1-\kappa\rho_\Phi}\hat{\gamma}_{\Phi,t} - \frac{\kappa(1-\rho_h)}{1-\kappa\rho_h}\ln h_t\end{aligned} \tag{26}$$

under a suitable transversality condition.

Imposing the cross-equation restriction (CER) (26) on the error-correction process (25) provides the equilibrium currency return

$$\Delta \ln S_t = \psi(1-\kappa)\tilde{b}_{t-1} + \frac{(1-\kappa)\rho_H}{1-\kappa\rho_H}\hat{\gamma}_{H,t-1} + \frac{1-\kappa}{1-\kappa\rho_H}\xi_{t-1} + \frac{\kappa(1-\kappa)}{1-\kappa\rho_H}\xi_t + \frac{(1-\kappa)\rho_{er}}{1-\kappa\rho_{er}}\hat{\gamma}_{er,t-1} + \frac{1-\kappa}{1-\kappa\rho_{er}}\eta_{t-1} + \frac{\kappa(1-\kappa)}{1-\kappa\rho_{er}}\eta_t - \frac{(1-\kappa)\rho_\Phi}{1-\kappa\rho_\Phi}\hat{\gamma}_{\Phi,t-1}\frac{(1-\kappa)(1-\rho_h)}{1-\kappa\rho_h}\ln h_{t-1} + u_{s,t}. \quad (27)$$

The most important implication of the equilibrium currency return equation (27) is that the logarithm of the exchange rate follows a Martingale difference sequence at the limit of $\kappa \to 1$ because

$$\lim_{\kappa\to 1} E_t \Delta \ln S_{t+1} = 0.$$

Therefore, in this paper, the exchange rate behaves like a random walk when the market discount factor approaches one along the equilibrium path of the two-country model. The equilibrium currency return equation (27) exhibits no dependence of the currency return on past information in this case. Hence, the equilibrium random walk property of the exchange rate, as found in Engel and West (2005), Nason and Rogers (2008), and Kano (2014), is also preserved in this extended model.[12]

In the limiting case with the unit market discount factor, the equilibrium currency return is dominated by the i.i.d. rational expectations error $u_{s,t}$. An advantage of working with a structural two-country model is that the rational expectations error $u_{s,t}$ is now fully interpretable as a linear combination of structural shocks. To see this, note that the rational expectations error $u_{s,t}$ in equilibrium is represented by

$$u_{s,t} = (E_t - E_{t-1})\Delta \ln S_t = \epsilon_{H,t} + \epsilon_{er,t} - \epsilon_{\Phi,t} + (E_t - E_{t-1})\hat{s}_t,$$

where $\epsilon_{H,t} \equiv \epsilon^h_{H,t} - \epsilon^f_{H,t}$, $\epsilon_{er,t} \equiv \epsilon^h_{er,t} - \epsilon^f_{er,t}$, and $\epsilon_{\Phi,t} \equiv \epsilon^h_{\Phi,t} - \epsilon^f_{\Phi,t}$. Appendix B shows that in the special case of two symmetric countries, assuming $\bar{d} = 0$ and $y_h = y_f$, the equilibrium de-trended exchange rate is determined by a linear function of $\tilde{b}_{t-1}$, $\hat{a}_t$, $\hat{h}_t$, $\hat{y}_t$, $\hat{q}_t$, ξ_t, $\hat{\gamma}_{H,t}$, $\hat{\gamma}_{er,t}$, $\hat{\gamma}_{\Phi,t}$, ξ_t, ξ_{t-1}, η_t, η_{t-1}:

$$\hat{s}_t = \frac{\beta\eta-1}{\beta p_h^* y^*}\tilde{b}_{t-1} + \frac{\beta\eta-1}{1-\beta\eta(1-\lambda)}\hat{a}_t + \frac{1-\kappa}{1-\kappa\rho_h}\hat{h}_t + \frac{\beta\eta-1}{1-\beta\eta\rho_y}\hat{y}_t - \frac{\beta\eta-1}{1-\beta\eta\rho_q}\hat{q}_t + \frac{\kappa\rho_H}{1-\kappa\rho_H}\hat{\gamma}_{H,t} + \frac{\kappa}{1-\kappa\rho_H}\xi_{t-1} + \frac{\kappa^2}{1-\kappa\rho_H}\xi_t + \frac{\kappa\rho_{er}}{1-\kappa\rho_{er}}\hat{\gamma}_{er,t} + \frac{\kappa}{1-\kappa\rho_{er}}\eta_{t-1} + \frac{\kappa^2}{1-\kappa\rho_{er}}\eta_t - \frac{\kappa\rho_\Phi}{1-\kappa\rho_\Phi}\hat{\gamma}_{\Phi,t} \quad (28)$$

[12] A caveat of the above result is that in this model, κ is given as a function of structural parameters β and γ_H: $\kappa = \beta/\gamma_H$. If $\gamma_H > 1$, as found in the postwar data on money growth rates in Japan and the United States, the admissible range of β between zero and one implies that κ is strictly less than one. In this paper, I assume that the limit of $\kappa \to 1$ is well approximated by the limit of $\beta \to 1$ because γ_H takes a value that is very close to one.

where the constant η approaches one at the limit of $\kappa \to 1$.[13] Hence, the surprise in the de-trended exchange rate between times t and $t-1$ is

$$(E_t - E_{t-1})\hat{s}_t = \frac{\beta\eta - 1}{1-\beta\eta(1-\lambda)}\epsilon_{A,t} + \frac{1-\kappa}{1-\kappa\rho_h}\epsilon_{h,t} + \frac{\beta\eta-1}{1-\beta\eta\rho_y}\epsilon_{y,t} - \frac{\beta\eta-1}{1-\beta\eta\rho_q}\epsilon_{q,t}$$
$$+ \frac{\kappa\rho_H}{1-\kappa\rho_H}\epsilon_{H,t} + \frac{\kappa^2}{1-\kappa\rho_H}\xi_t + \frac{\kappa\rho_{er}}{1-\kappa\rho_{er}}\epsilon_{er,t} + \frac{\kappa^2}{1-\kappa\rho_{er}}\eta_t - \frac{\kappa\rho_\Phi}{1-\kappa\rho_\Phi}\epsilon_{\Phi,t}$$

where $\epsilon_{h,t} \equiv \epsilon^h_{h,t} - \epsilon^f_{h,t}$ and $\epsilon_{y,t} \equiv \epsilon^h_{y,t} - \epsilon^f_{y,t}$. The rational expectations error is then given as an explicit linear function of the structural shocks:

$$u_{s,t} = \frac{\beta\eta - 1}{1-\beta\eta(1-\lambda)}\epsilon_{A,t} + \frac{1-\kappa}{1-\kappa\rho_h}\epsilon_{h,t} + \frac{\beta\eta-1}{1-\beta\eta\rho_y}\epsilon_{y,t} - \frac{\beta\eta-1}{1-\beta\eta\rho_q}\epsilon_{q,t}$$
$$+ \frac{1}{1-\kappa\rho_H}\epsilon_{H,t} + \frac{\kappa^2}{1-\kappa\rho_H}\xi_t + \frac{1}{1-\kappa\rho_{er}}\epsilon_{er,t} + \frac{\kappa^2}{1-\kappa\rho_{er}}\eta_t - \frac{1}{1-\kappa\rho_\Phi}\epsilon_{\Phi,t}$$

Notice that at the limit of $\kappa \to 1$, the model also implies the subjective discount factor $\beta \to 1$ under a positive deterministic money supply growth rate, $\gamma_H > 1$, which is close to one. In this limiting case, observe that the permanent monetary base shock $\epsilon_{H,t}$, the news shocks ξ_t and η_t, the excess reserve shock $\epsilon_{er,t}$, and the money demand shock $\epsilon_{\Phi,t}$ surely dominate the rational expectations error $u_{s,t}$ and, as a result, the random walk of the exchange rate:

$$\lim_{\kappa\to 1}\Delta \ln S_t = \lim_{\kappa,\beta,\eta\to 1} u_{s,t} = \frac{1}{1-\rho_H}\epsilon_{H,t} + \frac{1}{1-\rho_H}\xi_t + \frac{1}{1-\rho_{er}}\epsilon_{er,t} + \frac{1}{1-\rho_{er}}\eta_t - \frac{1}{1-\rho_\Phi}\epsilon_{\Phi,t}. \quad (29)$$

Therefore, no transitory shock matters for the total variations in the random-walk exchange rate. This is because when $\kappa \to 1$, or equivalently, $r^* \to 0$, the interest rate differential (23) becomes insensitive to the transitory money supply and consumption differentials. Hence, the exchange rate turns out to be neutral to any transitory monetary and real shocks.

4. A Bayesian unobserved component approach

4.1. The restricted UC model and posterior simulation strategy

Under the symmetric case with $\bar{d} = 0$ and $y = y_h = y_f$, FONCs (14)-(21) are degenerated to the following three expectational difference equations:

$$\hat{s}_t = \kappa E_t\hat{s}_{t+1} - (1-\kappa)(\hat{c}_t + \hat{a}_t - \hat{h}_t - \hat{q}_t) + \kappa E_t(\hat{\gamma}_{H,t+1} + \hat{\gamma}_{er,t+1} - \hat{\gamma}_{\Phi,t+1}) - \psi\kappa(1-\kappa)\tilde{b}_t,$$
$$\hat{a}_t + \hat{s}_t + \hat{c}_t - \hat{q}_t = \kappa E_t(\hat{a}_{t+1} + \hat{s}_{t+1} + \hat{c}_{t+1} - \hat{q}_{t+1}) + (1-\kappa)\hat{h}_t + \kappa E_t(\hat{\gamma}_{H,t+1} + \hat{\gamma}_{er,t+1} - \hat{\gamma}_{\Phi,t+1}),$$
$$\tilde{b}_t = \beta^{-1}\tilde{b}_{t-1} + p^*_h y^*(\hat{y}_t - \hat{c}_t), \quad (30)$$

[13] As defined in Appendix B, the constant η is one of the two roots of the expectational difference equation of the de-trended net foreign asset position $\tilde{b}_t$. A simple calculation shows that the equilibrium currency return (27) can be derived directly from the CER (28) once the approximated relation $\hat{s}_t \approx \ln S_t + \ln\Psi_t - \ln H^\tau_t - \ln(1-er_t)$ is recognized.

where $y^* = y/4$. Let $\mathbf{X}_t$ denote an unobserved state vector defined as

$$\mathbf{X}_t = [\hat{s}_t \ \hat{c}_t \ E_t\hat{s}_{t+1} \ E_t\hat{c}_{t+1} \ \tilde{b}_t \ \hat{\gamma}_{H,t} \ \xi_t \ \xi_{t-1} \ \hat{\gamma}_{er,t} \ \eta_t \ \eta_{t-1} \ \hat{\gamma}_{\Phi,t} \ \hat{a}_t \ \hat{h}_t \ \hat{y}_t \ \hat{q}_t]'.$$

Furthermore, let ϵ_t and ω_t denote random vectors consisting of nine structural shocks and two rational expectations errors: $\epsilon_t \equiv [\epsilon_{H,t} \ \epsilon_{A,t} \ \epsilon_{h,t} \ \epsilon_{y,t} \ \epsilon_{q,t} \ \epsilon_{\Phi,t} \ \epsilon_{er,t} \ \epsilon_{\xi,t} \ \epsilon_{\eta,t}]'$ and $\omega_t \equiv [\hat{s}_t - E_{t-1}\hat{s}_t \ \hat{c}_t - E_{t-1}\hat{c}_t]'$, respectively. In particular, for empirical investigation purposes, we presume that the structural shock vector ϵ_t is normally distributed, with a mean of zero and a diagonal variance-covariance matrix Σ: $\epsilon_t \sim i.i.d.N(\mathbf{0}, \Sigma)$ with $diag(\Sigma) = [\sigma_H^2 \ \sigma_A^2 \ \sigma_m^2 \ \sigma_y^2 \ \sigma_q^2 \ \sigma_\Phi^2 \ \sigma_{er}^2 \ \sigma_\xi^2, \sigma_\eta^2]'$.

Accompanied by the stochastic processes of the exogenous forcing variables, the linear rational expectations model (30) then implies that

$$\Gamma_0\mathbf{X}_t = \Gamma_1\mathbf{X}_{t-1} + \Lambda_0\omega_t + \Lambda_1\epsilon_t,$$

where Γ_0, Γ_1, Λ_0, and Λ_1 are the corresponding coefficient matrices. Applying Sims's (2001) QZ algorithm to the linear rational expectations model above yields a unique solution as the following stationary transition equation of the unobservable state vector:

$$\mathbf{X}_t = \mathbf{F}\mathbf{X}_{t-1} + \mathbf{Q}\epsilon_t, \tag{31}$$

where $\mathbf{F}$ and $\mathbf{Q}$ are confirmable coefficient matrices.

To construct this paper's UC model, we further expand the unobservable state vector $\mathbf{X}_t$ by the permanent monetary base differential $\ln H_t^\tau$, the excess reserve differential $\ln(1 - er_t)$, and the money demand differential $\ln \Phi_t$ to obtain the augmented state vector $\mathbf{Z}_t$: $\mathbf{Z}_t \equiv [\mathbf{X}_t' \ \ln H_t^\tau \ \ln(1 - er_t) \ \ln \Phi_t]'$. The stochastic processes of $\ln H_t^\tau$, $\ln(1 - er_t)$, and $\ln \Phi_t$ and the state transition (31) then imply the following non-stationary transition of the expanded state vector $\mathbf{Z}_t$:

$$\mathbf{Z}_t = \mathbf{G}\mathbf{Z}_{t-1} + \mathbf{D}\epsilon_t, \quad \epsilon_t \sim i.i.d.N(\mathbf{0}, \Sigma), \tag{32}$$

where $\mathbf{G}$ and $\mathbf{D}$ are confirmable coefficient matrices.

In this paper, I explore time-series data on the logarithm of the consumption differential $\ln C_t$, the logarithm of the output differential $\ln Y_t$, the logarithm of the monetary base differential $\ln H_t$, the logarithm of the non-excess reserve ratio differential $\ln(1 - er_t)$, the interest rate differential $r_t \equiv r_{h,t}^h - r_{f,t}^f$, and the logarithm of the bilateral exchange rate $\ln S_t$. Let $\mathbf{Y}_t$ denote the information set that consists of these six time series: $\mathbf{Y}_t \equiv [\ln C_t \ \ln Y_t \ \ln H_t \ \ln(1 - er_t) \ r_t \ \ln S_t]'$. It is then straightforward to show that the information set $\mathbf{Y}_t$ is linearly related to the unobservable state vector $\mathbf{Z}_t$ as

$$\mathbf{Y}_t = \mathbf{H}\mathbf{Z}_t, \tag{33}$$

where $\mathbf{H}$ is a confirmable coefficient matrix. The transition equation, the unobserved state (32), and the observation equation (33) jointly consist of a non-stationary state-space representation of the two-country model, which is the restricted UC model estimated in this paper.[14]

Given the data set $\mathbf{Y}^T \equiv \{\mathbf{Y}_t\}_{t=0}^T$, applying the Kalman filter to the UC model provides model likelihood $\mathbf{L}(\mathbf{Y}^T|\theta)$, where θ is the structural parameter vector of the two-country model. Multiplying the likelihood by a prior probability of the structural parameters, $\mathbf{p}(\theta)$, is proportional to the corresponding posterior distribution $\mathbf{p}(\theta|\mathbf{Y}^{\mathbf{T}}) \propto \mathbf{p}(\theta)\mathbf{L}(\mathbf{Y}^T|\theta)$ through the Bayes law. The posterior distribution $\mathbf{p}(\theta|\mathbf{Y}^T)$ is simulated by the random-walk Metropolis-Hastings algorithm, as implemented by Kano (2014).

4.2. Data and prior construction

In this paper, we examine post-Plaza accord quarterly data for Japan and the United States. The data span the period from Q1:1988 to Q4:2013. All the data included in the information set $\mathbf{Y}^T$, except nominal exchange rates, are seasonally adjusted annual rates.[15]

Table 1 reports the prior distributions of the structural parameters of the two-country model, $\mathbf{p}(\theta)$. We follow Kano (2014) to construct the prior distributions. In particular, we elicit a uniform prior distribution of κ and let the data tell the posterior position of κ given the identification of the restricted UC model. In so doing, on the one hand, the prior distribution of the mean gross monetary growth rate, γ_H, is intended to tightly cover its sample counterparts in both countries through the Gamma distribution, with a mean of 1.015 and standard deviation of 0.005. On the other hand, the prior distribution of the subjective discount factor β is uniformly distributed between zero and one. As a result, the prior distribution of the market discount factor κ is well approximated as the uniform distribution spread over the support of the unit interval.

4.3. Results

Table 2 reports the posterior distributions of the structural parameters. The second, third, and fourth columns correspond to the means, the standard deviations, and the 95 % Bayesian highest probability density (HPD) intervals of the the posterior distributions, respectively. An outstanding observation in the table should be found in the posterior distribution of the market discount factor κ. The posterior mean of the κ is 0.961, which is very close to the posterior means of the market discount factor around 0.960 observed by Nason and Rogers (2008), Sarno and Sojli (2005), Balke et al. (2013).[16] This empirically relevant inference on the market discount factor does

[14]The state-space form of the model, (32) and (33), decomposes the I(1) difference-stationary information set $\mathbf{Y}_t$ into permanent and transitory components exploiting the theoretical restrictions provided by the two-country model. Recursion of the Kalman filter for a non-stationary state-space model is explained in detail by Hamilton (1994).

[15]Appendix D provides a detailed description of the source and construction of the data examined in this paper.

[16]In contrast, Kano (2014) conducts the similar posterior simulation of the two-country general equilibrium model

not depend on any prior information because the prior distribution of κ is uniform. The reasonable size of the discount factor is identified by the model's restrictions imposed on the data.

The estimated market discount factor close to one, indeed, fits the model to the near random-walk Japanese yen/U.S. dollar rate. According to the model's theoretical implication (29), the currency return should be well approximated by $\epsilon_{H,t}$, $\epsilon_{\xi,t}$, $\epsilon_{er,t}$, $\epsilon_{\eta,t}$, and $\epsilon_{\Phi,t}$ under the discount factor close to one. Figure 3 plots the currency return in the data (the solid black line) and the sum of the smoothed inferences of $\epsilon_{H,t}$, $\epsilon_{\xi,t}$, $\epsilon_{er,t}$, $\epsilon_{\eta,t}$, and $\epsilon_{\Phi,t}$ through the Kalman smoother (the dashed blue line). Observe that the bumpy depreciation rate of the Japanese yen against the U.S. dollar is almost perfectly tracked by the sum of the smoothed inferences of these permanent shocks. Therefore, our model successfully explains the near random-walk behavior of the Japanese yen/U.S. dollar rate.

Is the model successful in mimicking the success and failure of the Soros chart? Figures 4(a) and (b) just adds the model's smoothed inferences of the two versions of the Soros chart to Figures 1(a) and (b). More precisely, in Figure 4(a), the smoothed inference of the differential in the permanent component of the monetary base between the two countries, $\ln H_t^\tau$, is plotted as the dotted dashed red line, while in Figure 4(b) the sum of the smoothed inferences of the differentials in the permanent component of the monetary bases and the non-excess reserve component, $\ln H_t^\tau + \ln(1-er_t)$, is displayed as the dotted dashed red line. Notice that the model's smoothed inferences on the cross-country differentials in the permanent components of the non-augmented and augmented monetary base almost perfectly replicate both the failure of the first Soros chart and the success of the second simultaneously. Hence, in our model, the augmented Soros chart is identified as a common stochastic trend that explains the slow-moving low-frequency component of the post-Plaza Accord Japanese yen/U.S. dollar exchange rate.[17]

Given the augmented Solos chart as the slow-moving stochastic trend of the Japanese yen/U.S. dollar, what is the main driver of the transitory component of the corresponding exchange rate? Figure 5 conducts a historical decomposition of the currency return into the structural shocks. Specifically, each small window in the figure plots the actual currency return as the solid black line and the smoothed inference of the corresponding structural shock as the dashed blue line. The middle center window corresponds to the permanent money demand shock $\epsilon_{\Phi,t}$. Observe that the identified money demand shock traces the transitory movement of the exchange rate, i.e., the cur-

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with the post-Bretton Woods sample of Canada and the United State and found a much smaller posterior mean of κ of 0.525.

[17]It is worth noting that Figures 4(a) and (b) plot only the *permanent component* of the monetary base differential from the Kalman smoothed inference of the model using the actual data of the monetary base differential. Hence, it is not obvious *a priori* that we can obtain a perfect replication of the Soros chart. The result depends on the estimation of the transitory component of the monetary base differential, which also is affected by the other cross-equation restrictions of the model imposed on the data of the monetary base differential as well as the interest rate differential. See Kano (2014) for the whole derivation of the cross-equation restrictions of the model in more detail.

rency return, very tightly. Hence, the money demand shock is identified as the main generator of the transitory component of the Japanese yen/U.S. dollar rate.

Our model also is able to describe the disconnection of the Japanese yen/U.S. dollar exchange rate with real economic variables. Again Figure 5 conducts a historical decomposition of the currency return into the structural shocks. Observe that none of real shocks, i.e., the TFP shock $\epsilon_{A,t}$ in the upper left window, the transitory output shock $\epsilon_{y,t}$ in the upper right window, the PPP deviation shock $\epsilon_{q,t}$ in the middle left window, plays a significant role in driving the currency return. Figure 6, on the other hand, plots the same historical decomposition of the real consumption growth rate into the structural shocks. It is clear that the TFP shock $\epsilon_{A,t}$ in the upper left window and the PPP deviation shock $\epsilon_{q,t}$ in the middle left window are the main drivers of the consumption differential.[18] Therefore, our model interprets the disconnection of the exchange rate from the real variables in the data plausibly.

Figure 7 reports the historical decomposition of the interest rate differential into the structural shocks. An outstanding smoothed inference the figure reveals is that the news shock to the permanent component of the monetary base $\epsilon_{\xi,t}$ in the lower middle window is the only shock to affect the interest rate differential instantaneously. This result implies that the interest rate differential is determined by the forward-looking anticipated information about the monetary base differential in near future. The tight linkage of the interest rate differential with the currency return in the data is generated by the current news about future permanent shifts in the relative size of the monetary base between the two countries.

5. Conclusions

The paper's successful explanation of the major statistical properties of the post-Plaza Accord Japanese yen/U.S. dollar exchange rate with a simple two-country model is conditional on an inevitable caveat. As shown in Figure 5, our paper identifies the dominant driver of the currency return, i.e., the short-run transitory component of the Japanese yen/U.S. dollar exchange rate, as the money demand shock $\epsilon_{\Phi,t}$ which is then a mixture of the money-in-utility preference shock and the money multiplier shock. Because there is no theoretical restriction the model imposes on this structural shock and the data, the money demand shock indeed acts as a free parameter in our posterior simulation of the restricted unobserved component model. Hence, it is still too ambitious to interpret $\epsilon_{\Phi,t}$ as the permanent money demand shock literally.

The identified shock $\epsilon_{\Phi,t}$, indeed, backs the sharp depreciation of the Japanese yen against the U.S. dollar that occurred after 2012Q4 when most market participants expected that the ex-

[18]This independence of the consumption growth rate from the monetary disturbances stems from the monetary super-neutrality with the money-in-utility lifetime utility function and the model's absence from price stickiness.

tremely easing monetary policy of the BOJ would be conducted by Governor Kroda in accordance with Prime Minister Abe as his new economic policy subsequently known as the "Abenomics." The paper's absence from a shaper structural identification of money demand shocks makes it difficult to understand any possible effect of the Abenomics on the sudden depreciation of the Japanese Japanese yen against the U.S. dollar between 2012Q4 and 2013Q4 within our model. A reason for this failure of our model in uncovering the major source of the sharp deprecation of the Japanese yen may be purely empirical: there have been only a short sample with four quarters since the beginning of the Abenomics. It, however, is more desirable to find another theoretical restriction to extract pure money demand shocks from the data by augmenting our model further. Is so doing, the most promising way might be to include a more realistic monetary policy framework with an interest rate setting rule. A correctly specified interest rate setting rule might lead to not only a better structural interpretation of the money demand shocks identified by this paper but also a more profound understanding of the implications of the recent unconventional monetary policy measures taken by the BOJ and Fed such as the zero lower bound of the policy rate, the forward guidance, and the QEs on the Japanese yen/U.S. dollar exchange rate. We leave these empirical and theoretical tasks as meaningful but quite challenging future research agendas.

Appendix A. Stochastically de-trended system

The stochastic de-trended versions of the home country's FONCs, (1), (3), (4), (5), and (12), are

$$p_{h,t}c_{h,t} + b^h_{h,t} + s_t b^f_{h,t} = (1 + r^h_{h,t-1}) b^h_{h,t-1}\gamma^h_{\Gamma,t}/\gamma^h_{M,t} + (1 + r^f_{h,t-1}) s_t b^f_{h,t-1}\gamma^f_{\Gamma,t}/\gamma^f_{M,t} + p_{h,t}y_{h,t},$$

$$\frac{1}{p_{h,t}c_{h,t}} = \beta(1 + r^h_{h,t})E_t\left(\frac{\gamma^h_{\Gamma,t+1}}{\gamma^h_{M,t+1}p_{h,t+1}c_{h,t+1}}\right),$$

$$s_t(1 + r^h_{h,t})E_t\left(\frac{\gamma^h_{\Gamma,t+1}}{p_{h,t+1}c_{h,t+1}\gamma^h_{M,t+1}}\right) = (1 + r^f_{h,t})E_t\left(\frac{s_{t+1}\gamma^f_{\Gamma,t+1}}{p_{h,t+1}c_{h,t+1}\gamma^f_{M,t+1}}\right),$$

$$\frac{m_{h,t}}{p_{h,t}} = c_{h,t}\left(\frac{1 + r^h_{h,t}}{r^h_{h,t}}\right),$$

$$r^h_{h,t} = r^h_{w,t}[1 + \psi\{\exp(-b^h_{h,t} + \bar{d}) - 1\}],$$

and

$$r^f_{h,t} = r^f_{w,t}[1 + \psi\{\exp(-b^f_{h,t} + \bar{d}) - 1\}].$$

Similarly, the stochastically de-trended versions of the FONCs of the foreign country, (2) (6), (7), and (8), are

$$\begin{aligned} q_t p_{h,t}c_{f,t} - a_t s_t b^f_{h,t} - a_t b^h_{h,t} = &-(1 + r^f_{w,t-1}) a_t s_t b^f_{h,t-1}\gamma^f_{\Gamma,t}/\gamma^f_{M,t} \\ &- (1 + r^h_{w,t-1}) a_t b^h_{h,t-1}\gamma^h_{\Gamma,t}/\gamma^h_{M,t} + q_t p_{h,t}y_{f,t}, \end{aligned}$$

$$\frac{a_t s_t}{q_t p_{h,t} c_{f,t}} = \beta(1+r^f_{w,t})E_t \frac{a_{t+1}s_{t+1}\gamma^f_{\Gamma,t+1}}{\gamma^f_{M,t+1}q_{t+1}p_{h,t+1}c_{f,t+1}},$$

$$(1+r^h_{w,t})E_t\left(\frac{a_{t+1}\gamma^h_{\Gamma,t+1}}{q_{t+1}p_{h,t+1}c_{f,t+1}\gamma^h_{M,t+1}}\right) = \frac{(1+r^f_{w,t})}{s_t}E_t\left(\frac{a_{t+1}s_{t+1}\gamma^f_{\Gamma,t+1}}{q_{t+1}p_{h,t+1}c_{f,t+1}\gamma^f_{M,t+1}}\right),$$

and

$$\frac{a_t s_t m_{f,t}}{q_t p_{h,t}} = c_{f,t}\left(\frac{1+r^f_{w,t}}{r^f_{w,t}}\right).$$

If the TFP differential a_t is I(1) as assumed in NR, the above system of stochastic difference equations becomes nonstationary through the home and foreign budget constraints and there is no deterministic steady state to converge. Notice that neither the cross-country permanent money supply differential $\ln M^\tau_{h,t}/M^\tau_{f,t}$ nor the cross-country preference shock differential $\ln \Gamma_{h,t}/\Gamma_{f,t}$ appears in the stochastically de-trended system of the FONCs. In contrast to the TFP differential a_t, the I(1) properties of $\ln M^\tau_{h,t}/M^\tau_{f,t}$ and $\ln \Gamma_{h,t}/\Gamma_{f,t}$ do not matter for the closing of the model. This might be an obvious result of the model's property that the super-neutrality of money holds in the money-in-utility model: Money growth does not matter for the deterministic steady state. Notice that at the deterministic steady state, the TFP differential a^* is one. Because of the stationarity of the system of equations, the deterministic steady state is characterized by constants c^*_h, c^*_f, p^*_h, s^*, b^{h*}_h, b^{f*}_h, r^{h*}_h, r^{f*}_h, r^{h*}_w, and r^{f*}_w that satisfy

$$\begin{aligned}
b^{h*}_h &= b^{f*}_h = \bar{d},\\
r^* \equiv r^{h*}_h &= r^{f*}_f = r^{h*}_w = r^{f*}_w = \gamma_H/\beta - 1,\\
s^* &= \frac{y_f(\gamma_H)^{-1}r^* + (y_h+y_f)(1-\beta^{-1})\bar{d}}{y_h(\gamma_H)^{-1}r^* - (y_h+y_f)(1-\beta^{-1})\bar{d}},\\
p^*_h y_h &= (1-\beta^{-1})(1+s^*)\bar{d} + (\gamma_H)^{-1}r^*,\\
p^*_h c^*_h &= (\gamma_H)^{-1}r^*,\\
c^*_f &= s^* c^*_h.
\end{aligned}$$

Appendix B. Derivation of the saddle path (28)

To understand the equilibrium transitory dynamics of the exchange rate in this model, it is informative to scrutinize a simpler version of the model that includes two symmetric countries. For this purpose, I set the parameter $\bar{d}$ to zero and assume that the transitory output components of the two countries, y_h and y_f, are equal to y. Notice that the deterministic steady state in this case is characterized by $s^* = 1$, $c^*_h = c^*_f = y$, and $p^*_h = (\gamma_M)^{-1}r^*$, where $r^* = \gamma_M/\beta - 1$.

I combine the log-linearized Euler equations of the home and foreign countries, (15) and (19), with those of the home country's interest rates (22) to yield the first-order expectational difference equation of $\hat{a}_t + \hat{s}_t + \hat{c}_t$:

$$\hat{a}_t + \hat{s}_t + \hat{c}_t - \hat{q}_t = \kappa E_t(\hat{a}_{t+1} + \hat{s}_{t+1} + \hat{c}_{t+1} - \hat{q}_{t+1}) + \kappa E_t(\hat{\gamma}_{H,t+1} + \hat{\gamma}_{er,t+1} - \hat{\gamma}_{\Phi,t+1}) + (1-\kappa)\hat{h}_t.$$

Since κ takes a value between zero and one, the above expectational difference equation has a forward solution of

$$\hat{a}_t + \hat{s}_t + \hat{c}_t - \hat{q}_t = \frac{\kappa\rho_H}{1-\kappa\rho_H}\hat{\gamma}_{H,t} + \frac{\kappa}{1-\kappa\rho_H}\xi_{t-1} + \frac{\kappa^2}{1-\kappa\rho_H}\xi_t + \frac{\kappa\rho_{er}}{1-\kappa\rho_{er}}\hat{\gamma}_{er,t} + \frac{\kappa}{1-\kappa\rho_{er}}\eta_{t-1} + \frac{\kappa^2}{1-\kappa\rho_{er}}\eta_t - \frac{\kappa\rho_\Phi}{1-\kappa\rho_\Phi}\hat{\gamma}_{\Phi,t} + \frac{1-\kappa}{1-\kappa\rho_h}\hat{h}_t$$

under a suitable transversality condition. By, exploiting this forward solution and the stochastic processes of both countries' TFPs (13), I rewrite the foreign UIP condition (20) as

$$E_t\hat{s}_{t+1} - \hat{s}_t = \psi(1-\kappa)\tilde{b}_t - \frac{\kappa\rho_H(1-\rho_H)}{1-\kappa\rho_H}\hat{\gamma}_{H,t} - \frac{\kappa(1-\rho_H)}{1-\kappa\rho_H}\xi_{t-1} + \frac{\kappa(1-\kappa)}{1-\kappa\rho_H}\xi_t - \frac{\kappa\rho_{er}(1-\rho_{er})}{1-\kappa\rho_{er}}\hat{\gamma}_{er,t} - \frac{\kappa(1-\rho_{er})}{1-\kappa\rho_{er}}\eta_{t-1} + \frac{\kappa(1-\kappa)}{1-\kappa\rho_{er}}\eta_t + \frac{\kappa\rho_\Phi(1-\rho_\Phi)}{1-\kappa\rho_\Phi}\hat{\gamma}_{\Phi,t} - \frac{(1-\kappa)(1-\rho_h)}{1-\kappa\rho_h}\hat{h}_t, \quad \text{(B.1)}$$

Furthermore, taking a difference between the log-linearized budget constraints of the home and foreign countries, (14) and (18), I find the law of motion of the international bond holdings

$$\tilde{b}_t = \beta^{-1}\tilde{b}_{t-1} + p_h^* y^*(\hat{a}_t + \hat{s}_t - \hat{q}_t + \hat{y}_t) - \frac{p_h^* y^* \kappa\rho_H}{1-\kappa\rho_H}\hat{\gamma}_{H,t} - \frac{p_h^* y^*\kappa}{1-\kappa\rho_H}\xi_{t-1} - \frac{p_h^* y^*\kappa^2}{1-\kappa\rho_H}\xi_t - \frac{p_h^* y^*\kappa\rho_{er}}{1-\kappa\rho_{er}}\hat{\gamma}_{er,t} - \frac{p_h^* y^*\kappa}{1-\kappa\rho_{er}}\eta_{t-1} - \frac{p_h^* y^*\kappa^2}{1-\kappa\rho_{er}}\eta_t + \frac{p_h^* y^*\kappa\rho_\Phi}{1-\kappa\rho_\Phi}\hat{\gamma}_{\Phi,t} - \frac{p_h^* y^*(1-\kappa)}{1-\kappa\rho_h}\hat{h}_t, \quad \text{(B.2)}$$

where $y^* = y/4$ and $\hat{y}_t \equiv \hat{y}_{h,t} - \hat{y}_{f,t}$.

Combining equation (B.1) with equation (B.2) then yields the following second-order expectational difference equation with respect to international bond holdings:

$$E_t\tilde{b}_{t+1} - [1 + \beta^{-1} + p_h^* y^*\psi(1-\kappa)]\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1} = -p_h^* y^*\lambda\hat{a}_t + p_h^* y^*(1-\rho_q)\hat{q}_t - p_h^* y^*(1-\rho_y)\hat{y}_t \quad \text{(B.3)}$$

It is straightforward to show that equation (B.3) has two roots, one of which is greater than one and the other of which is less than one.[19] Without losing generality, let η denote the root that is less than one. Solving equation (B.3) by forward iterations then shows that the equilibrium international bond holdings level is determined by the following cross-equation restriction (CER):

$$\begin{aligned}\tilde{b}_t &= \eta\tilde{b}_{t-1} + \beta\eta\lambda p_h^* y^*\sum_{j=0}^{\infty}(\beta\eta)^j E_t\hat{a}_{t+j} + \beta\eta p_h^* y^*(1-\rho_y)\sum_{j=0}^{\infty}(\beta\eta)^j E_t\hat{y}_{t+j} - \beta\eta p_h^* y^*(1-\rho_q)\sum_{j=0}^{\infty}(\beta\eta)^j E_t\hat{q}_{t+j}, \\ &= \eta\tilde{b}_{t-1} + \frac{\beta\eta\lambda p_h^* y^*}{1-\beta\eta(1-\lambda)}\hat{a}_t + \frac{\beta\eta p_h^* y^*(1-\rho_y)}{1-\beta\eta\rho_y}\hat{y}_t - \frac{\beta\eta p_h^* y^*(1-\rho_q)}{1-\beta\eta\rho_q}\hat{q}_t. \end{aligned} \quad \text{(B.4)}$$

Substituting equation (B.4) back into equation (B.2) provides the CER for the exchange rate (28):

$$\hat{s}_t = \frac{\beta\eta-1}{\beta p_h^* y^*}\tilde{b}_{t-1} + \frac{\beta\eta-1}{1-\beta\eta(1-\lambda)}\hat{a}_t + \frac{1-\kappa}{1-\kappa\rho_h}\hat{h}_t + \frac{\beta\eta-1}{1-\beta\eta\rho_y}\hat{y}_t - \frac{\beta\eta-1}{1-\beta\eta\rho_q}\hat{q}_t + \frac{\kappa\rho_H}{1-\kappa\rho_H}\hat{\gamma}_{H,t} + \frac{\kappa}{1-\kappa\rho_H}\xi_{t-1} + \frac{\kappa^2}{1-\kappa\rho_H}\xi_t + \frac{\kappa\rho_{er}}{1-\kappa\rho_{er}}\hat{\gamma}_{er,t} + \frac{\kappa}{1-\kappa\rho_{er}}\eta_{t-1} + \frac{\kappa^2}{1-\kappa\rho_{er}}\eta_t - \frac{\kappa\rho_\Phi}{1-\kappa\rho_\Phi}\hat{\gamma}_{\Phi,t}$$

Therefore, in this symmetric case, the competitive equilibrium along the balanced growth path is characterized by a lower dimensional dynamic system of $(\hat{s}_t, \tilde{b}_t, \hat{a}_t, \hat{\gamma}_{H,t}, \xi_t, \hat{\gamma}_{er,t}, \eta_t, \hat{\gamma}_{\Phi,t}, \hat{h}_t, \hat{y}_t, \hat{q}_t)$.

[19] To characterize the roots of the second-order expectational difference equation, see, for example, Sargent (1987).

Appendix C. Derivation of the error correction representation (25)

Let n_t denote the fundamental of the DSGE-PVM (24): $n_t \equiv \ln M_t - \ln \Gamma_t - \ln C_t - \psi\kappa\tilde{b}_t + \ln q_t$. Consider the currency return $\Delta \ln S_t$ adjusted by the fundamental $(1-\kappa)n_{t-1}$: $\Delta \ln S_t + (1-\kappa)n_{t-1}$. The DSGE-PVM (24) then implies:

$$
\begin{aligned}
\Delta \ln S_t + (1-\kappa)n_{t-1} &= (1-\kappa)\sum_{j=0}^{\infty}\kappa^j (E_t - E_{t-1})n_{t+i} + (1-\kappa)\sum_{j=0}^{\infty}\kappa^j E_{t-1}n_{t+i} \\
&\quad - (1-\kappa)\sum_{j=0}^{\infty}\kappa^j E_{t-1}n_{t+i-1} + (1-\kappa)n_{t-1}, \\
&= (1-\kappa)\sum_{j=0}^{\infty}\kappa^j (E_t - E_{t-1})n_{t+i} + \frac{(1-\kappa)^2}{\kappa}\sum_{i=0}^{\infty}\kappa^i E_{t-1}n_{t+i-1} - \frac{(1-\kappa)^2}{\kappa}n_{t-1}, \\
&= (1-\kappa)\sum_{j=0}^{\infty}\kappa^j (E_t - E_{t-1})n_{t+i} + \frac{1-\kappa}{\kappa}\ln S_{t-1} - \frac{(1-\kappa)^2}{\kappa}n_{t-1}.
\end{aligned}
$$

This result means that the currency return has the following error correction representation, given by equation (25):

$$
\begin{aligned}
\Delta \ln S_t = \frac{1-\kappa}{\kappa}(\ln S_{t-1} - \ln M_{t-1} + \ln \Gamma_{t-1} + \ln C_{t-1} + \psi\kappa\tilde{b}_{t-1} - \ln q_{t-1}) \\
+ (1-\kappa)\sum_{j=0}^{\infty}\kappa^j (E_t - E_{t-1})n_{t+i}.
\end{aligned}
$$

Appendix D. Data description and construction

All data for the United States are distributed by Federal Reserve Economic Data (FRED), operated by the Federal Reserve Bank of St. Louis (http://research.stlouisfed.org/fred2/). The consumption data are constructed as the sum of the real personal consumption expenditures on non-durables and services. FRED, however, distributes only the nominal values of two categories of personal consumption expenditures as *Personal Consumption Expenditure on Non-Durables (PCND)* and *Personal Consumption Expenditure on Services (PCESV)*. To construct the real series of two categories of personal consumption expenditure $C_{us,t}$, we first calculate the share of the two nominal consumption categories in the nominal total personal consumption expenditure *Personal Consumption Expenditure* and then multiply the real total personal consumption expenditures, *Real Personal Consumption Expenditures at Chained 2005 Dollars (PCECC96)*, by the calculated share. The output $Y_{us,t}$ is employed *Real Gross Domestic Product (GDPMC1)*. As the aggregate monetary supply $M_{us,t}$ and the excess reserve, we employ *St. Louis Adjusted Monetary Base (BASE)* and *Excess Reserves of Depository Institutions (EXCSRESNS)*. The nominal interest rate $r_{us,t}$ is provided by *three-month Treasury Bill (TB3MS)*.

As for the Japanese data, the series of the real consumption expenditures on non-durables and services, and real GDP are distributed by the Systems of National Accounts (SNA) database, released by

Cabinet Office, Government of Japan. We combine the series from the data whose benchmark year is 2000 and the one whose benchmark year is 2005 in the first quarter of the year 1994 using the growth rate of the series of the benchmark year being equal to 2000. The Japanese monetary data are obtained from Bank of Japan website. We use *Monetary Base (Reserve Requirement Rate Change Adjusted)/Seasonally Adjusted (X-12-ARIMA)/Average Amounts Outstanding* as the money supply $M_{jpn,t}$, and calculate the excess reserve by subtracting *Required Reserve (Average Outstanding)* from *Reserves/Average Outstanding.* Only the nominal interest rate $r_{jpn,t}$ is downloaded *Interest Rates, Government Securities, Treasury Bills for Japan (INTGSTJPM193N)* from FRED.

Finally, the nominal exchange rate between the United States and Japan is employed *Japan / U.S. Foreign Exchange Rate (EXJPUS)* in the FRED database.

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Table 1: Prior Distributions of Structural Parameters

	Parameters	Distribution	Mean	S.D.	95 % Coverage
β	Household Subjective Discount Factor	Uniform(0,1)	—	—	[0.025 0.975]
ψ	Debt Elasticity of Risk Premium	Gamma	0.010	0.001	[0.008 0.012]
γ_H	Deterministic (Gross) Monetary Base Growth	Gamma	1.015	0.005	[1.005 1.024]
λ	Technology Diffusion Speed	Beta	0.010	0.001	[0.008 0.012]
ρ_H	Monetary Base Growth AR(1) Coef.	Beta	0.100	0.010	[0.081 0.120]
ρ_q	RER AR(1) Coef.	Beta	0.850	0.200	[0.739 0.933]
ρ_Φ	Money Demand Growth AR(1) Coef.	Beta	0.100	0.010	[0.081 0.120]
ρ_{er}	Non Excess Reserve Growth AR(1) Coef.	Beta	0.100	0.010	[0.081 0.120]

Note 1. The AR(1) coefficients of the transitory money and output shocks, ρ_h and ρ_y respectively, have the mass points of zero for identification.

Note 2. The standard deviations of all the structural shocks, σ_H, σ_A, σ_h, σ_y, σ_q, σ_Φ, σ_{er}, σ_ξ, σ_η have the identical inverse Gamma prior distribution, with a mean of 0.01 and standard deviation of 0.01 for the benchmark information set.

Table 2: Posterior Distributions of Structural Parameters

Parameters	Mean	S.D.	95 % Interval
κ	0.961	0.003	[0.955 0.967]
β	0.974	0.005	[0.963 0.984]
ψ	0.010	0.001	[0.008 0.012]
γ_H	1.013	0.005	[1.003 1.022]
λ	0.010	0.001	[0.008 0.013]
ρ_H	0.091	0.009	[0.074 0.108]
ρ_q	0.944	0.014	[0.918 0.967]
ρ_Φ	0.091	0.007	[0.077 0.106]
ρ_{er}	0.094	0.008	[0.079 0.110]
σ_H	0.051	0.004	[0.042 0.059]
σ_A	0.009	0.001	[0.008 0.011]
σ_h	0.006	0.001	[0.004 0.009]
σ_y	0.005	0.001	[0.004 0.006]
σ_q	0.011	0.001	[0.008 0.014]
σ_Φ	0.053	0.004	[0.046 0.061]
σ_{er}	0.049	0.004	[0.042 0.057]
σ_ξ	0.018	0.002	[0.013 0.022]
σ_η	0.017	0.002	[0.012 0.021]
Marginal Likelihood	1528.968		

Note 1: The marginal likelihoods are estimated based on Geweke's (1999) harmonic mean estimator.

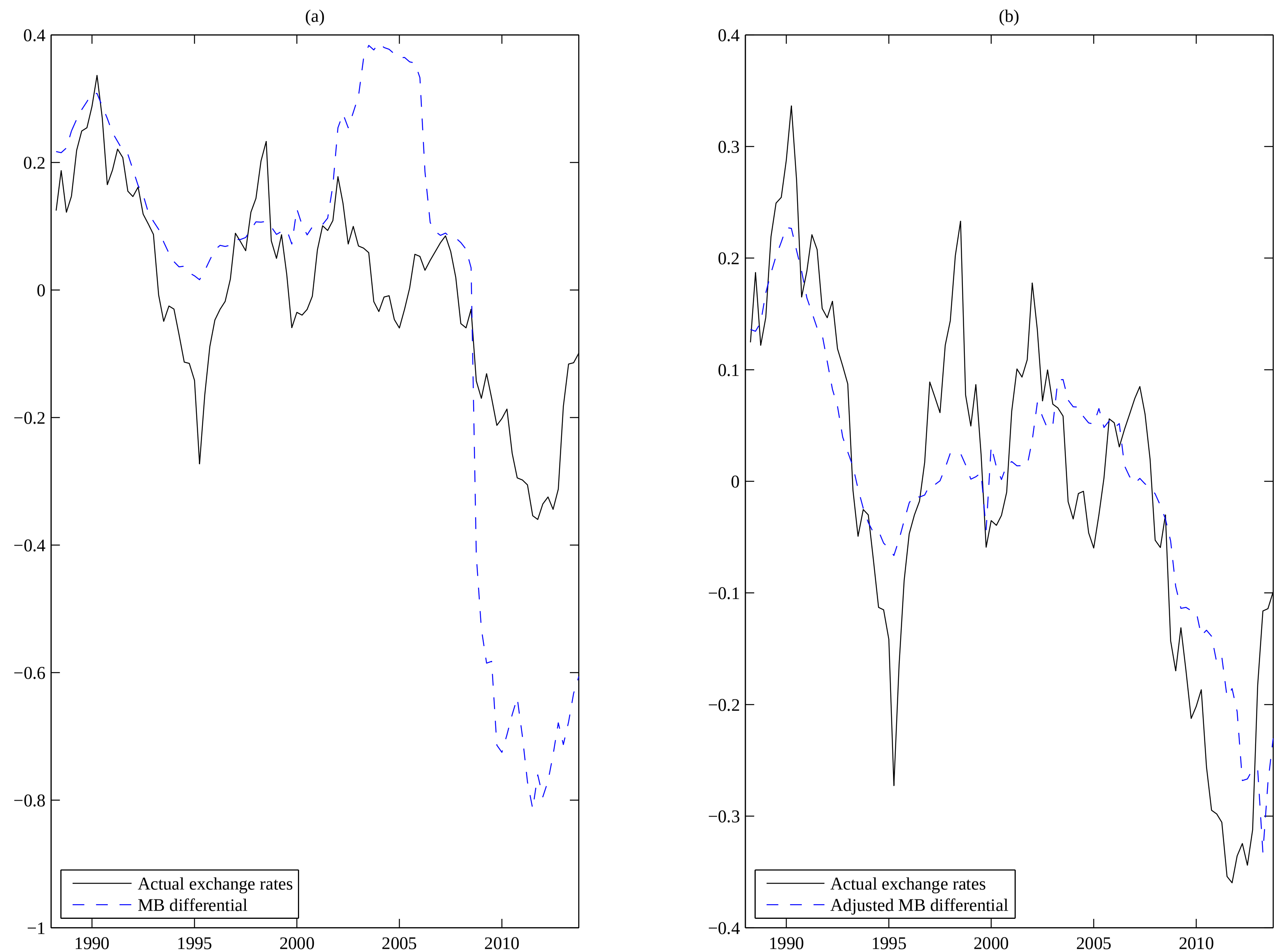

Figure 1: The Soros Chart

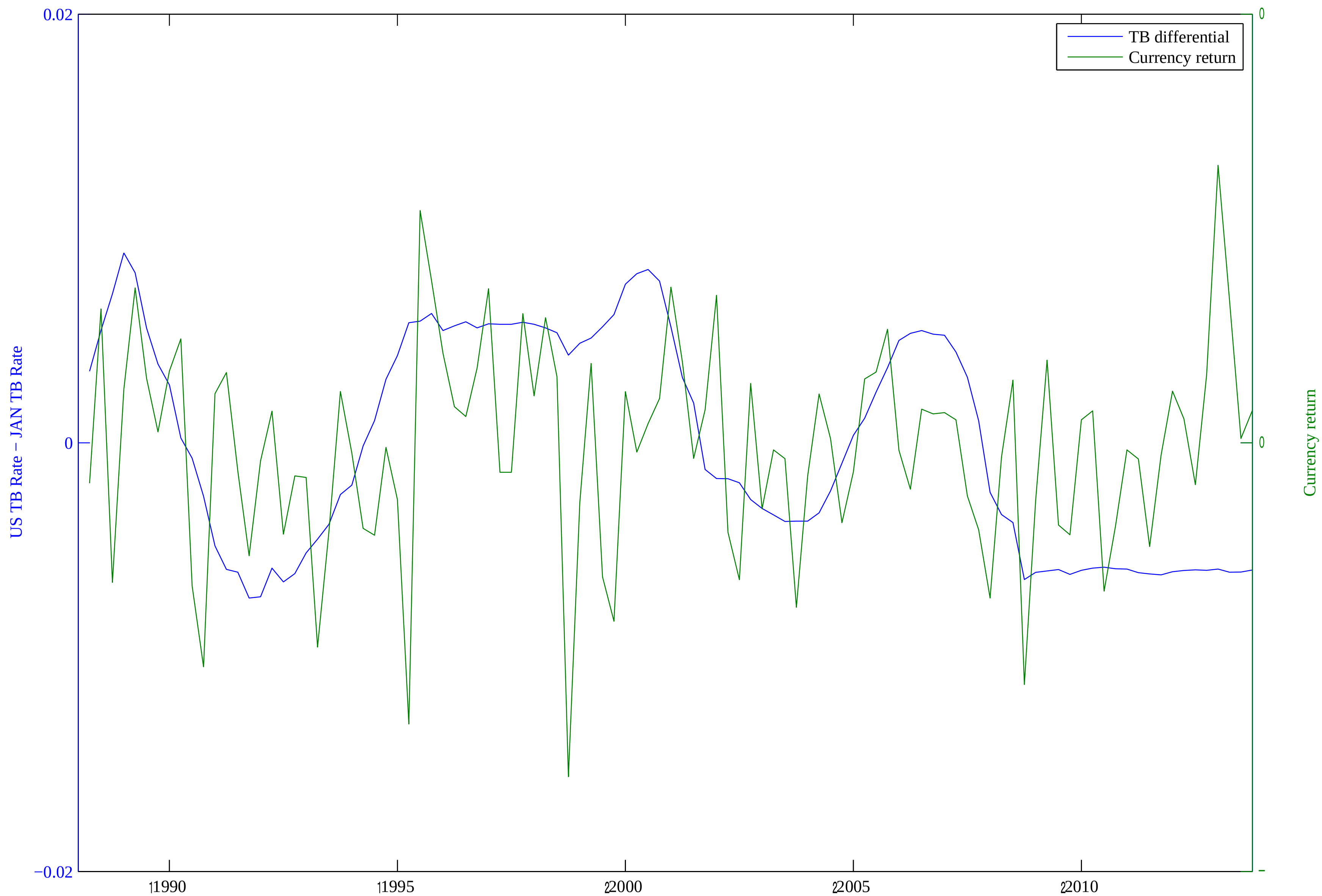

Figure 2: Three-month TB Differential and Japanese yen/U.S. dollar Currency Return

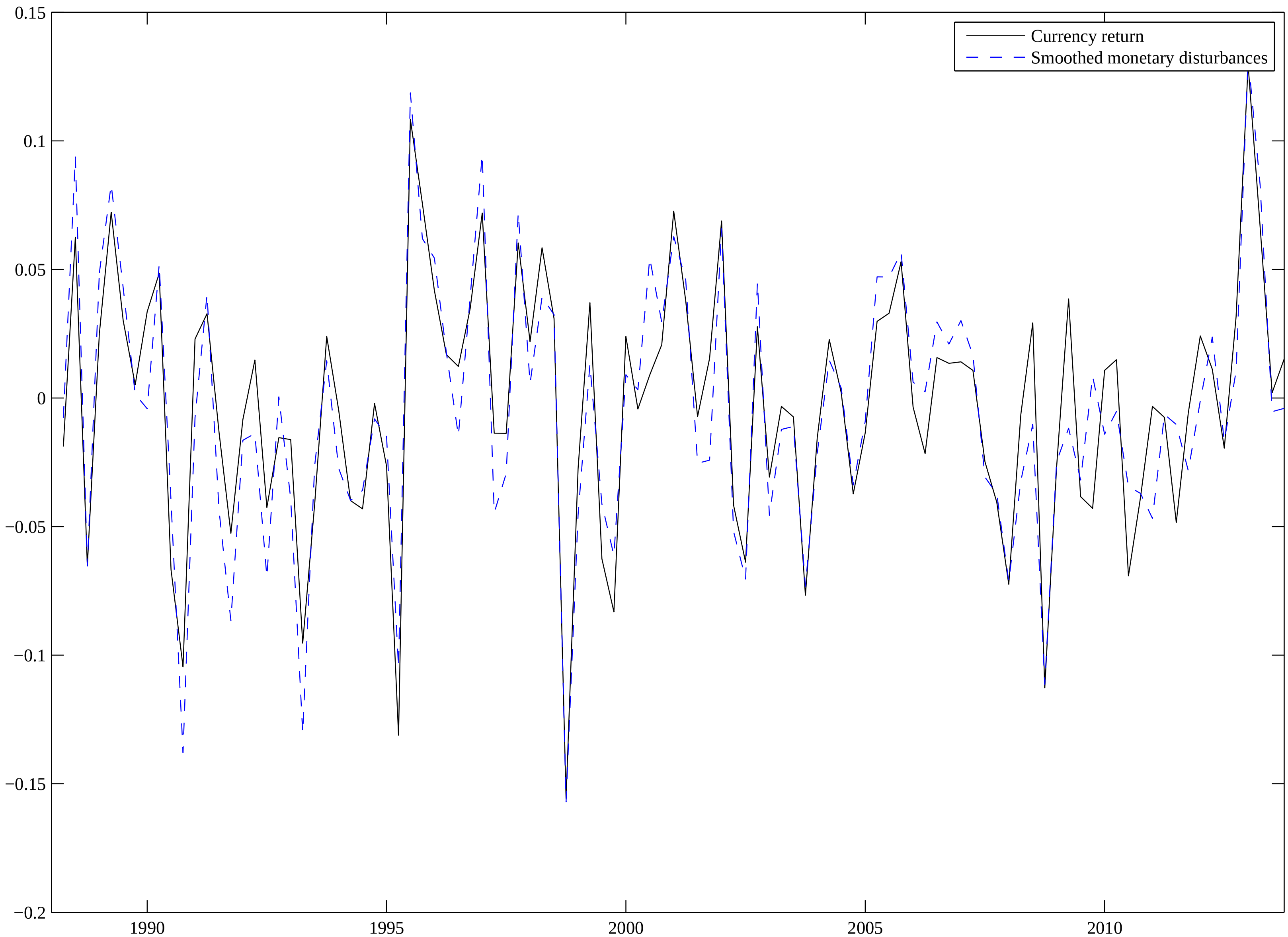

Figure 3: Currency Return and Smoothed Monetary Disturbances

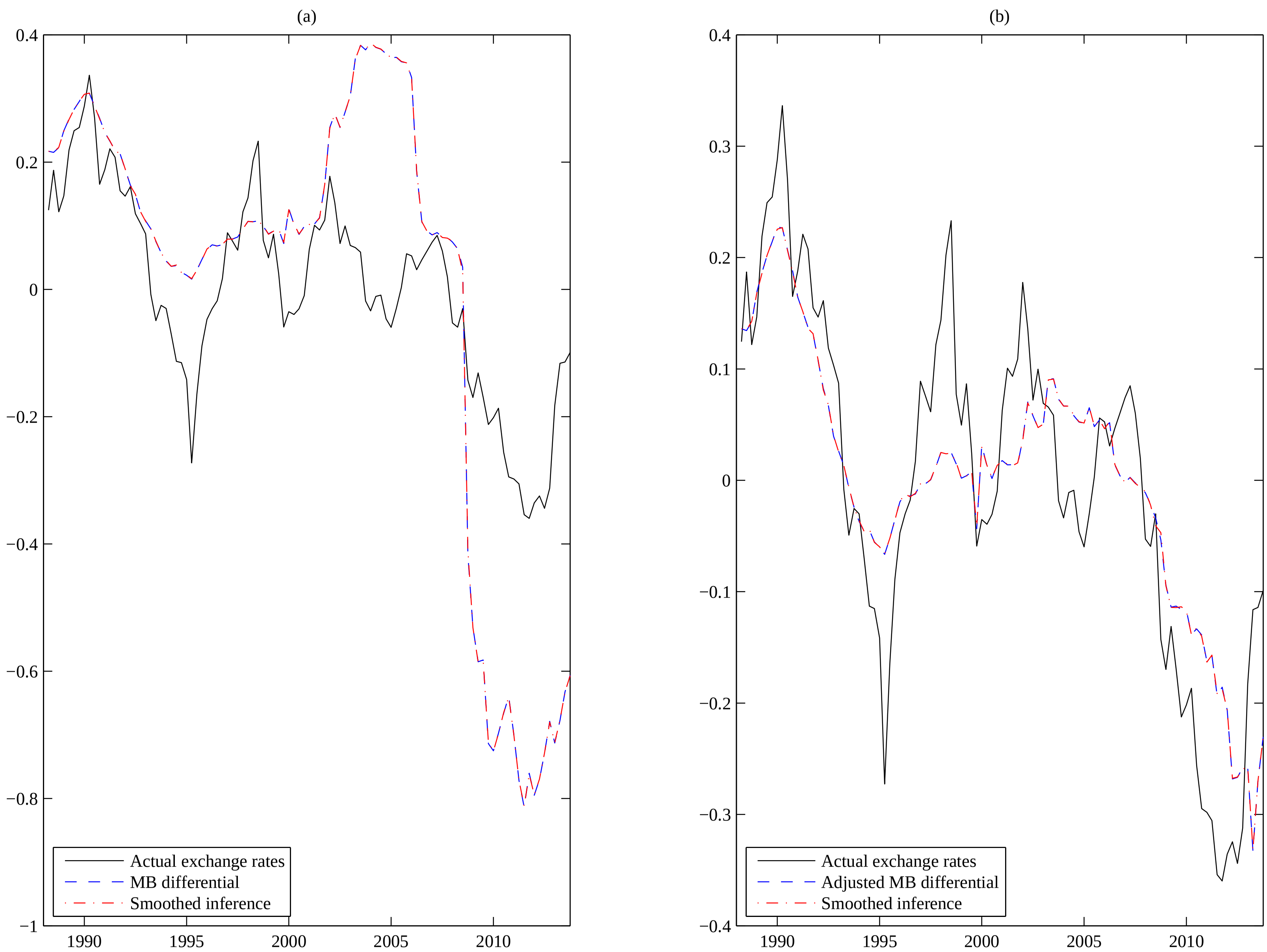

Figure 4: Smoothed Inferences on the Soros Chart

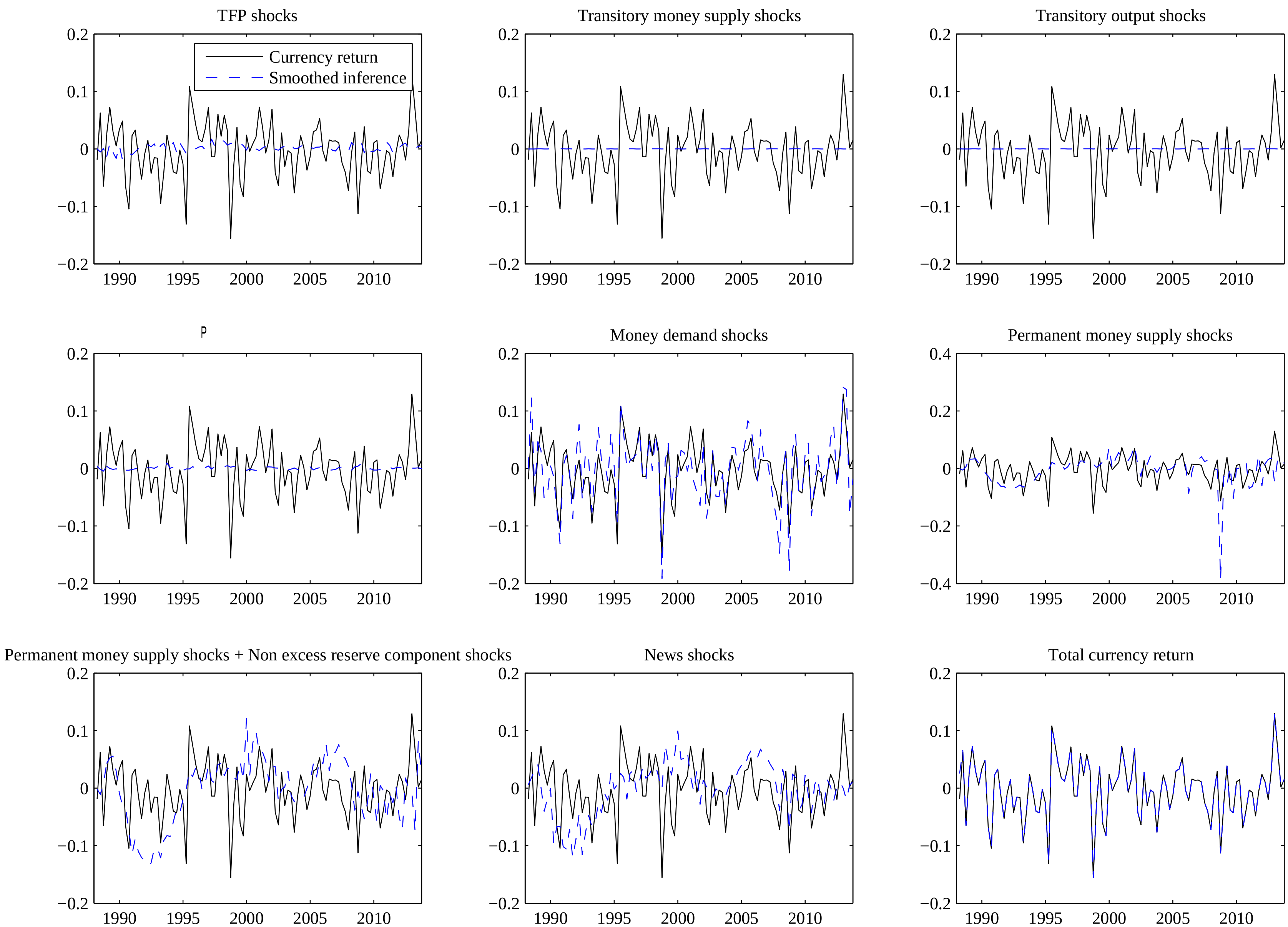

Figure 5: Historical Decomposition: Currency Return

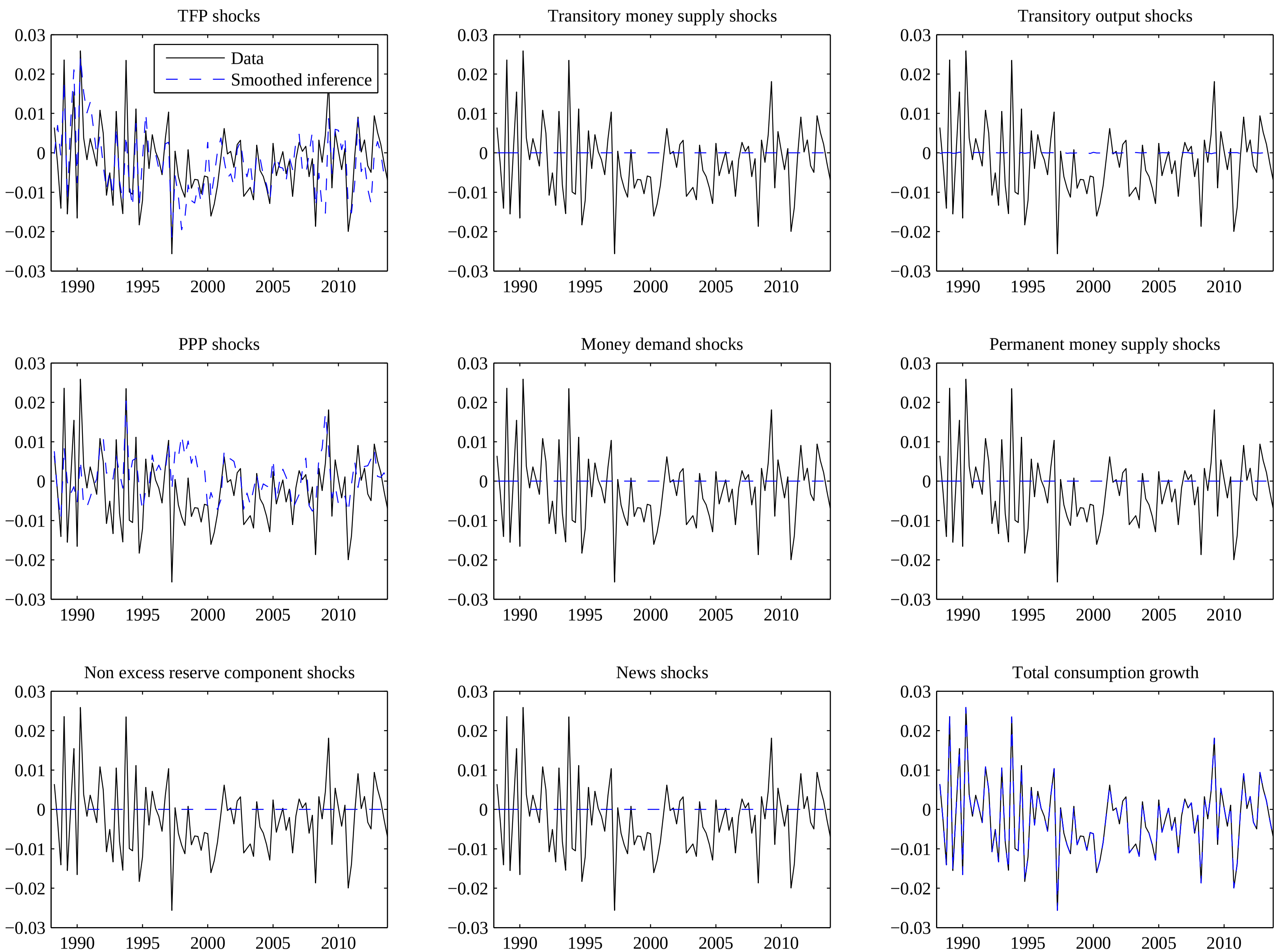

Figure 6: Historical Decomposition: Consumption Growth

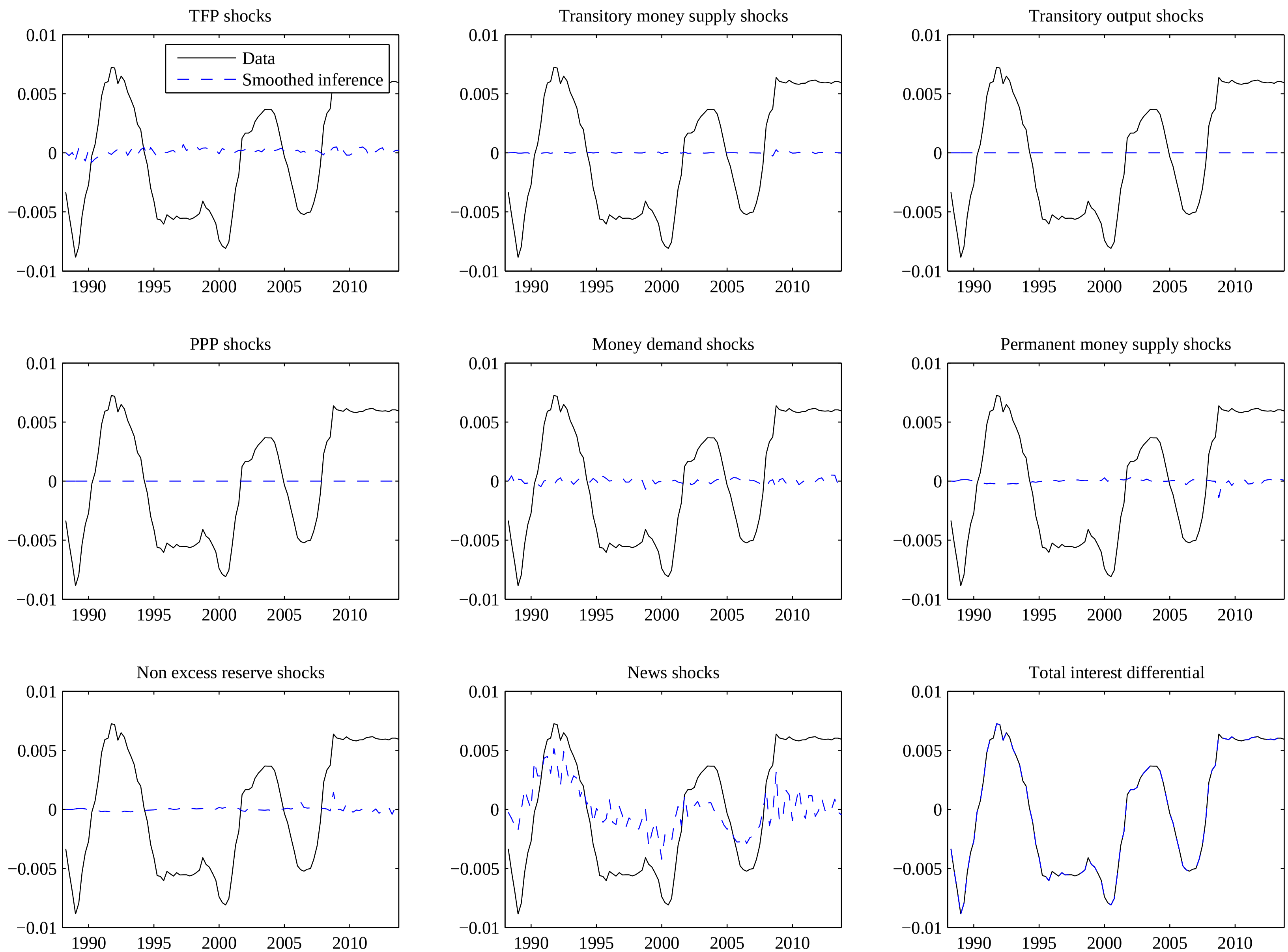

Figure 7: Historical Decomposition: Interest Rate Differential